DOCUMENT RESUME

ED 315 309	SE 051 204

TITLE	Senior High Mathematics 10/13/14: Interim Teacher Resource Manual.
INSTITUTION	Alberta Dept. of Education, Edmonton. Curriculum Branch.
PUB DATE	89
NOTE	331p.; Some pages have text against a gray background, which will not reproduce well.
PUB TYPE	Guides - Classroom Use - Guides (For Teachers) (052)
EDRS PRICE	MF01/PC14 Plus Postage.
DESCRIPTORS	Curriculum Guides; Foreign Countries; High Schools; Mathematics Curriculum; Mathematics Education; *Mathematics Instruction; Mathematics Materials; *Resource Materials; Secondary Education; *Secondary School Mathematics; Teaching Guides
IDENTIFIERS	*Alberta

ABSTRACT

This resource material is intended to assist teachers in translating the intentions of the Senior High Mathematics program into classroom practice. An overview of the "Course of Studies" for Mathematics 10, 13, and 14 is provided at the beginning of each course section in this manual. The overview includes the specific learner expectations, the attitudinal expectations, and the problem-solving expectations for each course. Activities for teaching concepts related to various topics are provided. Comments, process/problem solving and context, technology integration, and suggestions are provided for each concept. Basic and recommended textbooks are identified for each concept. Learning resource books are listed for each course. Appendices include template sheets, resources, suggestions for using writing assignments in teaching mathematics, guidelines for integrating media, and a list of consultants in the province. (YP)

****
*   Reproductions supplied by EDRS are the best that can be made   *
*   from the original document.   *
****

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

☒ This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

- Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

S. Wolodko

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

SENIOR HIGH

MATHEMATICS 10/13/14

INTERIM TEACHER RESOURCE MANUAL

Curriculum Support Branch
1989

Copyright © 1989, the Crown in Right of Alberta, as represented by the Minister of Education. Alberta Education, Curriculum Support Branch, 11160 Jasper Avenue, Edmonton, Alberta, T5K 0L2.

Permission is hereby given by the copyright owner for any person to reproduce this Teacher Resource Manual or any part thereof for educational purposes and on a non-profit basis.

4

ACKNOWLEDGMENTS

Alberta Education expresses its appreciation to those individuals and groups who assisted in the development and production of this manual and acknowledges the contribution of the following:

Writers

Florence Glanfield	Curriculum Support Branch, Alberta Education
Jim Neilsen	Curriculum Design Branch, Alberta Education

Research Assistant

Doug Franks	Curriculum Support Branch, Alberta Education

Mathematics 10 Field Test Teachers

Alex Brazeau	William Aberhart HS, Calgary SD #19
Les Cameron	St. Patrick's HS, Yellowknife
Archie Coderre	Foremost School, County of Forty Mile #8
Helen Dubé	École Notre Dame Senior High, Lakeland RCSSD #150
Don Ewing	Drumheller CHS, Drumheller Valley SD #62
Richard Fownes	Spirit River Secondary School, Spirit River SD #51
Fern Heinan	Lethbridge Collegiate Institute, Lethbridge SD #47
Chris Korte	Blessed Sacrament School, Wainwright RCSSD #31
Gerald Krabbe	Crescent Heights HS, Calgary SD #19
Mark Milner	Western Canada HS, Calgary SD #19
Ronald Patrick	Sexsmith Secondary School, County of Grande Prairie #1
Vick Pysyk	J. Percy Page HS, Edmonton RCSSD#7
Ron Rajotte	Louis St. Laurent HS, Edmonton RCSSD #7
Louise Sabourin	William Aberhart HS, Calgary SD #19
Denise Schweitzer	Central High Sedgewick Public School, County of Flagstaff #29
Yvonne Tschofen	École J.H. Picard, Edmonton RCSSD #7
Brian Walsh	École Notre Dame Senior High, Lakeland RCSSD #150

Mathematics 13 Field Test Teachers

Blanche Andrew	Three Hills School, Three Hills SD #60
Jim Braaksma	Crescent Heights HS, Calgary SD #19
Marj Farris	LaCrete Public School, Ft. Vermilion SD #52
Vern Hemeyer	Ponoka CHS, County of Ponoka #3
Bruce Kabaroff	Queen Elizabeth CHS, Edmonton SD #7
Drew Kurrie	Bev Facey CHS, County of Strathcona #20
Mary Jo Maas	F.P. Walshe School, Willow Creek SD #28
Kim Mathews	Sir John Franklin Territorial HS, Yellowknife
Ed Perepelitza	Lindsay Thurber CHS, Red Deer SD #104
Clint Sopchyshyn	St. Joseph CHS, Edmonton RCSSD #7
Ted Venne	Father Patrick Mercredi Community School, Ft. McMurray RCSSD #32

Mathematics 14 Field Test Teachers

Hilger Fast	Sir Winston Churchill HS, Calgary SD #19
Fay Harke	Leduc CHS, Leduc SD #297
Vada Lindstrand	Camrose CHS, Camrose SD #1315
Mildred Sake	Delnorte School, County of Minburn #27
Rob Tullikopf	Trochu Valley School, Three Hills SD #60

Mathematics 10/20/30 Ad Hoc Committee

Linda Brost, Department Head	Sir Winston Churchill HS, Calgary SD #19
Ken Brown, Teacher	Lacombe CHS, County of Lacombe #14
Jim Ellefson, Teacher	County of Thorhild Central HS, County of Thorhild #7
Mary Lou Forest, Consultant	Edmonton RCSSD #7
Florence Glanfield, Coordinator	Curriculum Support Branch, Alberta Education
John Hider, Department Head	Medicine Hat HS, Medicine Hat SD #76
Jim Neilsen, Program Manager	Curriculum Design Branch, Alberta Education
Steve Willard, Professor	Department of Mathematics, University of Alberta

Mathematics 13/23/33 Ad Hoc Committee

Russell Donnelly, Consultant	Calgary RCSSD #1
Florence Glanfield, Coordinator	Curriculum Support Branch, Alberta Education
Gary Glubish, Teacher	Grand Centre HS, Lakeland SD #5460
Joan Mick, Teacher	South Central HS, Acadia SD #8
Jay Nayyer, Teacher	Paul Rowe Jr-Sr HS, Peace River SD #10
Jim Neilsen, Program Manager	Curriculum Design Branch, Alberta Education
Mike Stone, Professor	Department of Mathematics, University of Calgary
Blair Wagner, Principal	St. Mary's RCS School, Taber RCSSD #54
Marilyn Woytiuk, Teacher	Paul Kane HS, St. Albert PSSD #6

Mathematics 14/24 Ad Hoc Committee

Ken Chipman, Principal	Senator Gershaw School, County of Forty Mile #8
P.J. Clooney, Teacher	M.E. Lazerte CSH, Edmonton SD #7
Richard Gilchrist, Assistant Principal	Memorial CHS, County of Parkland #31
Florence Glanfield, Coordinator	Curriculum Support Branch, Alberta Education
Rick LaGrange, Teacher	Grande Prairie CHS, Grande Prairie SD #2357
Barney McKay, Department Head	Shaughnessy HS, Calgary SD #19
Jim Neilsen, Program Manager	Curriculum Design Branch, Alberta Education
Doug Parsons, Department Head	Van Horne HS, Calgary SD #19

Senior High Mathematics Advisory Committee

Bob Dearborn, Test Development Specialist	Student Evaluation Branch, Alberta Education
Florence Glanfield, Coordinator	Curriculum Support Branch, Alberta Education
Dr. Bruce Harrison, Professor	Faculty of Education, University of Calgary
Lois Marchand, Mathematics Specialist	Calgary SD #19
Elaine Manzer, Teacher	Peace River HS, Peace River SD #10
Ken May, Teacher	Winston Churchill HS, Lethbridge SD #51
Dr. Jim Muldowney, Professor	Department of Mathematics, University of Alberta
Jim Neilsen, Program Manager	Curriculum Design Branch, Alberta Education
Randy Pack, Mathematics Program Head	Northern Alberta Institute of Technology
Klaus Puhlmann, Superintendent	Yellowhead SD #12
Hugh Sanders, Associate Director	Grande Prairie Regional Office, Alberta Education
Ken Schade, Department Head	Alberta Correspondence School, Alberta Education
Ted Thresher, Assistant Principal	Strathcona CHS, Edmonton SD #7
John Weatherhead, Teacher	Crescent Heights HS, Medicine Hat SD #76

Alberta Education also recognizes with appreciation the following former committee members:

Senior High Mathematics Advisory Committee

Don Cameron, Principal	St. Thomas Acquinas School, Provost RCSSD #65 (1986–1987)
Terry Cooke, Associate Dean (Resources)	Northern Alberta Instititute of Technology (1986–1988)
George Ditto, Mathematics Supervisor	Calgary SD #19 (1986–1988)

Mathematics 10/20/30 Ad Hoc Committee

Patricia Johnson, Teacher	John Maland HS, Devon SD #4972 (1987)
Stuart McMillan, Teacher	Jasper Jr/Sr HS, Jasper SD #3063 (1987–1988)

Mathematics 14/24 Ad Hoc Committee

Duncan McDougall, Teacher	Father Patrick Mercredi Community School, Ft. McMurrary RCSSD #32 (1987–1988)

Administration – Curriculum Support Branch, Alberta Education

The Mathematics 10/13/14 Interim Teacher Resource Manual was developed under the direction of the following:

Keith Wagner	Director
Merv Thornton	Associate Director, Resource Development and Implementation (Core) Unit
Florence Glanfield	Coordinator, Mathematics
Janice Leonard	Associate Director, Resource Development and Implementation (Complementary) Unit

Editor

Virginia Durksen

Copy Editor

Barb McCord

Art Work

Jaclyn Peebles

Desktop Publishing

Eileen Boyd
Joyce Depano-Yap
Teresa Hansen
Tania Pander
Ruby Watmough

TABLE OF CONTENTS

Acknowledgments iii
Purpose 1
Program Rationale and Philosophy 1
General Learner Expectations 2
Specific Learner Expectations 3
Course Sequences 7
Writing in Mathematics 9
Learning Resources 10
How to Use the Teacher Resource Manual 14

MATHEMATICS 10

Program Emphasis 17
Overview of the Course of Studies 18
Attitudes 25
Problem Solving 25
Number Systems 27
Operations on Polynomials 40
Factoring Polynomials 57
Equations and Inequalities 70
Coordinate Geometry and Graphing 76
Systems of Linear Equations 94
Statistics 98
Trigonometry 108

Reverse Textual Correlation

    Mathematics 10 116
    Holtmath 10 117
    Mathematics Principles and Process 10 118
    Mathématiques 10 119

MATHEMATICS 13

Program Emphasis 123
Overview of the Course of Studies 124
Attitudes 131
Problem Solving 131
Number Skills 133
Polynomials and Factoring 146
Coordinate Geometry and Graphing 168
Relations 183
Statistics 190
Geometry 202

8

Reverse Textual Correlation

    Mathematics for a Modern World (Book 2) 209
    Math Matters (Book 2) 210
    Mathématiques pour un monde moderne (livre 2) 211

MATHEMATICS 14

Program Emphasis 215
Overview of the Course of Studies 216
Attitudes 222
Problem Solving 222
Numeration 223
Ratio and Proportion 226
Statistics and Probability 235
Geometry 242
Measurement 260
Algebra and Graphing 276
Reverse Textual Correlation
    Applications of Mathematics 287

APPENDICES

Appendix A: 289
    Template: Binomial Grid 290
    Template: Algebra Tiles 291
Appendix B: 293
    Recommended Resources: 295
        (A) Print 295
        (B) Video 297
        (C) Computer Software 298
    Supplementary Resources: 299
        (A) Computer Software 299
    Other Resources 301
        (A) Print 301
        (B) Journals 306
Appendix C: 311
    Using Writing to Learn Mathematics 313
    Explorative Writing and Learning Mathematics 319
Appendix D: 327
    Guidelines for Effective Media Integration 328
    Resource/Media Centres
        (A) Regional 328
        (B) Urban 329
Appendix E: 231
    Alberta Education Mathematics Consultants 233

PURPOSE

This Teacher Resource Manual (TRM) is designed to help teachers implement the Mathematics 10, 13 and 14 courses. The TRM is a support document that provides helpful information to classroom teachers. The teaching strategies presented are suggestions only and are not legally binding. The Course of Studies for Mathematics 10, 13 and 14 outlines the courses which teachers are legally bound to teach.

An overview of the Course of Studies for each of Mathematics 10/13 and 14 is provided at the beginning of each course section in the TRM. The overview includes the specific learner expectations, the attitudinal expectations and the problem–solving expectations for each course. In addition, the Mathematics 14 overview includes the specific numeration expectations. As well, concepts in each course are clearly set out in boxes throughout the TRM. All of these components are part of the Course of Studies. The remaining portions of the Course of Studies: Program Rationale and Philosophy, General Learner Expectations, Specific Learner Expectations are stated at the beginning of the document and shaded.

The Mathematics 10/13/14 TRM is intended to assist teachers in translating the intentions of the Senior High Mathematics program into classroom practice.

PROGRAM RATIONALE AND PHILOSOPHY

In order for students to be able to set goals and make informed choices, they need an array of thinking and problem-solving skills. Fundamental to this is an understanding of mathematical techniques and processes that will enable them to apply the basic skills necessary to address everyday mathematical situations, as well as acquire higher order skills in logical analysis and methods for making valid inferences.

A knowledge of mathematics is essential for a well-educated citizenry. However, the need for and use of mathematics in the life of the average citizen is changing. Emphasis has shifted from the memorization of mathematical formulae and algorithms toward a more dynamic view of mathematics as a precise language, used to reason, interpret and explore. There continues to be a need for the logical development of concepts and skills as a basis for the appropriate use of mathematical information to solve problems. Moreover, the use of available technology along with techniques such as estimation and simulation, incorporated with more traditional problem-solving techniques, are the tools by which mathematical problems are solved.

Change in the way in which mathematics is used is necessitating a concurrent change in the emphases of mathematics education. Students need an expanded list of fundamental concepts but will also need to understand the ideas that make up those concepts and how they are related. They also require a familiarity with their applications. Most important, students have to be able to solve problems using the mathematical processes developed, and be confident in their ability to apply known mathematical skills and concepts in the acquisition of new mathematical knowledge. In addition, the ability of technology to provide quick and accurate computation and manipulation, to enhance conceptual understanding and to facilitate higher order thinking, should be recognized and used by students.

The majority of students who enter high school exhibit mainly concrete operational behaviours with regard to mathematics. It is recognized that high school mathematics courses include many abstract understandings which students are expected to acquire. The course content of the high school mathematics program is cognitively appropriate for the students and should be presented in a way which is consistent with the students' ability to understand.

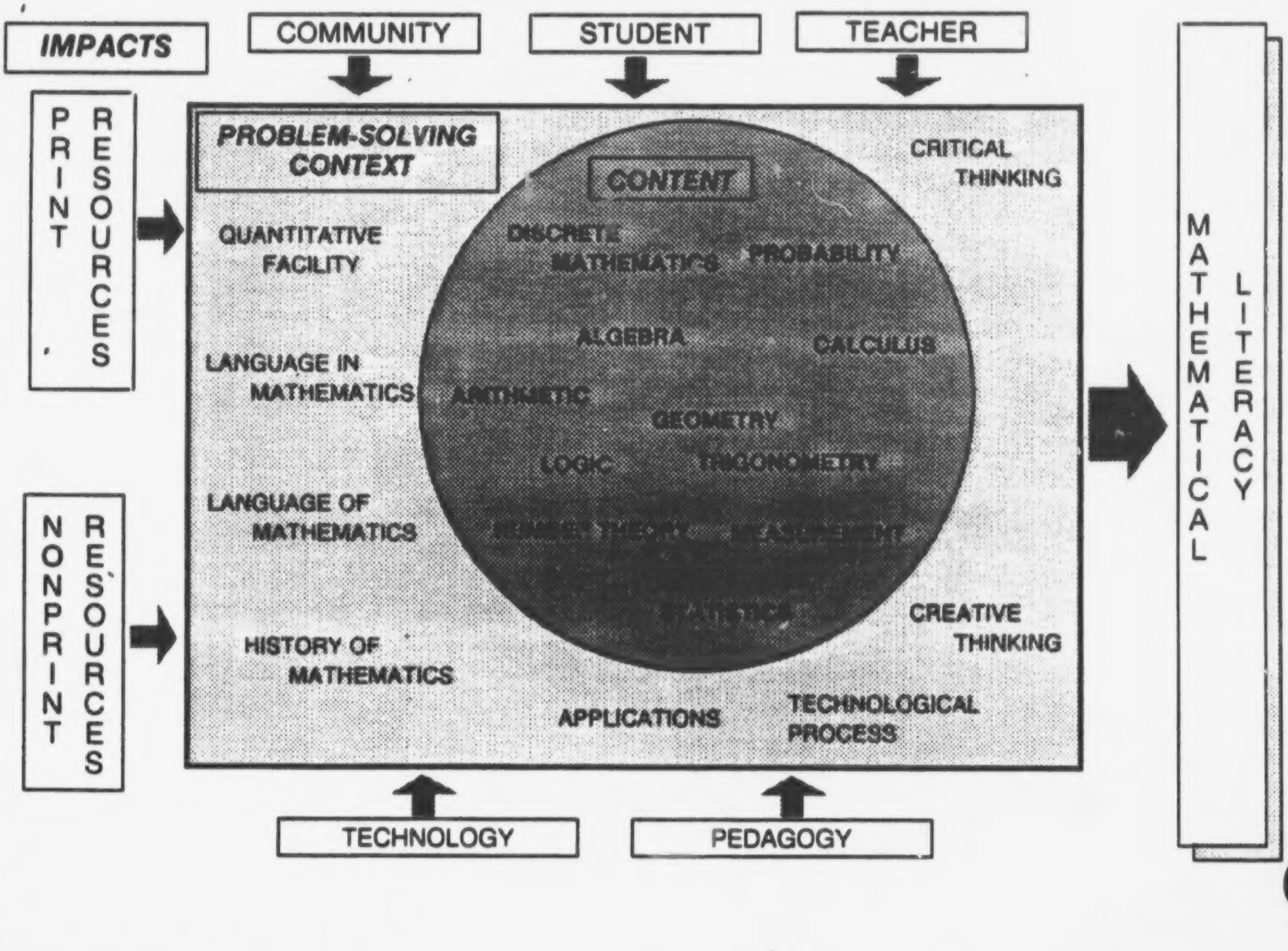
The Senior High School Mathematics Program, by providing course sequences commensurate with differing abilities, interests and aspirations, is designed to enable students to have success in mathematics. As well, it reflects the changing needs of society, and provides students with the mathematical concepts, skills and attitudes necessary to cope with the challenges of the future.

GENERAL LEARNER EXPECTATIONS

Upon conclusion of their senior high mathematics education, students will be expected to be mathematically literate. More specifically, *students will be expected to*

- have achieved understanding of the basic mathematical concepts, and developed the skills and attitudes needed to become responsible and contributing members of society
- apply basic mathematical skills and concepts in practical situations
- have developed the skills, concepts and attitudes that will ensure success in the mathematical situations that occur in future educational endeavours, employment and everyday life
- have developed the skills, concepts and attitudes that will enable the acquisition of mathematical knowledge beyond the conclusion of secondary education
- have developed critical and creative thinking skills
- be able to communicate mathematical ideas effectively
- understand how mathematics can be used to investigate, interpret and make decisions in human affairs
- understand how mathematics can be used in the analysis of natural phenomena
- understand and appreciate the positive contributions of mathematics, as a science and as an art, to civilization and culture.

A Model for Mathematical Literacy


IMPACTS
COMMUNITY
STUDENT
TEACHER
PRINT RESOURCES
NONPRINT RESOURCES
PROBLEM-SOLVING CONTEXT
CONTENT
CRITICAL THINKING
QUANTITATIVE FACILITY
DISCRETE MATHEMATICS
PROBABILITY
ALGEBRA
CALCULUS
LANGUAGE IN MATHEMATICS
ARITHMETIC
GEOMETRY
LOGIC
TRIGONOMETRY
LANGUAGE OF MATHEMATICS
NUMBER THEORY
MEASUREMENT
STATISTICS
CREATIVE THINKING
HISTORY OF MATHEMATICS
APPLICATIONS
TECHNOLOGICAL PROCESS
TECHNOLOGY
PEDAGOGY
MATHEMATICAL LITERACY

A Model for Mathematical Literacy outlines the factors that effect what and how students learn as they become mathematically literate. The model is fluid in that the content can be learned within a problem-solving context which engages any of a number of vehicles as the learning focus. At the same time, the students involved in the learning situation are affected in what and how they learn by forces which have impact upon them.

The Content of the high school mathematics program is that body of knowledge that is to be acquired by students. In the various high school mathematics courses, it is made up of topics that can be categorized into one or more of the listed strands.

Within each course in the high school mathematics program, students will focus on problem solving. **The Problem-Solving Context** refers to the instructional emphases within which the specific content expectations can be acquired. The various entries indicated within the model suggest processes that belong to the problem-solving context and may be used by students as vehicles for learning the content.

The Impacts on the Problem-Solving Context are those skills, attitudes and experiences that are possessed by the students and teachers involved, as well as the resources they may use throughout the learning process. They include the influence exerted by the culture and beliefs of the community as reflected by the school. The effectiveness of the context in enabling a student to acquire the content is dependent upon the skilful management by the teacher of those items that impact upon a student's learning.

Mathematical Literacy refers to the ability to use mathematical skills, concepts and processes to address successfully the mathematical situations which one encounters.

SPECIFIC LEARNER EXPECTATIONS

The specific learner expectations are what a student is expected to accomplish in a particular course. For mathematics these include attitudes, concepts and skills, as well as expectations about problem solving and the use of technology.

Attitudes are general dispositions toward mathematics education and social issues. Attitude expectations are listed at the beginning of each course and stated as "Students will be encouraged to develop ...".

A concept is an abstract or general idea about specific instances that have common properties or an identifiable relationship to one another. Each unit is broken into a number of concepts which are the major ideas to be addressed. Concepts may be presented as mathematical definitions or theorems, or as statements of mathematical ideas or abstractions. In the course of study, concepts are numbered with a single digit and stated as "Students will be expected to demonstrate an understanding that...".

Supporting each concept are a number of skills. Skills are intellectual or physical capabilities that will be developed within the context of the particular concept. In the program of studies, skills are numbered with two digits separated by a period, (n.m) and stated as "Students will be expected to...".

In addition, related to particular concepts and skills, are problem-solving expectations. These expectations are of two general types. Some identify specific types of problems which must be solved while others identify problem-solving behaviours or techniques to be learned by students. Problem-solving expectations occur throughout the specific learner expectations. They suggest that students should have the background skills and knowledge necessary to achieve the expectations successfully, using problem-solving techniques. In the program of studies, problem-solving expectations are numbered with three digits separated by periods (n.m.l).

Technology expectations identify specific areas in which calculator and/or computer technology will be applied by students as tools to be used for calculation, manipulation or graphing, or to aid in the analysis of problems. In the program of studies, technology expectations are numbered with three digits separated by periods (n.m.l).

Throughout the learner expectations, the words verify and prove appear. For the purposes of the Senior High Mathematics Program, they are interpreted as:

- Verify: to substantiate the validity of an operation, solution, formula or theorem through the use of examples which may or may not be generalized;
- Prove: to substantiate the validity of an operation, solution, formula or theorem in general and to provide logical arguments for each step in the process.

The major part of the content of each high school mathematics course consists of topics required of all students who take the course. The required content comprises 80% of the course and contains the concepts, skills and attitudes that all students are expected to acquire. As well, the required portion of all courses includes specific expectations in regard to problem solving and the use of technology.

Each course will include a compulsory component comprising 20% of the course, made up of elective material which will be consistent with the content and expectations of the required component. The purpose of the elective is to provide for enrichment, remediation, or innovative or experimental presentations or activities. It is not intended to provide acceleration or advanced placement. However, horizontal enrichment and extension is appropriate and students should have access to elective material that serves their individual needs and interests.

PROBLEM SOLVING

Through the use of problems and applications, students will be able to see the need for certain concepts and mathematical procedures, to understand concepts and acquire facility with procedures, and to apply and review concepts and procedures already learned. The emphasis on problem solving in high school mathematics has shifted from finding answers to routine "word problems", to the application of mathematical concepts, skills and processes to develop and discover mathematics within a wide variety of problem situations.

Problem solving has been integrated into all of the courses in the Senior High Mathematics Program and is explicitly mentioned throughout the learner expectations. In addition, specific expectations for problem solving are included at the beginning of each course. It is not intended that problem solving be presented as units of study; however, it is expected that upon completion of the course, students will have acquired all of the listed skills and concepts.

Problem solving is the focus of the concept development as well as the exercise portion, of a lesson. New material should be presented as a problem to be solved using previously acquired skills, concepts and processes. Teachers must be open to accepting non-rigorous strategies such as modeling, working backward and simulations and should emphasize the processes used and the type of thinking in which the student is engaged at least as much as the answer to the problem. Teachers must remember that what may be a problem to some students may be only a routine exercise to others, and should be prepared to adapt their presentations and expectations to reflect this. Above all, teachers must model problem-solving behaviour; students will think problem solving is important if they are encouraged to solve problems by their teachers and if they see their teachers solving problems.

At all levels of problem solving, students should be taught how and provided with the opportunities to ask questions and to analyse the thinking processes with which they tackle a problem. An exercise that provides students with this opportunity is outlined below.

"Archie's Envelopes"
Students work in three situations:

1. A group of four students is involved in solving a problem. Each student is given only one piece of information from the problem, in an envelope. Each student must explain their piece of information to the group. The group must
    a) identify "the problem";
    b) plan a solution;
    c) try out the plan; and,
    d) analyse the "plan" and method of solution.
2. Same as situation 1, with two students.
3. Students work individually on solving a problem.

(Thanks to A. Coderre, Math 10 Field Test Teacher, for this idea.)

As students become more adept at problem solving, they will begin to use more complex problem-solving processes. Such higher level processes will include making and testing conjectures, determining valid inferences, using inductive and deductive processes and developing and using algorithms to apply to a particular situation.

Through the course of studies, verbs such as "verify" and "prove" are used. These expectations suggest explicitly to the teacher that a student should have the background skills and knowledge necessary to achieve the expectation successfully using problem-solving techniques. Teachers are encouraged to adhere to the spirit of these expectations by treating them as problems to be solved. Students are more likely to internalize the skills or concepts if they are encouraged to develop them themselves.

TECHNOLOGY

Students will be expected to learn how and when to use a calculator and have a demonstrated proficiency in estimation and mental arithmetic.

To use calculators effectively, they must be able to judge the reasonableness of an answer and understand the importance of making a judgment about the result of a calculation.

Throughout the learner expectations, specific applications of calculators and computers are identified. These define explicitly the areas in which students are expected to use technology in their study of mathematics. Many of these expectations indicate that a particular technology be used in an investigation or for analysis. It is in these situations that the use of technology enables students to engage in critical and creative thinking and problem solving.

The integration of calculators into all courses in the high school mathematics program, in support of instruction, in class work, homework and evaluation, is essential. By using calculators appropriately, time previously spent on computation can be used for problem solving.

The use of computers by teachers and students in mathematics classes should be expanded. Teaching using computers allows concepts to be presented visually through graphics that can be manipulated by the teacher according to input supplied by the students. Students should be taught to use prepared software and to use simple programs written for particular purposes in order to use the power of the computer to create geometric displays, organize, analyse and present data, simulate real-life situations and create and run algorithms, as well as to do symbolic manipulations and calculations.

Electronic technology will continue to play an important role in the teaching of mathematics in both what is taught and how it is taught. Teachers should encourage students to use the available technologies as much and as well as possible and should adapt their teaching techniques and evaluation emphases to reflect this.

STUDENT EVALUATION

Evaluation of students in the Senior High Mathematics Program will involve assessment of the level of achievement of all of the learner expectations, including concepts, skills and attitudes, as well as problem solving and technology expectations. For more information regarding evaluation consult the Teacher Resource Manual for Senior High Mathematics.

Formative evaluation in the high school mathematics program involves processes which assess the levels of understanding that students have acquired within their mathematics study. As such, it is more than testing. Its basic purpose is to determine the nature of students' thinking about mathematics and from that assessment to adapt instruction to enable students to improve achievement. Further, it involves the assessment by the teacher, of changes in the dispositions of students toward the study of mathematics.

The primary goal of summative assessment is to determine the level of understanding that students have of mathematical processes and the interrelationship of mathematical topics and from that assessment to make judgments as to a student's achievement and likelihood of success in further mathematics curricula, the level of competence with concepts and procedures and the assessment of a student's ability to solve problems, to apply mathematics and to reason and analyse logically.

Because of the emphasis on problem solving and the acquisition of process skills, the use of techniques in addition to tests and quizzes for evaluation is recommended Other methods may include written reports of projects completed, formal and informal interviews, observations of individual and group work and evaluation of oral reports and presentations given as part of class activities. Regardless of the evaluation methods used, students must be familiar with the course expectations and the indicators of success. The criteria for evaluation must be based on the stated objectives of the course and must be reasonable and appropriate for the students involved.

For examples of evaluation strategies, consult:

1) Curriculum and Evaluation Standards for School Mathematics
2) How to Evaluate Progress in Problem Solving.

Both resources are available through the LRDC and a description of the resources can be found in Appendix B of this TRM.

COURSE SEQUENCES

To address the diverse needs, abilities and interests of students, the senior high mathematics program is composed of three sequences. Each of the sequences has the development of mathematical literacy as a primary goal and presents curricular content within a problem-solving context.

Mathematics 14/24

This sequence of courses is designed for that group of students whose needs, interests and abilities focus on basic mathematical understanding. Topics will be dealt with using a thematic approach, and applications will be a priority. Some topics from junior high mathematics will be covered in Mathematics 14 to improve the skill level of students who may wish to transfer to the Mathematics 13/23/33 sequence. Upon successful completion of Mathematics 24, students will have fulfilled the mathematics components for qualification for the General High School Diploma.

Mathematics 13/23/33

This sequence covers material from the areas of algebra, geometry, trigonometry, statistics and consumer mathematics. It is designed for students who require a non-rigourous academic mathematics program to prepare them for community and technical colleges, trades and employment. Upon successful completion of Mathematics 23, students will have fulfilled the mathematics component for qualification for the General High School Diploma.

Mathematics 10/20/30

This sequence of courses is designed for students intending to pursue postsecondary studies at a university or in a mathematics-intensive program at a technical school or college. Mathematics 10/20/30 emphasizes the theoretical development of topics from the areas of algebra, geometry, trigonometry and statistics up to a level acceptable for entry into universities and other postsecondary institutions. Upon successful completion of Mathematics 30, students will have fulfilled the mathematics component for qualification for the Advanced High School Diploma.

Mathematics 31

This course is designed for highly motivated, academic students who wish to pursue a first course in calculus and vectors. Mathematics 30 must be taken as a pre- or a co-requisite course.

Transfer

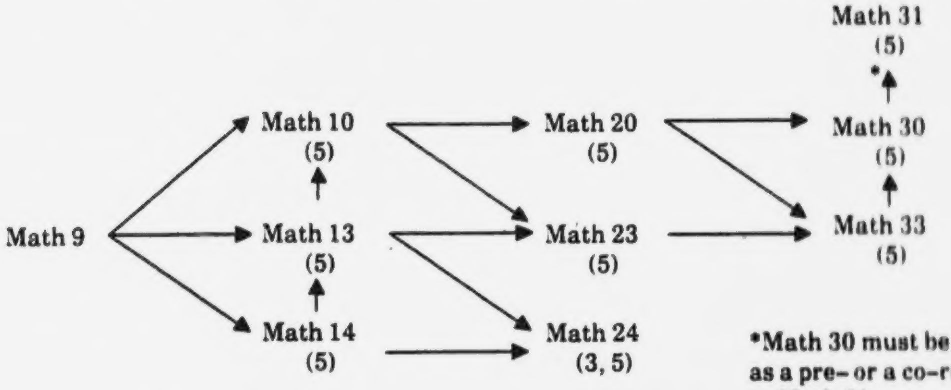
According to their interests, needs and abilities, students may choose to begin their high school mathematics education by enrolling in any of the three Grade 10 mathematics courses. Some students may wish to complete a more basic Grade 10 mathematics course in order to increase the likelihood of success in a more academic course. For this reason, students may enroll in Mathematics 14 and upon successful completion of that course transfer to the Mathematics 13/23/33 sequence by enrolling in Mathematics 13. Others may wish to complete Mathematics 13 successfully before transferring to the Mathematics 10/20/30 sequence by enrolling in Mathematics 10.

Students, however, will not be granted credits in a course when they have already been granted credits in a same-grade level course from a lower numbered sequence. For example, a student with credits in Mathematics 13 could not subsequently be granted credits in Mathematics 14.

Upon successful completion of Mathematics 33, students may enroll in Mathematics 30 in order to qualify for the Advanced High School Diploma. Students seeking to transfer should be aware of the increase in academic rigour and should have done sufficiently well in Mathematics 33 to make such a transfer feasible. For this reason, teachers may wish to suggest a level of achievement higher than a minimal pass in Mathematics 33 as an indicator of probable success in Mathematics 30.

Some students may find that they are in the wrong sequence for their needs or ability. Upon successful completion of any course, a student may enroll in the next higher course in a higher-numbered sequence. For example, a student who successfully completes Mathematics 10 may enroll in Mathematics 23. For other transfer options and the awarding of retroactive credits, please refer to the Guide to Education, Senior High School Handbook (from the Curriculum Design Branch, Alberta Education).

SENIOR HIGH MATHEMATICS - COURSE SEQUENCE



*Math 30 must be taken as a pre- or a co-requisite to Math 31

(CREDIT VALUES INDICATED IN BRACKETS)

WRITING IN MATHEMATICS

Communication, both written and oral, plays a vital role in the implementation of this mathematics program. Students should not only be involved in group problem–solving activities where they must share ideas with one another but should also have the opportunity to express their ideas in writing.

Writing in mathematics requires students to reflect on their reasoning, enhancing cognitive development. Writing also helps the teacher: more than just filling in answers, it reveals the student's level of understanding. Since you will have a good idea of where your students "are" (in their understanding), you will have the background information necessary to plan lessons that promote mathematical thinking and understanding.

This type of writing – writing to describe thinking and reasoning – will be new to students. Hence, students will need to spend time in going "beyond the right answer." This emphasis on writing will help our students obtain the goal of mathematical literacy as it promotes thinking and understanding about how a problem is solved.

For more information and examples of writing in high school mathematics, see Appendix C.

LEARNING RESOURCES

Introduction

Learning resources fall into four categories: Basic, Recommended, Supplementary and Other learning resources. These categories include print, nonprint and electronic software materials used by teachers or students to facilitate teaching and learning.

BASIC learning resources are those approved by Alberta Education as the most appropriate for meeting the majority of goals and objectives of the course, or substantial components of the course, as outlined in the program of studies. Basic resources are materials that all students should have.

RECOMMENDED learning resources are those approved by Alberta Education because they make an important contribution to the attainment of one or more of the major goals of the course as outlined in the program of studies. RECOMMENDED resources are further divided into: a) teacher recommended, and b) student recommended, where multiple copies (but not necessarily class sets) are desired.

SUPPLEMENTARY learning resources are those approved by Alberta Education because they support the course as outlined in the program of studies, by enriching or reinforcing the learning experience.

OTHER learning resources are those resources provided through the courtesy of other agencies or institutions. None of the titles have been evaluated by Alberta Education and their listing is not to be construed as an explicit or implicit departmental approval for use. These titles are provided as a service only to assist local jurisdictions to identify potentially useful learning resources. The responsibility for evaluating these resources prior to selection rests with the local jurisdiction.

The following learning resources support the Mathematics 10/13/14 courses. The abbreviations for the resources, used throughout the TRM, are noted in brackets. A description of the Recommended and Supplementary resources can be found in Appendix B.

MATHEMATICS 10

Basic

Holtmath 10 (HM10), Bye, M. et al., Toronto, Ontario: Holt, Rinehart and Winston, 1987.

Mathematics 10: Principles and Process (MPP10), Ebos, F. et al., Scarborough, Ontario: Nelson Canada, 1987.

Mathematics 10 (M10), Kelly, B. et al., Don Mills, Ontario: Addison-Wesley Publishers Ltd., 1987.

Ressource de Base

Mathématiques 10, Ebos, F. et al., Montréal, Québec: Éditions Beauchemin ltée, 1988.

Recommended

Activities for Implementing Curricular Themes from the Agenda for Action, Hirsch, C.R. (ed), Reston, Virginia: The National Council of Teachers of Mathematics, Inc., 1986.

Algebra Tiles for the Overhead Projector, Howden, H., New Rochelle, New York: Cuisenaire Company of America, 1985.

19

Curriculum and Evaluation Standards for School Mathematics, National Council of Teachers of Mathematics, Reston, Virginia: National Council of Teachers of Mathematics, 1989.

Exploring Data - Student Edition (ED), Landwehr, J.M. et al., Palo Alto, California: Dale Seymour Publications, 1986.

Exploring Data - Teacher's Edition, Landwehr, J.M. et al., Palo Alto, California: Dale Seymour Publications, 1987.

Holtmath 10 Teacher's Resource Manual (HM10 TRM), Webster, W. et al., Toronto, Ontario: Holt, Rinehart and Winston, 1988.

How to Evaluate Progress in Problem Solving, Charles, R. et al., Reston, Virginia: The National Council of Teachers of Mathematics, 1987.

Master Grapher, Demana, F. et al., Don Mills, Ontario: Addison-Wesley Publishers ltd., 1988.

Mathematics Dictionary Fourth Edition, James, G. et al., New York, New York: Van Nostrand Reinhold Company, 1976.

Mathematics 10: Principles and Process Alberta Supplementary Teacher's Resource Masters (MPP10 ASTRM), Ebos, F. et al., Scarborough, Ontario: Nelson Canada, 1989.

Mathematics 10: Principles and Process Teacher's Resource Masters (MPP10 TRM), Denison, P. et al., Scarborough, Ontario: Nelson Canada, 1988.

Mathematics 10 Teacher's Resource Book (M10 TRB), Kelly, B. et al., Don Mills, Ontario: Addison-Wesley Publishers Ltd., 1989.

The Language of Graphs, Swan, M., Nottingham, England: The Shell Centre for Mathematical Education, 1987.

Supplementary: see Appendix B.

MATHEMATICS 13

Basic

Mathematics for a Modern World Book Two Third Edition Revised (MMW2), Baxter, W.H. et al., Toronto, Ontario: Gage Educational Publishing Company, 1989.

Math Matters Book 2 Alberta Edition (MM2), Ebos, F. et al., Scarborough, Ontario: Nelson Canada, 1989.

Ressource de base

Mathématiques pour un monde moderne, livre 2 (MPMM2), Baxter, W.H. et al., Saint-Laurent, Québec: Éditions du Trecarre, 1988.

Recommended

Activities for Implementing Curricular Themes from the Agenda for Action, Hirsch, C.R. (ed), Reston, Virginia: The National Council of Teachers of Mathematics, Inc., 1986.

Activities for Junior High School and Middle School Mathematics, Easterday, K.E. et al. (eds), Reston, Virginia: The National Council of Teachers of Mathematics, Inc., 1981.

Algebra Tiles for the Overhead Projector, Howden, H., New Rochelle, New York: Cuisenaire Company of America, 1985.

Curriculum and Evalaution Standards for School Mathematics, National Council of Teachers of Mathematics, Reston, Virginia: National Council of Teachers of Mathematics, 1989.

Exploring Data – Student Edition (ED), Landwehr, J.M. et al., Palo Alto, California: Dale Seymour Publications, 1986.

Exploring Data – Teacher's Edition, Landwehr, J.M. et al., Palo Alto, California: Dale Seymour Publications, 1987.

How to Evaluate Progress in Problem Solving, Charles R., et al., Reston, Virginia: The National Council of Teachers of Mathematics, 1987.

Master Grapher, Demana, F. et al., Don Mills, Ontario: Addison-Wesley Publishers Ltd., 1988.

Mathematics Dictionary Fourth Edition, James, G. et al., New York, New York: Van Nostrand Reinhold Company, 1976.

The Language of Graphs, Swan, M., Nottingham, England: The Shell Centre for Mathematical Education, 1987.

Supplementary: see Appendix B.

MATHEMATICS 14

Basic

Applications of Mathematics (AM), Price, J. et al., Toronto, Ontario: Merrill Publishing Company, 1989.

Recommended

Activities for Implementing Curricular Themes from the Agenda for Action, Hirsch, C.R. (ed), Reston, Virgina: The National Council of Teachers of Mathematics, Inc., 1986.

Activities for Junior High School and Middle School Mathematics, Easterday, K.E. et al. (eds), Reston, Virginia: The National Council of Teachers of Mathematics, Inc., 1981.

Algebra Tiles for the Overhead Projector, Howden, H., New Rochelle, New York: Cuisenaire Company of America, 1985.

Curriculum and Evalaution Standards for School Mathematics, National Council of Teachers of Mathematics, Reston, Virginia: National Council of Teachers of Mathematics, 1989.

How to Evaluate Progress in Problem Solving, Charles, R. et al., Reston, Virginia: The National Council of Teachers of Mathematics, 1987.

Mathways, Kinetic, 1980.

MathWise, Agency for Instructional Television., 1981.

Mathematics Dictionary Fourth Edition, James, G. et al., New York, New York: Van Nostrand Reinhold Company, 1976.

The Language of Graphs, Swan, M., Nottingham, England: The Shell Centre for Mathematical Education, 1987.

Supplementary: see Appendix B.

22

HOW TO USE THE TEACHER RESOURCE MANUAL

This Teacher Resource Manual is designed to assist teachers in implementing the Mathematics 10/13/14 courses. It is specifically designed to be used on a regular basis. The design of the manual is as follows:

CONCEPT 1

The concept box includes the specific learner expectations taken from the course of studies.

COMMENTS	The **comments** section includes: comments to help clarify the intent of specific learner expectations; an overview of the material that students may have covered in junior high; notes regarding material that may be considered extraneous; and limiting examples.
PROCESS/PROBLEM-SOLVING CONTEXT	Suggestions for activities and examples of problems that are appropriate to the specific learner expectations are included, as well as references to other books and articles applicable to the specific learner expectations. Periodically, there will be problems or activities that apply to specific learner expectations; they will be preceded by **"Activity/Problem 2.2"** (this activity or problem refers to learner expectation 2.2). If the comment is preceded by **"Activity/Problem"** then the comment refers to the whole concept.
TECHNOLOGY INTEGRATION	Suggestions for specific calculator and computer exercises, video programs, and for authorized computer software are included.
ELECTIVE SUGGESTIONS	In some cases, elective suggestions are provided. This includes problems and activities that are extensions of the specific learner expectations.
RESOURCE CORRELATION	Provides a correlation of the Basic and Recommended textbooks that are approved for the Mathematics 10/13/14 courses.

MATH 10

Program Emphasis

Required Content

The major part of the content of each high school mathematics course consists of topics required of all students who take the course. The required content comprises 80% of the course and contains the concepts, skills and attitudes that all students are expected to acquire. As well, the required portion of all courses includes specific expectations in regard to problem solving and the use of technology.

Elective Material

Each course will include a compulsory component comprising 20% of the course, made up of elective material which will be consistent with the content and expectations of their required component. The purpose of the elective is to provide for enrichment, remediation, or innovative experimental presentations or activities. It is not intended to provide acceleration or advanced placement. However, horizontal enrichment and extension is appropriate and students should have access to elective material that serves their individual needs and interests.

Suggested Program Emphasis (in %) for Required Portion of Mathematics 10

Unit	Suggested Program Emphasis (in %)
Number Systems	10
Operations on Polynomials	15
Factoring Polynomials	14
Equations and Inequalities	10
Coordinate Geometry and Graphing	20
Systems of Linear Equations	10
Statistics	10
Trigonometry	11

Number Systems
Program Emphasis – 10%

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that rational numbers are those which can be written as $^a/_b$, $a, b \in I$, $b \neq 0$, and that they exhibit characteristics that are unique.	Students will be expected to identify repeating or terminating decimals as rational numbers. Students will be expected to change fractions to their equivalent decimal and percent value. Students will be expected to change terminating decimals to fractions and percent. Students will be expected to recognize the fractional equivalents of common repeating decimals (thirds, sixths, ninths).		pp. 27–30
Students will be expected to demonstrate an understanding that irrational numbers are those which cannot be written in the form $^a/_b$ $a, b \in I$, $b \neq 0$.	Students will be expected to recognize irrational numbers as infinite, non-repeating decimals. Students will be expected to find approximations for square roots of non-perfect squares. Students will be expected to recognize the square roots of negative integers as imaginary numbers. Students will be expected to recognize π as a special irrational number.	Students will be expected to use scientific calculators to find approximations for the square roots of rational numbers.	pp. 31–36
Students will be expected to demonstrate an understanding that real numbers are composed of rational and irrational numbers and that complex numbers are comprised of real and imaginary numbers.	Students will be expected to locate real numbers on a number line. Students will be expected to use absolute value to find the distance between points on a number line.	Students will be expected to represent rational and irrational numbers with measured and constructed line segments that correspond to the number.	pp. 37–39

Operations on Polynomials
Program Emphasis – 15%

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that polynomials can be used to describe some mathematical situations in which values vary.	Students will be expected to identify the following as they occur in the study of polynomials: term, variable, factor, monomial, binomial, trinomial, polynomial, coefficient, degree, exponent, base, power.		pp. 40–43
	Students will be expected to evaluate a polynomial for given values of the variables.	Students will be expected to write polynomials as rules which describe a mathematical situation.	
Students will be expected to demonstrate an understanding that calculations involving expressions with exponents are subject to laws and properties.	Students will be expected to use the multiplication law of exponents for powers with literal bases and integral exponents, $a^m \times a^n = a^{m+n}$.	Students will be expected to verify the multiplication law for exponents.	pp. 44–46
	Students will be expected to use the power law for exponents, $(a^m)^n = a^{mn}$ for expressions with literal bases and integral exponents.	Students will be expected to verify the power law for exponents.	
	Students will be expected to use the division law of exponents for powers with literal bases and integral exponents, $a^m/a^n = a^{m-n}$.	Students will be expected to verify the division law for exponents.	
Students will be expected to demonstrate an understanding that calculations and operations on polynomials depend on the application of the laws of operations for numbers and powers.	Students will be expected to add and subtract polynomials.		pp. 47–48
	Students will be expected to multiply polynomials by monomials, and binomials by binomials.		pp. 49–54
	Students will be expected to recognize the expansion of the square of a binomial and the product of conjugate binomials as special cases.		
	Students will be expected to divide polynomials by monomials.		pp. 55–56
	Students will be expected to divide polynomials by binomials of the form $ax + b$, $a, b \in I$.		

Factoring Polynomials
Program Emphasis - 14%

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that polynomials can be factored.	Students will be expected to factor polynomials that have a monomial common factor.		pp. 57–58
	Students will be expected to factor polynomials that have a binomial common factor.		
	Students will be expected to factor trinomials of the form $ax^2 + bx + c$; $a,b,c \in I$.		pp. 59–66
	Students will be expected to factor perfect square trinomials.		
	Students will be expected to factor difference of squares polynomials.		
	Students will be expected to factor polynomials that require a combination of methods in order to be fully factored.	Students will be expected to solve problems with factorable polynomials.	p. 67
Students will be expected to demonstrate an understanding that the values of the variables which make a polynomial equal to zero can be found by factoring.	Students will be expected to find rational zeroes of an integral quadratic polynomial in one variable by factoring.		pp. 68–69
	Students will be expected to solve quadratic equations with integral coefficients in one variable by factoring.	Students will be expected to solve problems by factoring which can be represented by quadratic equations.	

Equations and Inequalities
Program Emphasis - 10%

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that equations can be used to represent algebraically situations and relationships that involve equalities.	Students will be expected to solve and verify the solution of any first degree equation in one variable by applying formal algebraic procedures. Students will be expected to solve and verify the solution of simple quadratic and radical equations which can be reduced easily.	Students will be expected to represent problems with equations and solve the problems through the solution of the equations.	pp. 70–74
Students will be expected to demonstrate an understanding that some situations and relationships can be represented through the use of inequalities.	Students will be expected to solve, graph and verify the solution of any first degree inequality.	Students will be expected to verify the "reverse the sign" rule in the inequalities. Students will be expected to represent problems with inequalities and solve the problems through the solution of the inequalities.	p. 75

Coordinate Geometry and Graphing
Program Emphasis - 20%

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that points and lines can be represented on a Cartesian plane and that they have characteristics which can be defined and measured.	Students will be expected to use the formula for the distance between two points.	Students will be expected to deduce the distance formula from the Pythagorean theorem. Students will be expected to solve problems that involve the use of the distance formula.	pp. 76–77
	Students will be expected to use the formula for the coordinates of the midpoint of a line segment.	Students will be expected to verify the midpoint formula. Students will be expected to solve problems that involve the use of the midpoint formula.	pp. 78–79
	Students will be expected to use the formula for the slope of a line passing through two points.	Students will be expected to solve problems that involve the use of the formula for the slope of the segment between two points. Students will be expected to solve problems that involve the relationship between the slopes of parallel and perpendicular lines.	pp. 80–83
	Students will be expected to determine whether points in a plane are collinear.	Students will be expected to solve problems that involve the necessary and sufficient conditions for collinearity.	p. 84
Students will be expected to demonstrate an understanding that the ordered pairs which satisfy a linear equation, $Ax + By + C = 0$, correspond to coordinates on the Cartesian plane and can be used to graph the equation on the Cartesian plane.	Students will be expected to find sets of ordered pairs that satisfy linear equations.		pp. 85–86
	Students will be expected to graph linear equations by plotting sets of ordered pairs that satisfy the equation.		
	Students will be expected to find the x- and y- intercepts of linear equations.		p. 87
	Students will be expected to graph linear equations by use of the x- and y- intercepts.		
	Students will be expected to write linear equations in the slope–intercept form, $y = mx + b$.		pp. 88–90
	Students will be expected to graph linear equations using the slope and the y–intercept or another point that satisfies the equation.	Students will be expected to design and carry out an investigation involving the use of scientific calculators or computers to determine the effects of the parameters m and b on the graphs of linear equations.	
	Students will be expected to identify and graph equations of lines parallel to the x- and y–axes.		p.91
Students will be expected to demonstrate an understanding that lines in a plane can be uniquely determined.	Students will be expected to graph and write the equation of a line given the conditions which uniquely determine it.	Students will be expected to solve problems that involve conditions which uniquely determine a line.	pp. 92–93

Systems of Linear Equations
Program Emphasis - 10%

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that some mathematical circumstances can be represented algebraically by a system of two linear equations in two unknowns.	Students will be expected to solve simple systems of linear equations in two unknowns graphically. Students will be expected to solve systems of linear equations in two unknowns algebraically.	Students will be expected to solve problems that can be represented with a system of two equations in two unknowns.	pp. 94–96
Students will be expected to demonstrate an understanding that lines in a plane can relate to each other in various ways.	Students will be expected to recognize the characteristics of systems of linear equations with graphs that are parallel, coincident or intersecting.	Students will be expected to verify the conditions which are present in a system of equations that determine the relationships of the graphs.	p. 97

Statistics
Program Emphasis–10%

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that populations have certain characteristics which can be determined by analysing the characteristics of a suitable sample of that population.	Students will be expected to determine a sample that will adequately and accurately represent a population. Students will be expected to evaluate samples for bias, appropriateness of sample type, amount of data and randomness. Students will be expected to draw inferences about the population from which a sample was taken and express it using the language of probability.	Students will be expected to determine a valid sample, design and carry out a simple survey, and make a valid inference to the population based on the results. Students will be expected to defend or oppose a generalization that is made from a sample to a population.	pp. 98–100
Students will be expected to demonstrate an understanding that data collected from a sample must be organized, presented and analysed in order that valid inferences will be drawn.	Students will be expected to identify a situation, formulate and hypothesis and plan what data need to be collected. Students will be expected to organize data using stem-and-leaf plots and box plots. Students will be expected to choose and calculate appropriate measures of central tendency for sets of data. Students will be expected to represent data using appropriate graphs. Students will be expected to do simple visual analyses on sets of data organized in stem-and-leaf plots, box plots and graphs. Students will be expected to evaluate intuitively an analysis of data for the confidence with which the results can be accepted.	Students will be expected to derive and defend conclusions based on the visual analyses of sets of data.	pp. 101–107

Trigonometry
Program Emphasis - 11%

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that similar triangles have equal angles and that the corresponding sides are proportional.	Students will be expected to recognize and write the similarity relationship between similar triangles.		pp. 108–109
	Students will be expected to use the relationship between the lengths of the sides of similar triangles to calculate the lengths of unknown sides.	Students will be expected to generalize the properties of similar triangles through the use of inductive processes.	
		Students will be expected to solve problems involving the use of the properties of similar triangles.	
		Students will be expected to develop the sine, cosine and tangent ratios in similar right triangles through the use of the properties of similar triangles.	p. 110
Students will be expected to demonstrate an understanding that the sine, cosine and tangent ratios of a given angle are particular ratios between pairs of corresponding sides of similar, right triangles that contain the given angle.	Students will be expected to find the sine, cosine and tangent ratios and the measures of the acute angles within right triangles, given the measures of any two sides.	Students will be expected to use a calculator to find the sine, cosine and tangent ratios of any acute angle.	pp. 111–113
		Students will be expected to use a calculator to find the measure of an acute angle given the value of one of the trigonometric ratios of the angle.	
		Students will be expected to solve problems in which an angle measure in a right triangle must be found.	
	Students will be expected to find the measure of a side of a right triangle given the measure of a second angle and the length of a side.	Students will be expected to solve problems in which the length of a side of a right triangle must be found.	pp. 114–115

Attitudes

Within the study of the content of this course, it is expected that *students will be encouraged to*

- be confident in their mathematical knowledge and in their ability to acquire new knowledge;
- demonstrate persistence, resolve, flexibility and ingenuity in finding the solution to problems;
- develop an attitude of intellectual curiosity and openness to new ideas, insights and change in the pursuit of mathematical knowledge;
- exhibit an attitude of curiosity and spontaneity and appreciate creativity and innovation in representing situations mathematically;
- value and be aware of the importance of communication skills in mathematics;
- appreciate and develop a positive attitude toward the usefulness of computational competence, mathematical processes and problem–solving skills which are used in the decision making and modeling processes in our society;
- value the contributions of mathematics to our culture and civilization.

Problem Solving

Problem solving is to be integrated into all areas of the high school mathematics program because a major purpose of studying mathematics is to learn to solve problems. A student who has successfully completed this course will understand that thinking and problem-solving skills are required in order to explain the world, and that mathematics can provide some of the tools for problem solution.

Students will understand that problems can be solved in many ways, and that problems can be routine, requiring only the application of a known procedure or algoarithm, or non-routine, requiring the development of a process or the conducting of an investigation to solve them.

Students will be expected to use a variety of procedures to help them understand mathematical problems. In particular, they will be expected to

- read the problem thoroughly
- identify and clarify key components
- restate the problem using familiar terms
- evaluate the given information as to whether it is insufficient or extraneous
- interpret pictures, charts and graphs
- determine any hidden assumptions
- ask relevant questions
- identify given, needed and wanted information
- diagram or model the problem situation
- use suitable notation
- determine valid inferences.

Students will be expected to develop a variety of strategies for use in the solution of mathematical problems. In particular, they will be expected to

- conduct an investigation
- use estimation and approximation
- develop equations or use formulae
- use flow charts
- make lists and charts
- look for patterns
- work backward
- break the problem into smaller parts

- look for a simpler or related problem
- make diagrams or models
- use manipulatives
- choose and sequence a series of mathematical operations
- sketch the graph of a problem situation
- establish procedures to gather and organize data.

Students will be expected to develop a variety of skills which can be used to carry out the plan for the solution of a problem. In particular, they will be expected to

- apply selected strategies.
- present ideas clearly
- document the solution process
- utilize appropriate group behaviours
- use calculators and computers
- evaluate problem-solving strategies for effectiveness
- search for additional information
- ask questions
- be open to inspirations, intuitions and "bright ideas".

Students will be expected to employ a variety of skills to help them look back over the solution of a problem. In particular, they will be expected to

- determine the reasonableness of an answer
- explain the solution in oral or written form
- consider the possibility of additional solutions
- search for other strategies and processes of solution
- create and solve similar problems
- note the characteristics that will be identifiable in similar problems
- make a generalization.

CONCEPT 1

Students will be expected to demonstrate an understanding that rational numbers are those which can be written as $^a/_b$, a, $b \in I$, $b \neq 0$, and that they exhibit characteristics that are unique.

Students will be expected to:

1.1 **identify repeating or terminating decimals as rational numbers**
1.2 **change fractions to their equivalent decimal and percent value**
1.3 **change terminating decimals to fractions and percent**
1.4 **recognize the fractional equivalents of common repeating decimals (thirds, sixths, ninths).**

COMMENTS

Students' math skills should be reviewed as needed when introducing new material, rather than as a separate section at the beginning of a unit. (See **Elective Suggestions** for review suggestions and textbook correlation.)

Students will have done some previous work with rational numbers. In junior high they perform the operations of addition, subtraction, multiplication and division on positive and negative rational numbers; apply the rules of order of operations to evaluate expressions that involve rational numbers; convert rational numbers from $^a/_b$ to decimal form ($b < 10$ or b is a power of 10); convert rational numbers from decimal form to $^a/_b$ form (terminating decimals only); and convert whole number and fractional percents to fraction and decimal forms.

Although students have done this work, it is important to reinforce the definition of rational numbers and the meaning of percent. All calculation should be done by appropriate means. Paper and pencil calculations should be limited and the use of calculators emphasized along with the accompanying skills of estimation, mental arithmetic and alertness to the reasonableness of results.

Sets and set notation are not part of the junior high mathematics program. The textbooks mention sets; make sure students understand the terminology associated with set theory.

Reinforce proper notation for decimal equivalents of fractions, e.g., $^5/_6 = 0.8\dot{3}$. Reinforce the definition of rational numbers, the meaning of percent, and the transformation of any fraction or decimal to a percent. ($^2/_3 = 0.\dot{6} = 66.\dot{6}\%$). Students should recognize that $0.\dot{3} = {}^1/_3$.

These objectives need not be taught discretely. The "order" may be: change fractions to decimals, identify repeating or terminating decimals as rational numbers, recognize common fractional equivalents of common repeating decimals (thirds, sixths, ninths), change terminating decimals to fractions and percent.

35

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: 1.1 Have students use their calculators to change fractions to decimals. Students often think of rational numbers as only fractions because by definition they are numbers that can be written in the form $^a/_b$. In this activity, students should recognize that all of the decimals they have obtained are either repeating or terminating. Building from their understanding that fractions are rational numbers, they should see that repeating and terminating decimals are also rational numbers.

Activity: 1.4 Have students use their calculators to find the decimal equivalent of some common fractions. Then have students predict what multiples of those common fractions will look like, and check their predictions with the calculator. Have students write a generalization of the patterns they have discovered.

e.g. $^1/_3 = 0.3333333...$ What do you think $^2/_3$ will equal? Check with your calculator. What do you think $^4/_3$ will equal? The same activity can be done for sixths and ninths.

Activity: 1 In discussing the definition of rational numbers, discuss why $b \neq 0$. Have students attempt to find an answer for any number divided by zero. Students will get an error symbol on the calculator. A small proof for why $^a/_0$ does not work is:

> Is there such a number x where $^a/_0 = x$, and $a \neq 0$?
> If this is the case, then 0 times $x = a$.
> This would mean $0 = a$.
> We said that $a \neq 0$.
> So, $^a/_0 = x$ does not exist.

Activity: 1 Does $^0/_0$ exist?

> Is there such a number x where $^0/_0 = x$?
> This means that 0 times $x = 0$.
> This is true for any value of x, i.e., x could be any number.
> If $x = 3$, then $^0/_0 = 3$, or if $x = 10$, then $^0/_0 = 10$.
> From this, 3 could = 10 and this does not make sense.
> So, $^0/_0 = x$ does not exist.

Problem: 1 A rubber ball is known to rebound half the height it drops. If the ball is dropped from a height of 30 m, how far will it have travelled by the time it hits the ground for: the first time, the second time, the third time, the fourth time, the fifth time?

Activity: 1.3 A nice lead-in to the discussion of irrational numbers is to give students a decimal that is neither repeating nor terminating and ask them to come up with a fractional equivalent. This does not have to be a formal approach; have students work in groups of two or three to attempt to solve this.

TECHNOLOGY INTEGRATION

Activity: 1.1 Find decimal equivalents for fractions with a denominator of 7. Why are these unusual? Are there other fractions which behave in a similar fashion? Why will decimals repeat in a block which is a maximum size of 1 less digit than the denominator? (Notice also that the decimal equivalents for $^{1}/_{7}$, $^{2}/_{7}$, $^{3}/_{7}$, $^{4}/_{7}$, $^{5}/_{7}$ and $^{6}/_{7}$ all use the same six digits (1, 2, 4, 5, 7, 8) in the same order.) Have students work out the exact decimal equivalent of a fraction with a very long repeating block, using a calculator or computer.

One such procedure is as follows ($^{5}/_{17}$ used as an example using a calculator with an 8–digit display):

1. Divide the numerator by the denominator and write down the approximate decimal equivalent, leaving off the last digit. Why would we leave off the last digit?
   Example: 0.294117

2. Multiply 0.294117 by 17 and subtract the result from 5. Why?
   Example: $0.294117 \times 17 = 4.999989$
   $5 - 4.999989 = 0.000011$

3. Divide 11 by 17 and add all but the last digit to the approximate decimal equivalent from step one. What step in the long division process does this represent?
   $^{11}/_{17} = 0.647058$
   Approximation becomes 0.294117647058

4. Multiply 0.647058 by 17 and subtract the result from 11. Why?
   $0.647058 \times 17 = 10.999986$
   $11 - 10.999986 = 0.000014$

5. Divide 14 by 17 and add all but the last digit to the approximate decimal equivalent from step three.
   $^{14}/_{17} = 0.823529$
   Approximation becomes 0.2941176470588823529
   Since the last two digits in this approximation are the first two, the digits appear to have begun to repeat. How do we know that this is true?

6. $^{5}/_{17} = 0.2941176470588235$

For further discussion of using the calculator to find the period of repeating decimals when the period is long, see "Expanding the Limits of the Calculator Display," Mathematics Teacher, January 1986.

Activity: Multiply 1234567 × 9999999 by looking for a pattern using smaller numbers and then by using a calculator.

Activity: See HM10 TRM Lesson Notes, p. 2.

ELECTIVE SUGGESTIONS

Review:

M10:	pp. xiv–xxiii, 2–17
HM10:	pp. 1–17
MPP10:	pp. 10–17

Extension: Examine the algorithm to change repeating decimals to fractions and have students express the reasons this algorithm works.

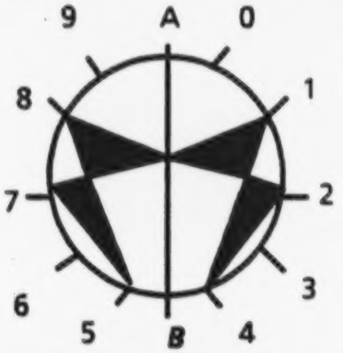
Problem Solving in Mathematics: Focus for the Future, pp. 54, 55: "The Handshake Problem," solutions #1, 2 and 3 provide an opportunity for a discussion of problem-solving strategies.

Problem: What is the 50th digit of the decimal fraction for $^{1}/_{13}$?

There are various approaches to solving this problem. For example, the students could divide until the 50th digit was determined. Students might also notice a pattern while dividing, and attempt to predict the answer before calculating the 50 digits.

Answer: The 50th digit is 7.

Activity: 1.1 Have the students determine the period of $^{1}/_{7}$ ($^{1}/_{7} = 0.142857$). Then draw a circle, and evenly divide the circumference into ten points. Label them from 0 to 9. On this circle, connect the digits found in the period: 1 to 4, 4 to 2, and so on, and finally from 7 to 1. What is observed? (A symmetric pattern, a circle.) It turns out that for rational numbers of the form $^{1}/_{p}$, where p is prime, which have an even number of digits in their repeating decimal, a symmetric geometric pattern as described is always obtained. This could become a poster activity.



For more information, see "Repeating Decimals, Geometric Patterns and Open-Ended Questions," Mathematics Teacher, October 1982.

See also "Periodic Pictures," Mathematics Teacher, February 1987, for a set of activities and a computer program designed to further deepen students' understanding of the graphical symmetry of repeating decimals.

RESOURCE CORRELATION

M10:	pp. 14–17
HM10:	pp. 9–11
MPP10:	p. 17

Mathématiques 10: p. 17
HM10 TRM D/A Master 1–2
MPP10 ASTRM: pp. 1–2

CONCEPT 2

Students will be expected to demonstrate an understanding that irrational numbers are those which cannot be written in the form $^a/_b$, a, b$\in$I, b$\neq$0.

Students will be expected to:

2.1 recognize irrational numbers as infinite, non-repeating decimals
2.2 find approximations for square roots of non-perfect squares
    2.2.1 use scientific calculators to find approximations for the square roots of rational numbers
2.3 recognize the square roots of negative integers as imaginary numbers
2.4 recognize π as a special irrational number.

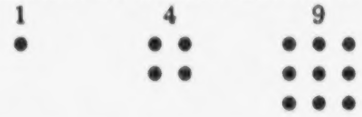
COMMENTS

Students have done little work with irrationals. They have worked with π as the ratio of the circumference to the diameter of a circle, and have computed the square roots of whole numbers using calculators and estimation, but have never discussed the irrational number system. You may wish to deal with the irrational numbers before working with the pure imaginary numbers.

The Pythagorean Relation is useful in discussing some uses for irrational numbers.

Irrational numbers are the infinite decimals that are not repeating. The square root of any number that is not a perfect square, the cube root of any number that is not a perfect cube, etc., are irrational numbers.

"Perfect square" numbers are numbers that can be arranged in squares. e.g.,



3, 5, 6, 7 are not "perfect squares" since they cannot be arranged in a square.

Students should continue to practise recognizing the square roots of perfect squares to 225 and should use this knowledge to develop the ability to estimate square roots of any number to 225. In addition, they should use calculators to find square roots, but be adept at estimation techniques to check the reasonableness of answers.

Pure Imaginary Numbers: Students can be introduced to pure imaginary numbers by finding the square roots of negative numbers. Time should not be spent on operations on complex numbers or on very sophisticated definitions. A **pure imaginary number** is a complex number of the form a + bi where a and b are real numbers, a = 0 and b $\neq$ 0 and $i^2 = -1$. Limiting examples: Find the square roots of the following: $\sqrt{-4}(=2i)$, $\sqrt{^{-16}/_9}\,(=\,^4/_3 i)$, and approximate the square root of $\sqrt{-24}(\approx 4.9i)$.

The history of π may be an interesting way to introduce it as an irrational number:

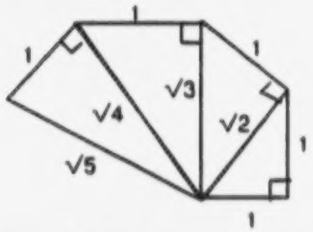
225 B.C. – Archimedes estimates π as $^{223}/_{71} < \pi < {}^{22}/_{7}$
480 A.D. – the Chinese use $^{355}/_{113}$ for π
530 A.D. – the Hindu value for π set at $^{3927}/_{1250}$
1220 A.D. – a European estimate of π is given as $^{864}/_{275}$
1761 – Johann Lambert proves that π is irrational
1986 – π computed to 29 360 128 decimal places

π is the first letter of the Greek word perimetron, meaning the distance around. Mathematician Leonhard Euler began using it in 1737 to represent the ratio of the circumference to the diameter of a circle.

PROCESS/PROBLEM-SOLVING CONTEXT

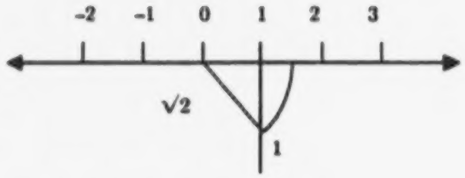
Activity: 2.1 Introduction to irrational numbers can be done following the discussion of rational numbers. If students have been taught to use the algorithm for changing repeating decimals to fractions, have them try to develop a similar algorithm to change irrationals (which repeat according to some visible pattern) to fractions. Have students explain why this sort of algorithm cannot work. This does not have to be a formal approach; have students work in groups of two or three in an attempt to solve this.

Activity: 2.2 Have students carefully construct the following right triangles and measure the hypotenuse in each triangle to find approximations for $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, etc.



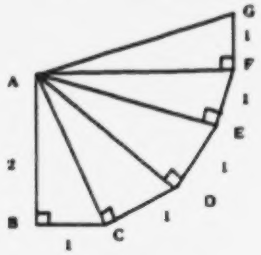
Note: This activity provides some good review of measurement techniques with both the protractor and the ruler. It is important to be accurate in order to get a good approximation.

Activity: 2.2 The Pythagorean Relation can be used to approximate $\sqrt{2}$ on a number line:



Note: Students could measure this to discover a numerical approximation of $\sqrt{2}$.

Problem: What is the length of segment $\overline{AG}$?



Answer: 3 units.

Activity: 2.2 In discussing why some "$\sqrt{\ }$" numbers are rational, the definition of rational numbers can be used.

$\sqrt{4}$ is rational because it can be expressed as a fraction $^{2}/_{1}$ and $^{2}/_{1} \times {}^{2}/_{1} = 4$. If $\sqrt{3}$ were rational then we would have to find a fraction, that when multiplied by itself would equal 3 (i.e., $^{a}/_{b} \times {}^{a}/_{b} = 3$). Have students try several combinations of fractions to see if they ever get an exact value of 3. This can be done with any irrational number and could lead into a discussion of the proof that $\sqrt{a}$ is irrational. See **Elective Suggestions** for the proof.

Problem: Mental Calculation: How many whole numbers are between $\sqrt{8}$ and $\sqrt{80}$?

Answer: 6.

For other **problems** see HM10, p. 42 and HM10 TRM, Lesson Notes, p. 62, "Mental Math."

Problem: A glazier wished to bring a huge mirror which measured 2.5 m by 2.25 m into a house through a doorway which measured 2.1 m by 0.9 m. Could he get the mirror into the house in one piece?

Answer: Yes.

Activity: 2.3 In introducing the square roots of negative numbers as imaginary, students could work together to "invent" or "imagine" a way in which the square roots of negative numbers could be represented. The following must exist in their method:

1. The root squared must equal the radicand.
2. The procedure must work for all the roots of all negative numbers.
3. The numbers used and the processes may be "invented" or "imaginary."

Activity: 2.4 Have groups of students draw sets of regular n-gons (n = 3, 4, 5, 6, 8, 10, 12, 15, 18, 20, 30) with equal radii (distance from centre to a vertex). Record the perimeter of each polygon and the ratios between the perimeter and the diameters.

n	P	P/d

Discuss with students what happens to the ratio as n becomes large. Can a circle be thought of as a regular polygon with an infinite number of sides? Remind them that in a circle $^c/_d = \pi$.

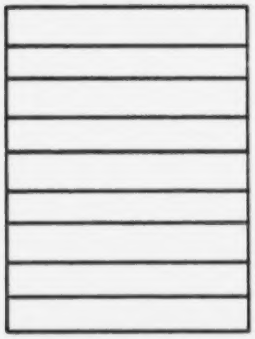
Activity: 2.4 In estimating the value of π, have students:

1. Draw a circle and measure its diameter.
2. Use a string to approximate the circumference and then measure the string.
3. Divide the measure of the diameter into the measure of the circumference. This is an approximation of π.
4. Use a different circle and compare the results.

Activity: 2.4 One way to compute a rough estimate of pi's value is to use Buffon's Needle Problem. (Buffon's Needle Problem is an example of the Monte Carlo method of probability, which will be studied in Math 20).

Buffon's Needle Problem:
You require four 1 cm pieces from toothpicks. From a reasonable height, randomly drop the four sticks onto a ruled surface (see below for an example). Count the number of times that the stick lands on or across one of the lines. Drop the sticks 25 times (total of 100 drops). Keep an accurate record of the number of hits and drops in all.

Note: the distance between the lines should be 2 cm.



1. Divide the total number of sticks dropped (100) by the total number of hits. What is the quotient?________________
2. Sum individual hits with those of the whole class, and divide that number into the total number dropped by the whole class. What is the pooled result?____________________
3. Are these good estimates of pi?____________________
4. Which estimate is the best?____________________
5. Why do you think this is the case?________________

Adapted with permission from "Pi: The Digit Hunt" in Student Math Notes, copyright 1987 by the National Council of Teachers of Mathematics. See also "Probability and Pi" in Activities from the Mathematics Teacher for a set of related activities.

Problem: Use a calculator to evaluate $\sqrt{2}$ using the continued fraction

$$1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\dots}}}}$$

Activity: Using the history of π, have students compare the approximations of π to the current value of π. At what digit do they begin to differ or at what digit do they become the same?

Problem: 2.4 In the Rhind papyrus, the area of a circle is said to be the square of $^{8}/_{9}$ of the diameter. What approximations were the Egyptians using for π? Give your answer as a common fraction.

Answer: $\pi \approx (^{16}/_{9})^2 = \frac{256}{81}$

A problem such as this also serves as a review of finding the area of a circle.

Problem: "The Telephone Pole Problem," solution #1, p. 57, in Problem Solving in Mathematics: Focus for the Future

TECHNOLOGY INTEGRATION

Computer simulation of Buffon's Needle Problem. The program is written in BASIC for the Apple computer. You'll notice in the graphic display that the needles are the same length as the distance between rules. The adjustment for this change has been made with the insertion of the 2 in the formula given in line 250.

```
10     REM COUNT DE BUFFON'S ESTIMATION OF PI
20     INPUT "HOW MANY NEEDLES TO DROP?"; N
30     HOME
40     HGR
50     HCOLOR=2
60     FOR Y=0 TO 159 STEP 10
70     HPLOT 0, Y TO 279, Y
80     NEXT Y
90     HIT=0
100    HCOLOR=3
110    FOR Z=1 TO N
120    X=INT(RND(Z)*280)
130    Y=INT(RND(Z)*160)
140    ANGLE=RND(Z)*1000
150    X1=X+10*COS(ANGLE)
160    Y1=Y+10*SIN(ANGLE)
170    IF X1<0 OR X1>279 THEN 200
180    IF Y1<0 OR Y1>159 THEN 200
190    HPLOT X,Y TO X1,Y1
200    IF Y=Y1 AND INT(Y/10)=Y/10 THEN HIT=HIT+1
210    IF Y/10=INT(Y/10) AND Y1/10=INT(Y1/10) THEN 230
220    IF INT(Y/10)< >INT(Y/10) THEN HIT=HIT+1
```

```
230    NEXT Z
240    VTAB 21
250    PRINT "PI ESTIMATE = ";2*N/HIT
260    END
```

Reprinted with permission from "Pi: The Digit Hunt" in Student Math Notes, copyright 1987 by the National Council of Teachers of Mathematics.

Calculator Practice: See MPP10, p. 174, "Calculator Tip."

ELECTIVE SUGGESTIONS

Extension: More able students may understand the indirect proof that $\sqrt{2}$ is irrational.

Let $\sqrt{2} = {}^{a}/_{b}$ (in lowest terms), where $a, b \in I$, $b \neq 0$, squaring both sides gives

$$2 = {}^{a^2}/_{b^2}$$

$$\text{and } 2b^2 = a^2$$

Now a^2 must be even, so a is even and can be written as 2n for $n \in I$.
Therefore, $a^2 = (2n)^2 = 4n^2$ and $4n^2 = 2b^2$
This means that b^2 and hence b must also be even. This is a contradiction from the first assumption, which said that ${}^{a}/_{b}$ was in lowest terms. Therefore, $\sqrt{2}$ cannot be written as ${}^{a}/_{b}$ and must be irrational.

Further discussion on π: MPP10, p. 155, "Calculator Tip" and MPP10, p. 177, "Math Tip."

RESOURCE CORRELATION

M10: pp. 18–29 (Note: Do not include the third, fourth, and fifth root problems.)
HM10: pp. 14–16, 71
MPP10: pp. 22, 43–45 (Note: Do not include problems which require operations with mixed radicals.)
Mathématiques 10: pp. 22, 43–45
HM10 TRM D/A Master 1-3
MPP10 ASTRM: pp. 3–5

CONCEPT 3

Students will be expected to demonstrate an understanding that real numbers are composed of rational and irrational numbers and that complex numbers are comprised of real and imaginary numbers.

Students will be expected to:

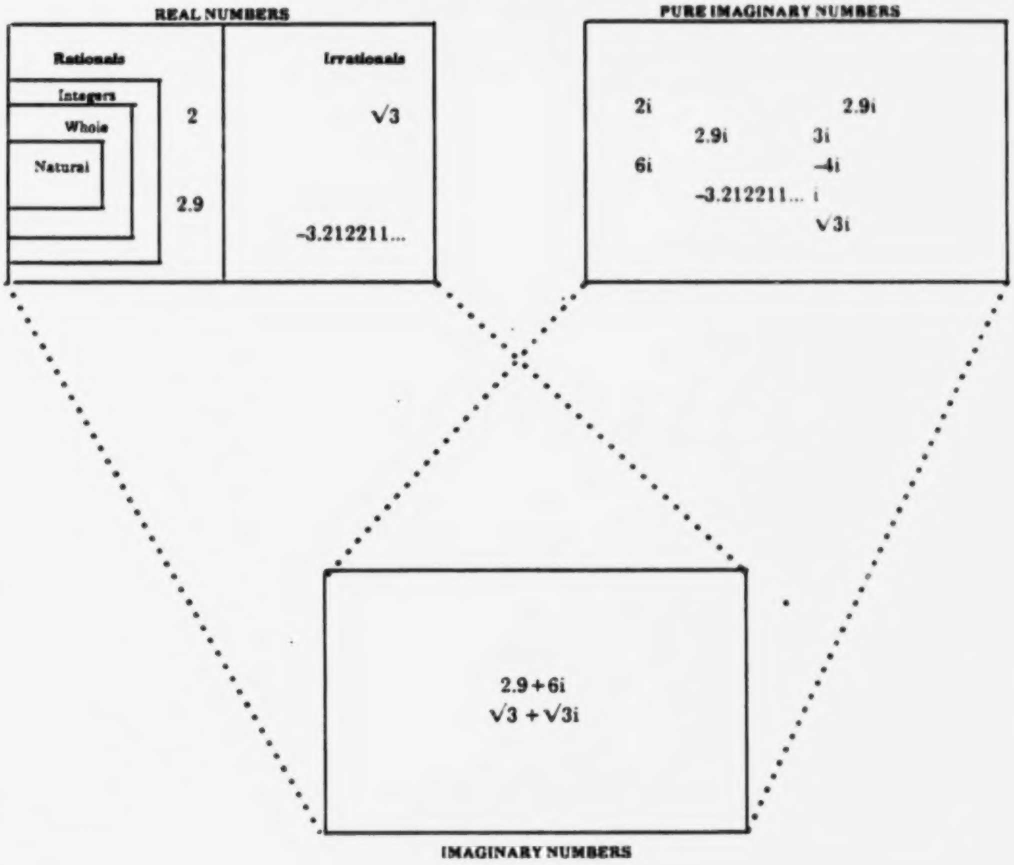
- **3.1 locate real numbers on a number line**
    - **3.1.1 represent rational and irrational numbers with measured and constructed line segments that correspond to the number**
- **3.2 use absolute value to find the distance between points on a number line.**

COMMENTS

Real Numbers consist of any rational or irrational number. Since students have been exposed to pure imaginary numbers, they should have little difficulty identifying real numbers as those that they can physically represent or for which they can find real examples.

Complex Numbers consist of any number, real or imaginary, of the form a + bi where a and b are real. The discussion should be limited to the square roots of negative numbers, in other words, in the form a + bi, where a = 0.

COMPLEX NUMBERS


REAL NUMBERS
Rationals
Irrationals
Integers
Whole
Natural
2
2.9
√3
-3.212211...
PURE IMAGINARY NUMBERS
2i
2.9i
2.9i
3i
6i
-4i
-3.212211... i
√3i
2.9 + 6i
√3 + √3i
IMAGINARY NUMBERS

Tell the students that if they doubt the meaningfulness and usefulness of "imaginary" numbers, they are not alone. For hundreds of years these numbers were simply ignored. It was accepted that there is no square root of a negative number because a negative number is not a square.

In the 16th century the $\sqrt{-1}$ was introduced in an important mathematics book written by Cardan. Over the next two-and-one-half centuries the mathematics of imaginary (complex) numbers was developed, but not without considerable doubt and controversy about the usefulness and especially the meaningfulness of this work. In fact, mathematicians found uses for imaginary numbers even while they debated their meaning! It was René Descartes who coined the word "imaginary."

Complex numbers are now used extensively in mathematics, science and technology (for example, quantum mechanics and electric circuitry).

For more information on the development of complex numbers, see, for example, "Thinking the Unthinkable: The Story of Complex Numbers (with a Moral)," Mathematics Teacher, October 1988.

Absolute Value of x is its undirected distance from zero. The symbol |x| is used for absolute value.

PROCESS/PROBLEM-SOLVING CONTEXT

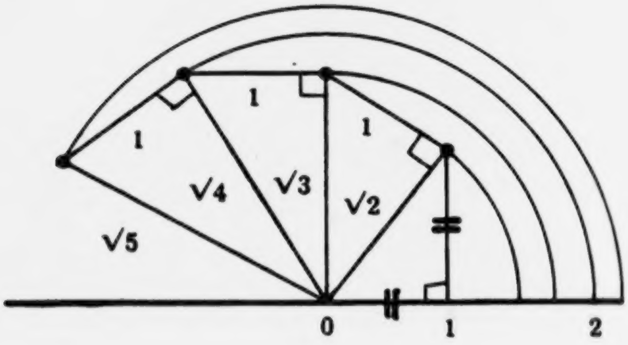
Activity: 3.1 It is interesting to compare rational and irrational numbers on the number line. This is a good visual representation of how rational numbers and irrational numbers compare. Using Activity 2.2 (Number Systems Concept 2), have students draw a number line where rationals and irrationals are noted. E.g., Have students plot $^1/_{10}$, $^1/_8$, $^1/_4$, $\sqrt{2}$, $\sqrt{5}$, 0.123..., 1.101101110, 1.1110,... on the same line. The relationship of these numbers to one another should be the focus of this activity.

Problem: 3.1 Use a calculator to order the following numbers from smallest to largest and locate them on a number line: $4\,^4/_9$, 4.45, $4.4\dot{5}$, 4.454445..., $^4/_3\pi$.

Problem: 3.1 Describe how you could use right triangles to find an estimate for $\sqrt{30}$, using only a pencil, protractor and ruler. What is the minimum number of triangles needed? Answer:

ELECTIVE SUGGESTIONS

Activity: 3.1 Using the Activity 2.2 (Number Systems, Concept 2), construction, have students mark with their compasses the approximate value of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, and $\sqrt{5}$ on a number line. Notice that although the consecutive numbers n are evenly spaced along the line, $\sqrt{n}$ values are placed closer together as n increases.



RESOURCE CORRELATION

M10: pp. 9–10, 22, 26–27
HM10: pp. 14–16, 24–25
MPP10: pp. 22–25
Mathématiques 10: pp. 22–25
HM10 TRM D/A Master 1–8
MPP10 ASTRM: pp. 3–5

CONCEPT 1

Students will be expected to demonstrate an understanding that polynomials can be used to describe some mathematical situations in which values vary.

Students will be expected to:

1.1 identify the following as they occur in the study of polynomials: term, variable, factor, monomial, binomial, trinomial, polynomial, coefficient, degree, exponent, base, power
1.2 evaluate a polynomial for given values of the variables
  1.2.1 write polynomials as rules which describe a mathematical situation.

COMMENTS

Introduction to the vocabulary associated with polynomials should be done within the appropriate context. See Mathematics Dictionary 4th Edition, by James & James for accurate definitions.

Students may have been previously introduced to the terms. This is an extension of the grade 9 program. Students have experience at solving equations in one variable and have evaluated polynomials in the sense of "checking" an answer after solving an equation.

It is important to emphasize what a polynomial is: a function which describes a situation in terms of one or more variables. A polynomial is a general rule which can then be evaluated for values of the variables as conditions and parameters change. The formula for the perimeter of a rectangle, $2L + 2W$, is an example of a polynomial. $2L + 2W$ describes the perimeter of any rectangle; substituting values for L and W we can describe the perimeter of a particular rectangle and hence describe a specific case.

It is also appropriate to mention to students that there will be some values of the variable which will make the polynomial equal zero. These special values are called zeroes.

Students have a difficult time understanding the concept of "variable." It is important to spend some time discussing it.

A Discussion of Variable:

The meaning of variable is variable; using the term differently in different contexts can make it hard for students to understand. As teachers, we should be sensitive to this multiple usage.

Complete the following activity yourself and then have the students do it. Discuss the list generated with the class.

Complete the following sentence:
A variable is a __________.

Use *the one* English word that, in your opinion, best captures the meaning of the term *variable*. (If you generate a number of possible candidates, please list them in order of preference.)

Mere synonyms are not acceptable; try to get to the heart of the concept.

Some of the words in the list may be better than others, but there is no *one right* word for "variable."

Here are three suggestions for teaching the concept of variable.

1. Variables are tools for expressing mathematical generalizations. It helps, therefore, if students have the habit of verbalizing such generalizations before they are asked to formalize those generalizations using the language of mathematics. It might be useful to make a habit of asking students to summarize some of their observations about arithmetic in their own words, in short exercises. ("What happens when you add three times a number to two times that number?" or "Can you say whether the sum of two odd numbers will necessarily be even or odd?") By asking students to observe patterns and to summarize them verbally, we may help make the transition from arithmetic to algebra.

2. The dynamic aspects of the variable concept should be stressed whenever it is appropriate and feasible. At the simplest level, students can be led to observe that the cost (variable) that appears on a gasoline pump is a (dynamic, linear) function of the amount of gasoline that emerges from the pump. Other dynamic relationships include time-dependent phenomena, for example, the temperature in a room at time t, the distance an object falls in t seconds, and so on.

3. Much of the algebra curriculum consists of solutions of equations independent of the context from which they were drawn. Many applications of algebra, for example, the standard word problems, are so neatly packaged, and taught in separate units, that students miss the point - that the language of algebra is a powerful means of capturing the mathematical essence of a wide variety of situations and not just the ones they've been taught to solve. We recommend that students be exposed to a wide range of problems for which algebra, especially in its role of a tool for mathematical generalization can be useful.

Reprinted with permission from "On the Meaning of Variable," Mathematics Teacher, (pp. 420, 425–426) copyright September 1988, by the National Council of Teachers of Mathematics.

For a discussion of the concept of variable that focuses on the similarities and differences between literal symbols (variables) and numbers, see "What are these Things called Variables?" Mathematics Teacher, October 1983. See also, "Conceptions of School Algebra and Uses of Variables," in The Ideas of Algebra, K–12 for further discussion on "variable."

For a discussion (with illustrative examples) of fostering greater understanding of algebraic concepts through the use of diagrams, see "Developing Algebraic Representation Using Diagrams," in The Ideas of Algebra, K–12.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: **1.1** In order to establish an understanding of "variable," students should draw from their own experience. This will lead into a discussion of how mathematics can be used to describe events that occur in their lives. For instance, use the discussion of $P = 2L + 2W$ to discuss how L and W are variables because this formula always describes the perimeter of a rectangle. Have students evaluate this polynomial for certain rectangles, given the dimensions of a rectangle.

Activity: **1.1** This is a good place to start modeling polynomials using tiles. See "Modeling Polynomials" and "The Zero Principle" in Algebra Tiles for the Overhead Projector, pp. 5–12.

Activity: **1.2** Have students write and evaluate expressions that describe recognizable situations such as the number of books in a library, etc., and have students identify the parts that "vary."

Problem: **1.2** The zone basketball final was held at Vista Heights High School. Ticket prices were as follows:

Adults $5
Students $3
Children $1

Write an expression to describe the income generated.
The gym seats 500 people. If all seats were filled and there were 100 adults and 346 students, what income was generated?

Activity: **1.2** A further discussion of variable: Write an expression that describes the number of tires of moving vehicles touching the pavement in your community when:

a) only the 4-wheeled motorized vehicles are presently being driven
b) only the 2-wheeled motorized vehicles are presently being driven
c) the combined number of 4- and 2-wheeled motorized vehicles are presently being driven
d) only the passenger/school buses are presently being driven.

Points to discuss: Does "presently being driven" include those vehicles stopped at an intersection? What difficulties are there in defining variables? Is precision necessary?

For a discussion on overcoming the misconceptions students sometimes demonstrate when translating from written language to mathematical language, see "From Words to Algebra: Mending Misconceptions," in The Ideas of Algebra, K–12.

TECHNOLOGY INTEGRATION

Have students use their calculators when evaluating polynomials with large numbers. Mental computation and estimation should be reinforced. For instance, have students estimate their answer prior to actual calculation.

See HM10, p. 110, "Computer Application" for a program to find the zeroes of a polynomial.

Evaluate polynomials using a spreadsheet. This reinforces the concept of variable. Since the spreadsheet will quickly calculate the value of an expression, values of the variable can be easily changed. Students will have an opportunity to predict what will happen to the value of a polynomial, if the variable increases, decreases, etc. Appleworks and Microsoft Works are Alberta Education Basic Resources that have spreadsheet capabilities.

ELECTIVE SUGGESTIONS

For a game designed to help increase understanding in algebra and logic see, "Using a Game as a Teaching Device," in Activities for Junior High School and Middle School Mathematics.

RESOURCE CORRELATION

M10: pp. 46–48, 156–157
HM10: pp. 80–83
MPP10: pp. 61, 236–237 (Note: This book integrates the evaluation of polynomials with polynomial operations.)
Mathématiques 10: pp. 61, 236–237
HM10 TRM D/A Master 4–9

CONCEPT 2

Students will be expected to demonstrate an understanding that calculations involving expressions with exponents are subject to laws and properties.

Students will be expected to:

- **2.1 use the multiplication law of exponents for powers with literal bases and integral exponents. $a^m \times a^n = a^{m+n}$**
  - **2.1.1 verify the multiplication law for exponents**
- **2.2 use the power law for exponents, $(a^m)^n = a^{mn}$, for expressions with literal bases and integral exponents**
  - **2.2.1 verify the power law for exponents**
- **2.3 use the division law powers with literal bases and integral exponents, $a^m/a^n = a^{m-n}$**
  - **2.3.1 verify the division law for exponents.**

COMMENTS

Discussion of integral exponents includes discussion of zero and negative exponents.

In junior high, students have discussed the exponent laws in relation to numeric bases only. In leading a discussion of literal bases, you may wish to review their skills with numeric bases.

"To Verify" requires the student to substantiate the validity of an operation, solution, formula or theorem through the use of examples which may or may not be generalized.

An example of a verification is:
Verify $a^m \cdot a^n = a^{m+n}$
The student might write:
1) $c^2 \cdot c^3 = c \cdot c \cdot c \cdot c \cdot c$
$= c^5$
$\therefore \quad c^2 \cdot c^3 = c^{2+3} = c^5$ or
2) $8^2 \cdot 8^1 = 8 \cdot 8 \cdot 8$
$= 8^3$
$\therefore \quad 8^2 \cdot 8^1 = 8^{2+1} = 8^3$

Emphasis should be on the process, not on difficult or complex calculations. Do not go beyond the level of difficulty in the textbooks.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: 2.2 Power Law **Emphasize that $(a^3)^4$ means a^3 used as a factor 4 times. Have students write an explanation of why the multiplication rule for exponents works. Zero and negative exponents should be used to demonstrate that the laws of exponents do not change if they are used. Zero and negative exponents play an important role in the evaluation of expressions.**

Problem: 2.1, 2.2 If $a = 2^{x+2}$, then find the simplified value of 8^x in terms of a.

Answer: $8^x = a^3/64$

Problem: 2.2 Evaluate

$$\left(\left(\left(\left(2^{-2}\right)^{2}\right)^{-2}\right)^{2}\right)^{-2}$$

Answer: 2^{-32}

Problem: 2.2 Although the following problem may be solved directly with the calculator, suggest that students search for a strategy to solve the problem without the calculator. (For example, to "decompose" a^{mn} into $(a^m)^n$).

Arrange the following in increasing numerical order:
$a = 2^{55}$
$b = 3^{44}$
$c = 5^{33}$
$d = 6^{22}$

Answer: $a < d < b < c$
$(2^5)^{11} < (6^2)^{11} < (3^4)^{11} < (5^3)^{11}$

Problem: 2.2 Suppose a = $\left(\left(\left(2^{2}\right)^{2}\right)^{2}\right)^{2}$ and $b = {}_{2}22^{2^{2}}$. Express the ratio of $^{b}/_{a}$ in the form 2^p, where *p* is an integer.

Answer: $^{b}/_{a} = 2^{65520}$

Problem: 2.3 Division Law This problem will also reinforce the use of order of operations.
Simplify the following:

$$\cfrac{x^{16}}{\cfrac{x^{8}}{\cfrac{x^{4}}{\cfrac{x^{2}}{x}}}}$$

Answer: x.

Activity: 2.3 Zero Exponents This is particularly helpful when explaining why $x^0 = 1$.

Have students try an exercise such as the following:
By expansion $x^6 = x \cdot x \cdot x \cdot x \cdot x \cdot x$

so $x^6 \div x^6 = \dfrac{x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x \cdot x} =$ ______________ ①

But, according to the division law $x^6 \div x^6 = x^{?-?} = x^{?}$ ②.

Therefore, from ① and ② we can conclude that_______ = _______.
Try this law out for various values of x then finish this sentence:
A power with an exponent of zero______________________________
____

Activity: 2.3 Negative Exponents By expansion,
$x^6 \div x^8 = \dfrac{x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x} =$ ____________ ③

but according to the division law, $x^6 \div x^8 = x^{?-?} = x^{?}$ ④
Therefore, from ③and ④ we can conclude

____________ = ________________
Try this out on your calculator for x = 2, 5, 10 and exponents of -1, -2, -3.

Finish this sentence: A power with a negative exponent

____

____

Problem: 2.3 Evaluate $x^0 - 0^x$ $(x \neq 0)$.

Answer: 1.

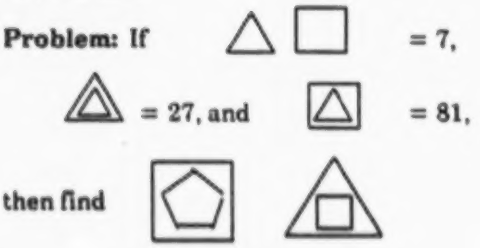
For more **Problems** see Problem Solving in Mathematics: Focus for the Future, p. 65, #5, 6.

TECHNOLOGY INTEGRATION

Software:
Algebra Drill and Practice II (Conduit), option "Integral Exponents," provides practice in simplifying exponential expressions.

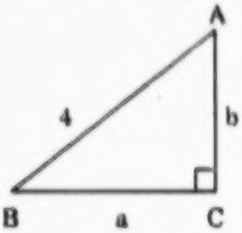
ELECTIVE SUGGESTIONS

Problem: If



Answer: $5^4 + 4^3 = 689$.

Problem: Find the area of △ ABC if $a + b = \sqrt{18}$.



Answer: A = $^1/_2$ square unit

Allow students time to explore possible solution strategies. They may need suggestions or hints such as: expression for the area of this triangle? ($A = {}^1/_2ab$) The expression $a + b$ is significant here. What is the algebraic expansion of $(a + b)^2$? How can we use the Pythagorean Relation here? ($a^2 + b^2 = 16$)

RESOURCE CORRELATION

M10:	pp. 52–58
HM10:	pp. 54–57
MPP10:	pp. 26–33 (Note: Do not include rational exponents.)

Mathématiques 10: pp. 26–33
HM10 TRM D/A Master 3-1

CONCEPT 3

Students will be expected to demonstrate an understanding that calculations and operations on polynomials depend on the application of the laws of operations for numbers and powers.

Students will be expected to:

3.1 add and subtract polynomials.

COMMENTS

Addition and subtraction of polynomials are no longer included in the junior high curriculum. Many students will not have experienced polynomials other than through their work with solving equations. If students have done some work with addition and subtraction of polynomials, then time should be spent on visualizing a polynomial and how addition/subtraction is a generalization of the addition/subtraction properties that were learned in earlier grades. Time should be spent on the connection between previous work and this work so that polynomials are not treated as completely different from number operations.

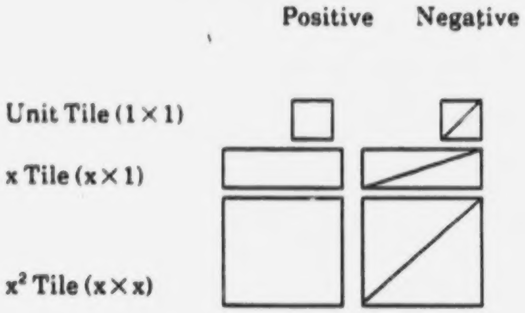
Manipulatives: The introductory study of polynomials provides an ideal opportunity for teachers to provide instruction geared to the students' cognitive levels. Students in this course will be at a concrete stage of development with respect to polynomials, and can benefit from thoughtful development of concepts using concrete manipulatives. However, the aim is not just to acquire facility with manipulatives, but rather to use them to progress from the concrete, through a transitional (pictorial) stage and finally to a formal understanding of the abstract concepts and processes involved in the study of polynomials.

The purpose of manipulatives is to provide students with a concrete example of an abstract concept of process. Work with manipulatives does not replace the development of a mathematical algorithm, but development of the algorithm should follow introduction of the concept using manipulatives. There is no use in a teacher demonstrating the concept to a class using manipulatives or in allowing students to work with manipulatives after presentation of a formal algorithm. Students at a concrete level must be allowed to internalize the concrete representation of the concept and must be given the opportunity to work with manipulative son their own for these methods to be effective.

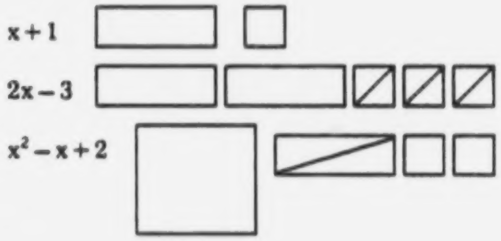
Emphasis should be on understanding, not on long or difficult computations with complicated polynomials. Questions should be no more difficult than those involving 2 trinomials of maximum degree 3 and 3 variables. Encourage students to check their calculations by substitution.

PROCESS/PROBLEM - SOLVING CONTEXT

Activity: Have students cut, from construction paper, algebraic tiles representing x^2, x and 1. (See template in Appendix A.) Mark the tiles on one side with a diagonal to represent negative. The tiles will appear as follows:



Using these tiles, polynomials can be represented as follows:



Have students practice representing various polynomials using these tiles.

These concrete representations of polynomials can be used to do additions and subtractions. Students will be able to see that addition and subtraction operations are possible only on like terms.

See Algebra Tiles for the Overhead Projector, pp. 13–16, for more details and suggestions.

RESOURCE CORRELATION

M10: pp. 72–74, 79–81
HM10: pp. 84–85
MPP10: pp. 66–69
Mathématiques 10: pp. 66–69
HM10 TRM D/A Master 4–1

CONCEPT 3

Students will be expected to demonstrate an understanding that calculations and operations on polynomials depend on the application of the laws of operations for numbers and powers.

Students will be expected to:

3.2 multiply polynomials by monomials, and binomials by binomials

3.3 recognize the expansion of the square of a binomial and the product of conjugate binomials as special cases.

COMMENTS

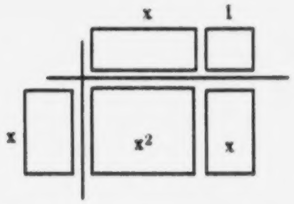
This concept is no longer included in the junior high curriculum. Many students will not have experienced polynomials other than through solving equations. If students have done some work with the multiplication of polynomials, then time should be spent on visualizing a polynomial and on how multiplication is a generalization of the multiplication properties that were learned in earlier grades. Time should be spent on the connection between previous work and this work so that polynomials are not treated as completely different from number operations.

Remind students of the distributive law. Emphasis should be on the process, not on long or complex calculations. Do not go beyond the difficulty level of the exercises in the textbooks.

Recognizing the square of a binomial $[(a+b)^2 = a^2 + 2ab = b^2]$ and the product of conjugate binomials $[(a+b)(a-b) = a^2 - b^2]$ should be done within the multiplication of other binomials and recognized as special cases only after the discussion of the expansion of binomials.

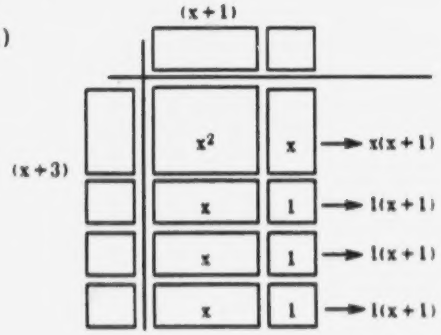
PROCESS/PROBLEM-SOLVING CONTEXT

Activity: When multiplying using manipulatives, set up a table and compare the multiplication to finding the area of a rectangle. Remember to multiply all terms on the side by all terms on the top.



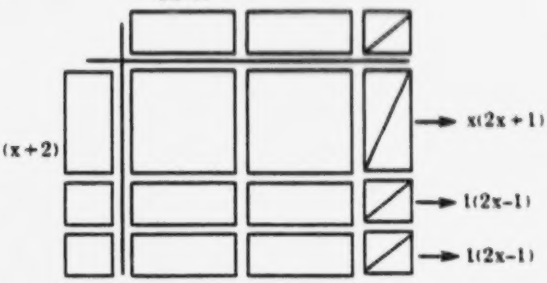
The product is denoted by the tiles inside the box, in this case $x^2 + x$.

Examples:
$(x+3)(x+1)$



The tiles inside the box show that the product is $x^2 + 4x + 3$. Notice that the product is a rectangle. This is important when we factor.

$(x+2)(2x-1)$



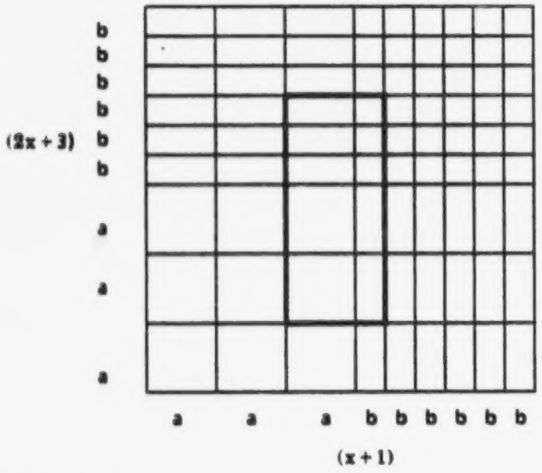
The tiles inside the box show that the product is $2x^2 + 4x - x - 2 = 2x^2 + 3x - 2$.

Is the product always a rectangle? What would $(x+3)(x+3)$ look like? Develop this with the students.

For more information see: Algebra Tiles for the Overhead Projector, pp. 17–23.

Activity: Another approach to teaching multiplication of binomials is to use binomial grids. The idea is to provide students with a sheet with a number of grids, formed by geometrically interpreting the multiplication of two binomials. (See template in Appendix A.) Particular multiplications can be accomplished by outlining a rectangular region on the grid with sides which are the lengths of the given binomials. The area of the rectangle will be the product of the two binomials.

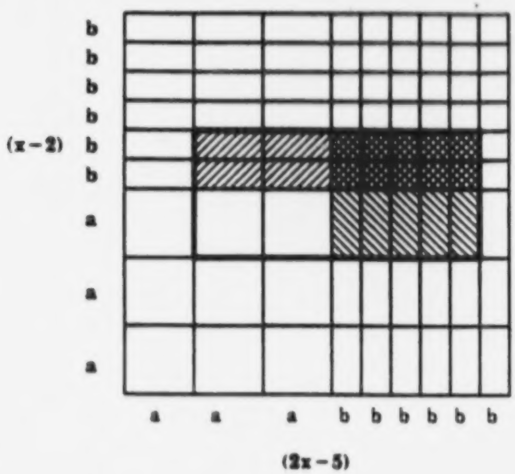
For example, the product of $(2x+3)(x+1)$ would be found as follows:



Product is $2x^2 + 5x + 3$

58

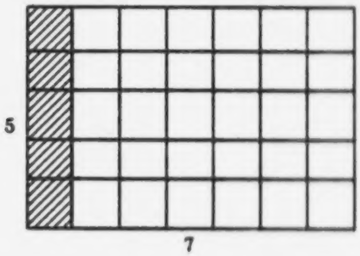
To find products which involve negatives, mark the areas derived from negative terms in some way. Those areas which are marked once only will be interpreted as negative in the final product. Those which are marked twice will be interpreted as positive. For example, the product of $(x-2)(2x-5)$ would be found as follows:



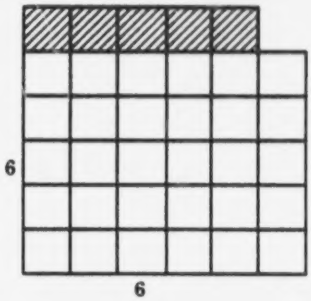
Product is $2x^2-9x+10$

Adapted with permission from "The Binomial Grid," Mathematics Teacher, May 1986, copyright by the National Council of Teachers of Mathematics.

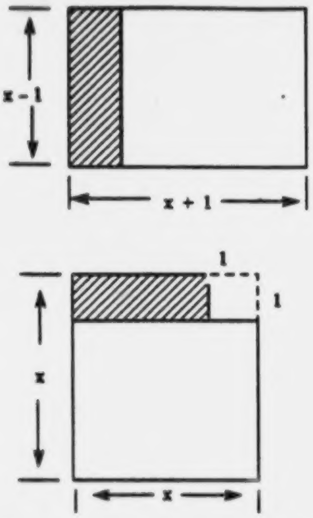
Here is another way of pictorially introducing the special case $(x-1)(x+1)=x^2-1$. Start with an example, for instance, 5 × 7, represented as a rectangle:



The 5 by 7 rectangle has thirty-five squares. If the five squares on the left are shifted to the top of the figure, the large figure can be described as "almost" a square, containing one less unit square than the thirty-six in a 6 by 6 square.



This geometric representation can be generalized to the case where, for all real numbers $x > 1$, the following holds:



$(x - 1)(x + 1) = x^2 - 1$

Problem: Consider sets of three consecutive integers, such as (3, 4, 5), (7, 8, 9), and (9, 10, 11). Ask students to multiply the first and third integers, and square the middle integer. What is the relationship between the two products? Does it work for all integers? What about (21, 22, 23), (–1, 0, 1), (–2, 1, 0), (–7, –6, –5)? Does it only work for integers? Have students investigate with numbers such as ($5\frac{1}{4}$, $6\frac{1}{4}$, $7\frac{1}{4}$) and (2.41, 3.41, 4.41).

Discuss with the class how the relationship between the three numbers might be described algebraically. What expressions develop when the products are considered?

Two possibilities are: $(x, x+1, x+2)$ and $(x-1, x, x+1)$.

The products in the first case are: $x(x+2) = x^2 + 2x$ and $(x+1)^2 = x^2 + 2x + 1$.

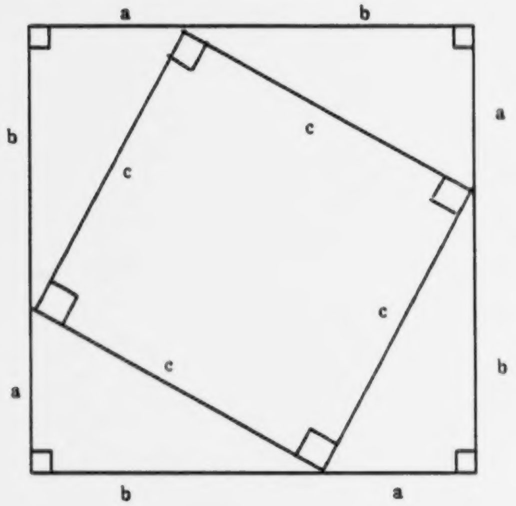
In the second case, they are: $(x-1)(x+1) = x^2-1$ and x^2.

In both cases it now becomes clear why the two products differ by one. Stress the value of algebra in generalizing.

For more **Problems** see: Problem Solving in Mathematics: Focus for the Future, p. 63, #22.

Problem 3.2: Show that $a^2 + b^2 = c^2$, given that the area of the four triangles plus area of the small square = area of the large square.

(An algebraic proof of the Pythagorean Relationship.)



ELECTIVE SUGGESTIONS

Activity: Explore how an understanding of binomial multiplication ($a = 1$) can be applied to mental calculations, such as 14 × 17. Can 14 × 17 be written as $(10 + 4) \times (10 + 7)$?

$$(10+4)(10+7) = (100+70+40+28) = (100+110+28)$$

What is the pattern of products which are multiples of 10? Does this always work with numbers?

For extension of this, see the **Elective Suggestions** in Factoring Polynomials, page 58.

Problem: 3.2 It has been observed that 39 × 62 = 93 × 26. That is, when you reverse the order of the digits in 39 and 62, the product of the reversed numbers is the same as the product of the two original numbers. Do any other pairs of two two–digit numbers have this property?

Let the students guess and test, then suggest that they can generalize their examination by expressing each two–digit number as a binomial, making use of place value.

Answer: Yes there are other two–digit numbers which have this property. The key is that the product of the tens digits (3 × 6) equals the product of the units digits (9 × 2). This can be derived algebraically.

Adapted with permission from "On the Meaning of Variable," in Mathematics Teacher, September 1988, copyright by the National Council of Teachers of Mathematics.

Problem: (3, 4, 5) and (5, 12, 13) are Pythagorean triples (a, b, c) because they fulfil the Pythagorean Relationship, $a^2 + b^2 = c^2$.

Now define b and c in terms of a in the following manner:

Let $b = \frac{a^2-1}{2}$, $c = \frac{a^2+1}{2}$

Verify with examples that the triples (a, b, c) so formed will always form Pythagorean triples. Do (3, 4, 5) and (5, 12, 13) fit this pattern? Prove this result algebraically.

See: M10, TRB, Teaching Notes, p. 15, section 3–4. for another polynomial multiplication activity.

RESOURCE CORRELATION

M10: pp. 76–78, 84–92
HM10: pp. 86–92
MPP10: pp. 58–61, 72–75
Mathématiques 10: pp. 58–61, 72–75
HM10 TRM D/A Master 4–2, 4–3, 4–4
MPP10 ASTRM: pp. 6–7

CONCEPT 3

Students will be expected to demonstrate an understanding that calculations and operations on polynomials depend on the application of the laws of operations for numbers and powers.

Students will be expected to:

3.4 divide polynomials by monomials

3.5 divide polynomials by binomials of the form $ax+b$, $a, b \in I$.

COMMENTS

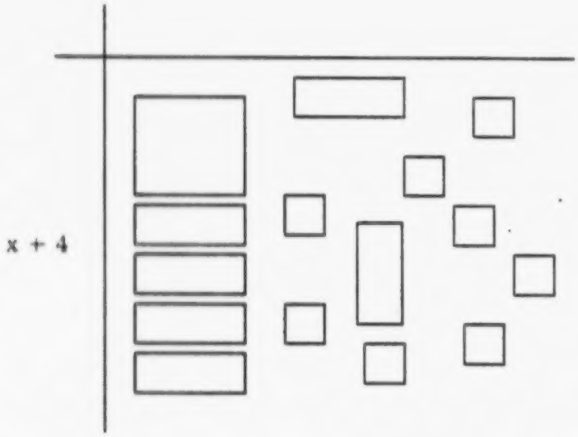
Students can be taught to write their results in the form of $P = D \times Q + R$, where P is the divident, D the divisor, Q the quotient and R the remainder.

The introduction to this should begin with dividing a polynomial by a whole number (e.g., $(3x^2+6x+9) \div 3$ and $(2x^2+4) \div 2$). This approach is concrete in the sense that students can use the algebra tiles to separate the polynomial into three groups. This is particularly useful as students have not yet discussed factoring.

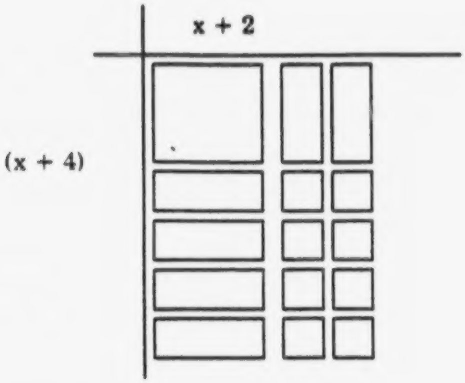
PROCESS/PROBLEM-SOLVING CONTEXT

Activity: A concrete approach (based on the experience of multiplying), to introducing division of a polynomial by a binomial is as follows: e.g., $(x^2+6x+8) \div (x+4)$

An x^2 tile, 6–x tiles and 8– one tiles must be arranged in a rectangle where $x+4$ is one dimension.



Arranging the remaining tiles; the rectangle looks like:



so: $(x^2 + 6x + 8) \div (x + 4) = (x + 2)$

See Algebra Tiles for the Overhead Projector, pp. 25–27, for a more detailed description.

RESOURCE CORRELATION

M10:	pp. 76–78
HM10:	pp. 93, 104–105
MPP10:	pp. 62–65
Mathématiques 10:	pp. 62–65
MPP10 ASTRM:	pp. 8–9

64

CONCEPT 1

Students will be expected to demonstrate an understanding that polynomials can be factored.

Students will be expected to:

1.1 factor polynomials that have a monomial common factor
1.2 factor polynomials that have a binomial common factor.

COMMENTS

This concept is no longer included in the junior high curriculum, and will be new to students. It may be taught in conjunction with multiplying polynomials by monomials and binomials.

Remind students of factoring numbers into the product of prime factors. Extend this to factoring monomials. Remind students of how to find the GCF of numbers and introduce "common factoring" as writing a polynomial as the product of the greatest common factor of its terms and a quotient.

For the purposes of this concept, the Greatest Common Factor will be the one which is made up of the GCF of the numeric coefficients and the literal coefficient(s) to the greatest exponent(s) which will divide into each term.

Use examples and assign questions which are of the same difficulty level students are liable to encounter in further study. Remind them that it is possible to check their work by reversing the procedure.

Examples:
$2x^2 + 6x = 2x(x-3)$
$2x(x+3) + 5(x+3) = (x+3)(2x+5)$

This is an appropriate time to introduce students to factoring by grouping, e.g., $x^3 + 2x^2 + 3x + 6$.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students do several multiplications involving monomials and polynomials and then try to establish a procedure to "undo" the multiplication. Have students write in their own words the rules they develop.

Problem: Why is $n^2 + n$, $n \in I$ always even? This is a good example of a problem that students can solve by working together.

Problem: 1.2 Take any three-digit number and write it down twice to make a six-digit number. Show that this number is exactly divisible by 7 or 11 or 13. Why?

65

Discussion:

Consider the number **abc**. Writing it twice produces **abcabc**. In expanded place–value notation, the number becomes:

$\text{abcabc} = (a \times 10^5) + (b \times 10^4) + (c \times 10^3) + (a \times 10^2) + (b \times 10) + c = 10^2a(10^3+1) + 10b(10^3+1) + c(10^3+1)$

Written in this way, what do we notice about each of the terms? What are the divisors of 1001?

TECHNOLOGY INTEGRATION

Algebra Drill and Practice II (Conduit), "Factoring Binomials" option, and Factoring Algebraic Expressions (Mindscape), "Common Factor" option, both provide drill and remedial opportunities in factoring the common factor in binomial expressions.

ELECTIVE SUGGESTIONS

Binomial Multiplication and Factoring/Mental Computation: The following discussion is an extension of the **Elective Suggestions** of Operations on Polynomials, Skill 3.2, on page 53. It is placed here because some factoring by group is involved.

Have students multiply $(10a+b)(10a+c)$. (b) Extend and simplify in the special case where $b+c=10$. (c) Then have the students explore and describe how this result can be used to help simplify the mental calculation of such products as 24×26, 61×69, etc. Will it work for 48×92, or 37×38? For 75^2? For 3.4×3.6? Why or why not?

a) $(10a + b)(10a + c) = 100a^2 + 10ab + 10ac + bc$
$\qquad = 100a^2 + 10a(b + c) + bc$

b) when $b + c = 10$
$100a^2 + 10a(b + c) + bc = 100a^2 + (10a)(10) + bc$
$= 100a^2 + 100a + bc = 100a(a + 1) + bc$

c) Using the first product, 24×26: $a = 2, b = 4, c = 6$
$100a(a + 1) + bc = 100(2)(3) + 24 = 600 + 24 = 624$

One description of this Mental Computation Pattern is: Multiply the tens digit (2) by its successor (3), and append to this product ($2 \times 3 = 6$) the product of the units digits ($4 \times 6 = 24$.) The answer to the multiplication 24×26 is 624.

> Multiply the first digit (2) by its successor (3), [product = 6] and append the product of the units digits ($4 \times 6 = 24$). The answer to the multiplication 24×26 is 624.
>
> Encourage students to describe orally or in writing what they believe the pattern is.

What patterns can be described for mental computation when $b+c \neq 10$?

What generalizations are evident in the product of numbers of the type 3.4×3.6?

RESOURCE CORRELATION

M10: pp. 93–96
HM10: pp. 93–95
MPP10: pp. 76–80
Mathématiques 10: pp. 76–80
HM10 TRM D/A Master 4–5

66

CONCEPT 1

Students will be expected to demonstrate an understanding that polynomials can be factored.

Students will be expected to:

1.3 factor trinomials of the form $ax^2 + bx + c$; $a, b, c \in I$
1.4 factor perfect square trinomials
1.5 factor difference of squares polynomials.

COMMENTS

It is important that students understand the relationship between particular numerical products and the algebraic generalization. These activities also provide opportunities to reinforce place value.

This concept is no longer included in the junior high curriculum, and will be new to students.

Emphasis should be on their understanding of the process, not on difficult or complicated examples. Limit assignments to the difficulty level in the textbook.

Refer to the multiplication of binomials and use the same manipulatives. See Algebra Tiles for the Overhead Projector, pp. 25–31.

Remind students that $-1 + 1 = 0$ and that ⧄+☐ = 0 as well. This concept will be used to factor trinomials where the constant term is negative.

Make sure that students have an opportunity to develop factoring skills on their own. For example, before any factoring algorithm is taught, have students try to factor by using only what they know about the multiplication of binomials, i.e., the product of the numerical coefficients of the degree one terms is the numerical coefficient of the degree two term and that the product of the constants is the constant term of the polynomial. A thorough understanding of these facts will provide a better understanding of any algorithm which is developed.

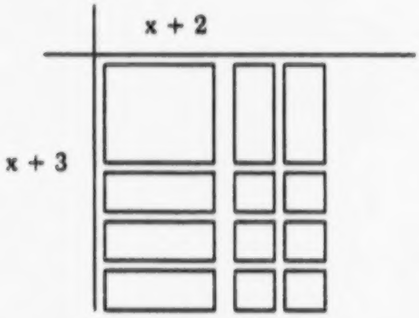
PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Students can be asked to arrange manipulatives representing polynomials into rectangles in order to find the dimensions (factors).

Examples:

Factor $x^2 + 5x + 6$

Arrange the tiles within a table like the one used for multiplication. There is only one way in which they can be arranged into a rectangle.

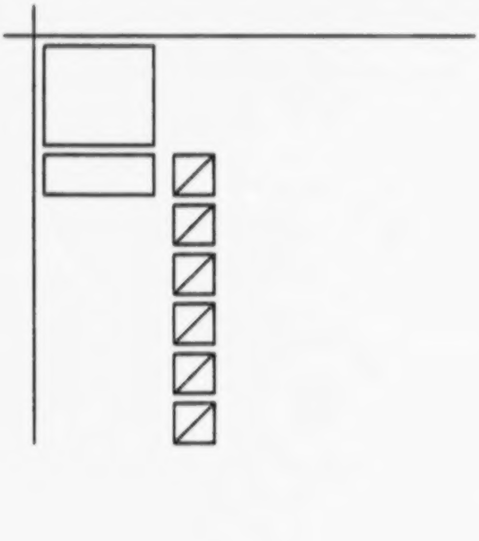


$\therefore\ x^2 + 5x + 6 = (x + 3)(x + 2)$

Factor $x^2 + x - 6$:

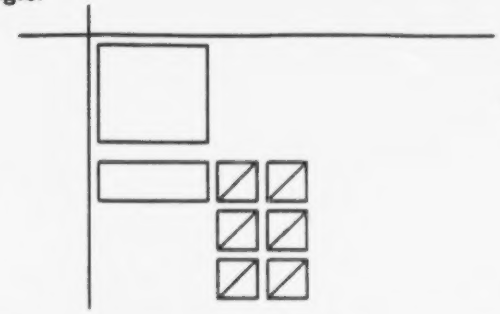
One x^2 tile, one x tile, and 6 negative unit tiles must be formed into a single rectangle. This rectangle is impossible to form. However, adding the same number of positive and negative x tiles will allow a rectangle to be formed.

Form the units tiles into a rectangle in the lower right-hand corner. There are only two alternatives for this, either a 6 by 1 rectangle:

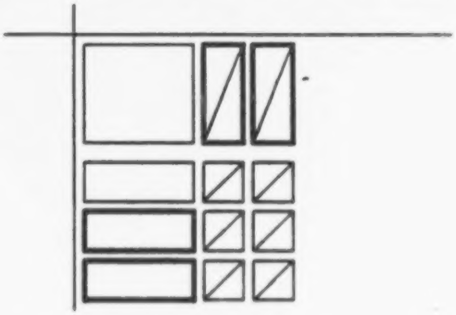


68

or a 3 by 2 rectangle:



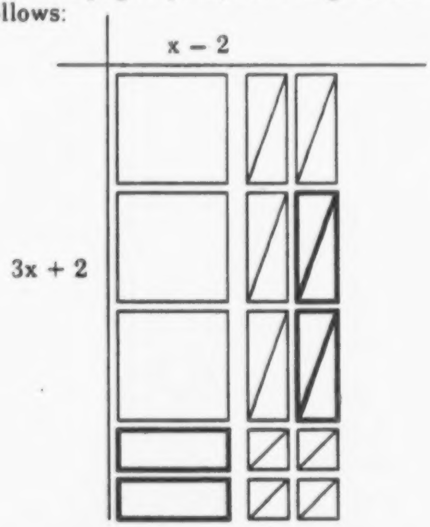
Only the 3 by 2 rectangle will leave the same number of spaces (2) to be filled by "x" tiles along both the top and the side. By placing two positive x tiles along the side (next to the original x tile), and then providing 2 negative x tiles along the top, the overall polynomial doesn't change but the rectangle is completed.



$\therefore\ x^2 + x - 6 = (x + 3)(x - 2)$

Factor $3x^2 - 4x - 4$

Again, the tiles will not form a rectangle unless some additional tiles are introduced. The page layout, including the additional tiles (bold) is as follows:



Adapted with permission from "Finding Factors Physically," in Activities for Implementing Curricular Themes from the 'Agenda for Action', copyright 1986 by the National Council of Teachers of Mathematics. See also Algebra Tiles for the Overhead Projector, pp. 25–31.

Activity: Factoring trinomials can also be examined using the Binomial Grid (as described for Operations on Polynomials, Concept 3, pp. 50–51). The above discussion can be used with the Binomial Grid.

Problem: 1.4 If $a^2 + b^2 = 15$ and $ab = 5$, find the value of $(a+b)$.

Answer: One solution is:

$$\begin{aligned} (a+b)^2 &= a^2 + 2ab + b^2 \\ &= (a^2 + b^2) + 2ab \\ &= 15 + 2(5) \\ &= 25 \\ \therefore a + b &= 5 \end{aligned}$$

Activity: 1.5 see HM10 TRM, pp. 39–40 for a numerical and pictorial introductory activity to difference of squares factoring.

Problem: 1.5 Compute *n* if $(10^{12} + 25)^2 - (10^{12} - 25)^2 = 10^n$.

Answer: One solution is:

$$\begin{aligned} &(10^{12} + 25)^2 - (10^{12} - 25)^2 \\ &= [(10^{12} + 25) + (10^{12} - 25)][(10^{12} + 25) - (10^{12} - 25)] \\ &= [(2)(10^{12})](50) \\ &= (100)(10^{12}) \\ &= 10^{14} \end{aligned}$$

$$\therefore n = 14$$

TECHNOLOGY INTEGRATION

Software:
Algebra Drill and Practice II (Conduit), "Factoring Trinomials" option, and
Solving Quadratic Equations and Factoring Algebraic Expressions (both through Mindscape) provide practice in factoring.

70

ELECTIVE SUGGESTIONS

Activity: 1.3 Alternative Approaches to Factoring Trinomials of the Form ax^2+bx+c.

a) Consider the theorem which states:

Given a trinomial ax^2+bx+c and integers m and n such that $m+n=b$ and $mn=ac$, then

$$ax^2+bx+c=\frac{(ax+m)(ax+n)}{a}$$

Encourage students to verify this statement with examples. For example, factor the expression $2x^2-7x-15$. In this case, $b=-7$ and $ac=(2)(-15)=-30$. The procedure is to look for two numbers, the sum and product of which are respectively -7 and -30. These two numbers (m and n) are $+3$ and -10.

The theorem tells us that we can now directly write down the factored expression as:

$$2x^2-7x-15=\frac{(2x+3)(2x-10)}{2}$$

In this case, one of the binomials contains a common factor, 2. Simplifying, then, we have:

$$2x^2-7x-15=\frac{(2x+3)(2x-10)}{2}=(2x+3)(x-5)$$

As an activity, encourage the students to prove this theorem.

b) A second, related, theorem states:

Given a trinomial ax^2+bx+c and integers m and n such that $m+n=b$ and $mn=ac$, then

$$ax^2+bx+c=\frac{(mx+c)(nx+c)}{c}$$

Have the students also verify this theorem with examples. Using the previous example, $2x^2-7x-15$, where $m=+3$ and $n=-10$, and substituting according to the theorem, produces:

$$2x^2-7x-15=\frac{(3x-15)(-10x-15)}{-15}$$

In this case each binomial contains a common factor. Simplifying gives:

$$2x^2-7x-15=\frac{(3)(x-5)(-5)(2x+3)}{-15}=(x-5)(2x+3)$$

As an activity, encourage the students to prove this theorem.

(a) and (b) adapted with permission from "Factoring Trinomials," Mathematics Teacher, January 1986, copyright by the National Council of Teachers of Mathematics.

c) Another approach makes use of a "box" or "cell" technique.

A multiplication example will help introduce the approach:

Consider $(2x-1)(x+5)$

Arrange the box as shown and multiply cell-by-cell; then combine like terms:

	x	$+5$
$2x$	① $2x^2$	② $10x$
-1	③ $-x$	④ -5

Therefore $(2x-1)(x+5) = 2x^2 + 10x - x - 5 = 2x^2 + 9x - 5$.

Encourage the students to examine the process and products.

Notice that:
1) the products of the diagonal elements are the same ($-10x^2$);
2) cell ① holds the x^2 term of the trinomial,
3) the sum of cells ② and ③ yields the term in x, and
4) cell ④ holds the constant.

Factoring:
Consider the previous example, $2x^2-7x-15$. Set up the box and place the x^2 and constant terms as shown:

	① $2x^2$	②
	③	④ -15

Cells ② and ③ contain terms in x, the sum of which is $-7x$ and the product of which is $-30x^2$. These terms are $+3x$ and $-10x$. Place them in cells ② and ③, in no particular order.

	① $2x^2$	② $+3x$
	③ $-10x$	④ -15

Determine the common factors of each row and column and place them as shown:

	$2x$	$+3$
x	$2x^2$	$+3x$
-5	$-10x$	-15

Therefore $2x^2-7x-15=(2x+3)(x-5)$.

Note: Remove any common factor in the trinomial before proceeding.

How does this approach compare to other strategies for factoring trinomials, such as factoring by decomposition or using the tiles and binomial grid?

Adapted with permission from "Box Technique for Factoring," Mathematics Teacher, February 1987, copyright by the National Council of Teachers of Mathematics.

Activity: Have students examine and write about the "Investigate" problems at the bottom of page 100, in M10. Apply this understanding of perfect square trinomials to problems 6 and 11 on pages 82 and 83 in MPP10. Relate this examination to that on page 102 of HM10.

Activity: See M10, TRB, Teaching Notes, p. 16, section 3–8, for another approach to factoring trinomials.

Activity: **1.5** Explore the patterns present in the products of two numbers whose sum is 20.

$9\times11=99=100-1=$ ____________________
$8\times12=96=100-4=$ ____________________
$7\times13=91=100-9=$ ____________________, etc.

How can the knowledge of factoring a difference of squares be applied here? Can the pattern be generalized to the product of other pairs of numbers?

Problem: **1.5** Express the number 96 as the difference of two squares in four different ways.

Answer: 10^2-2^2
11^2-5^2
14^2-10^2
25^2-23^2

This can be done in a guess–and–check manner, but suggest that the students look for patterns. One pattern that can be suggested if the students do not discover it for themselves is to consider the pairs of factors of 96. How can $a^2-b^2=(a+b)(a-b)$ be related to these pairs? Does it work for all such pairs of factors of 96?

The factor pairs of 96 are (12,8), (16,6), (24,4), (32,3) and (48,2). Four of these pairs of numbers can be expressed as the sum and difference of two other positive integers. For example: (12,8) = (10 + 2, 10–2).

Is there a way to determine these a and b values? Which pair of factors cannot be treated in this way (a, b $\in$ I)?

RESOURCE CORRELATION

M10:	pp. 98–106
HM10:	pp. 96–102
MPP10:	pp. 81-91
Mathématiques 10:	pp. 81-91
HM10 TRM D/A Master 4–6, 4–7	

CONCEPT 1

Students will be expected to demonstrate an understanding that polynomials can be factored.

Students will be expected to:

1.6 factor polynomials that require a combination of methods in order to be fully factored
    1.6.1 solve problems with factorable polynomials.

COMMENTS

Remind students that factoring is a way to write polynomials in a way which may be more understandable or appropriate. Encourage them to examine polynomials for common factors and general forms.

It is not particularly useful to create complex polynomials which can be factored only with difficulty. Provide exercises involving polynomials students might reasonably see at some future time.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Evaluating polynomials by first factoring and grouping into a nested form represents a useful application of factoring, and an opportunity to make use of the calculator.

For example: Evaluate
$x^4 + 4x^3 - 5x^2 - 12$ for $x = 4$.
In nested form, the expression becomes:
$x^4 + 4x^3 - 5x^2 - 12 = x\{x[x(x+4)-5]\}-12$
Check your answer using a calculator.

Problem:
If $m^6n^2 + 2m^4n^2 + m^2n^2 = 25m^2n^4$, $m, n \neq 0$, then $m^4 = ?$

Answer:

$m^6n^2 + 2m^4n^2 + m^2n^2 = 25m^2n^4$
$m^2n^2[m^4 + 2m^2 + 1] = m^2n^2(25n^2)$
$[m^2 + 1]^2 = 25n^2$
$m^2 + 1 = \pm 5n.$
Therefore:
$m^2 = 5n - 1$ or $m^2 = -5n - 1$
$m^4 = (5n - 1)^2$ $m^4 = (-5n - 1)^2$
$m^4 = 25n^2 - 10n + 1$ $= 25n^2 + 10n + 1$

For more **Problems** see: Problem Solving in Mathematics: Focus on the Future, p. 63, #24, 25.

TECHNOLOGY INTEGRATION

Software:
Factoring Algebraic Expressions (Mindscape) "Mixed" option, provides drill and practice.

RESOURCE CORRELATION

M10:	p. 108
HM10:	p. 103
MPP10:	p. 92
Mathématiques 10:	p. 92
HM10 TRM D/A Master 4–8	

CONCEPT 2

Students will be expected to demonstrate an understanding that the values of the variables which make a polynomial equal to zero can be found by factoring.

Students will be expected to:

- **2.1 find the rational zeros of an integral quadratic polynomial in one variable by factoring**
- **2.2 solve quadratic equations with integral coefficients in one variable by factoring**
  - **2.2.1 solve problems by factoring which can be represented by quadratic equations.**

COMMENTS

Zeros are the values of the variable which give the polynomial a value of 0. As a way of finding the zeros of a polynomial, introduce the following:
If $ab = 0$, then $a = 0$ or $b = 0$.

PROCESS/PROBLEM-SOLVING CONTEXT

Problem:
Find all values of x which satisfy the following:

$$(x^2 - 2x + 1)^{x^2 - 4x + 3} = 1$$

Possible Solutions:

a) $x^2 - 4x + 3 = 0$
$\therefore (x - 3)(x - 1) = 0$
$x = 3$ or $x = 1$
(When $x = 3$, have 4^0
When $x = 1$, have 0^0
$\therefore x = 1$ not a solution)

b) $x^2 - 2x + 1 = 1$
$\therefore x^2 - 2x = 0$
$\therefore x(x - 2) = 0$
$x = 0$ or $x = 2$
(When $x = 0$, have 1^3
When $x = 2$, have 1^{-1})
Solutions: $x = 0, 2, 3$

c) $x^2 - 2x + 1 = -1$ and
$x^2 - 4x + 3$ is even

d) Other possibilities?

Problem:
A garden has an area of 240 m^2. What are its dimensions if the length is 8 m more than the width? Answer: 20m by 12m.

Problem: The sum of twice a fraction and half that fraction, times that fraction equals that fraction. What is that fraction?

Answer: 0, or $^2/_5$

Problem: What positive number is equal to its square added to its opposite?

Answer: 2.

Note: This problem can be solved numerically with guess-and-test. It can also be solved by setting up a simple, factorable quadratic. The value of the latter situation is that it represents the general case, showing that the only solution is 2. Does the condition that the number is positive limit the number of solutions?

Problem: When I open my mathematics book, there are two pages that face me. If the product of the two pages is 342, what are the two page numbers?

For more **Problems** see: Problem Solving in Mathematics: Focus for the Future, pp. 65, 66, #10, 13. (Note: These problems involve rational expressions.)

Problem Solving Challenge for Mathematics, p. 49, #1, 2 (involves rational expression).

TECHNOLOGY INTEGRATION

Software:
Algebra Drill and Practice II (Conduit), "Roots of Quadratic Equations" option, and
Solving Quadratic Equations (Mindscape) provide practice in solving quadratic equations by factoring.
Use a graphing program such as Equation Math (MECC), Green Globs and Graphing Equations (Sunburst) or MasterGrapher (Addison-Wesley) to explore what happens to various factorable polynomials if one of the numerical coefficients is changed just a little. The polynomial may now no longer be factorable but does the factorable polynomia continue to provide a reasonable estimate of the roots?

See HM10, p. 110, "Computer Application" for a program to find the zeros of a polynomial.

ELECTIVE SUGGESTIONS

Problem: If x and y are consecutive odd integers, $x < y$. When $3x^2-2y = 129$, find the value of $x + y$.

Answer:

$y = x + 2$

$3x^2-2y = 129$

$\therefore 3x^2-2(x+2) = 129$

$3x^2-2x-133 = 0$

$(3x+19)(x-7) = 0$

$\therefore x = 7, y = 9$, and $x + y = 16$.

RESOURCE CORRELATION

M10:	pp. 180–184
HM10:	pp. 106–110
MPP10:	pp. 93–96
Mathématiques 10:	pp. 93–96

CONCEPT 1

Students will be expected to demonstrate an understanding that equations can be used to algebraically represent situations and relationships that involve equalities.

Students will be expected to:

- **1.1 solve and verify the solution of any first degree equation in one variable by applying formal algebraic procedures**
    - **1.1.1 represent problems with equations and to solve the problems through the solution of the equations**
- **1.2 solve and verify the solution of simple quadratic and radical equations which can be reduced easily.**

COMMENTS

Students have spent a great deal of time solving linear equations that involve all forms of rationals. Equations of the form: $x + a = b$, $ax = b$, $ax + b = c$, $ax + bx = c$, $x/_a = b/_c$, $ax + b = cx$, $a(x + b) = c$, and $ax + b = cx + d$ have been studied in junior high. Equations of the form $a/_x = b$ have not been solved.

It is important to stress what an equation is: it is a statement of equality that may be true, may be false or may depend on the values of the variable. An example of an equation that is true is $3 + 6 = 9$. An example of an equation that is false is $6 + 9 = 17$ and an example of an equation that depends on the values of the variable is $2 + x = 18$. This then helps to reinforce the concept of variable. Although x represents a number, it could be any number (so it varies) and in the process of solving the equation, a particular value for x is determined.

Students should be given ample opportunity to translate English into algebraic expressions and should have the opportunity to solve problems for which they can write equations. Solving problems by type, e.g., coin, mixture, age, distance, should be avoided. Students should not experience these as "distinct" problems and should not believe that there is only one way to solve the problem.

Stress that quadratic equations will have two solutions and that radical equations may yield roots which will not solve the original equation. The verification of both radical and quadratic equations is important for this reason. Limiting examples:

1) $x^2 = 20$

2) $(2x + 5)^2 = 25$

3) $\sqrt{x} = 4$

4) $\sqrt{3x-5} = 1$

78

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: 1.1 To encourage flexible thinking in problem solving, have students solve equations for a specified algebraic expression rather than just for the variable. For example, solve $3(x+4)-8=13$ for $x+4$ and solve $x+7=10$ for $5x$, etc. This allows students to practice their solving equation skills, yet takes it one step farther to a more abstract form of problem. Students may solve these problems by first solving for the variable and then substituting into the given expression. They can be led to solving for the expression by the use of formula manipulation, for example, solve $P=2(l+w)$ for $l+w$.

Problem: 1.1 Solve for: $^{(4x-7)}/_{3}$
if $6(4x-7)=18$

Many students will want to solve for x, and then substitute in $^{(4x-7)}/_{3}$

How many considered the following:
$6(4x-7)=18$

$^{6(4x-7)}/_{18}={}^{18}/_{18}$

$^{(4x-7)}/_{3}=1$

Provide other similar opportunities for the students.

Adapted with permission from "Flexibility and Algebraic Problem Solving," Mathematics Teacher, April 1987, copyright by the National Council of Teachers of Mathematics.

Problem: 1.1 The two equations below both have 2 as a solution:

$^{(7p-2)}/_{4}=3 \quad 3(17n-30)=12$

Create two equations that have the same form as these but have 9 as a solution. This emphasizes the strategy of working backwards when solving problems.

Reprinted with permission from "Activities: Teaching Problem-Solving Skills," in Activities for Implementing Curricular Themes from Agenda for Action, copyright 1986 by the National Council of Teachers of Mathematics.

Problem: 1.1 Let a, b, c, and d be non-zero integers between –10 and 10. What are the largest and smallest solutions possible for $ax+b=cx+d$? This activity not only involves students in solving problems, but it will reinforce estimation and assist students in seeing patterns that may form in reaching this solution.

Answer: largest solution: $x=20$; smallest solution: $x=-20$.

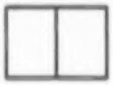
Reprinted with permission from: "Reasons for Estimating", in Estimation and Mental Computation, copyright 1986 by the National Council of Teachers of Mathematics.

Activity: 1.2 **Have students use equivalencies to demonstrate that squaring or taking the square root of both sides of an equation are legitimate operations.**

e.g., $5 = {}^{10}/_{2}$
$5^2 = ({}^{10}/_{2})^2$
$25 = {}^{100}/_{4}$

Problem: 1.2 Metric paper sizes have the property that a sheet of paper cut in half gives two smaller sheets of the same shape as the original sheet. Find the ratio of the sides of the sheets.

Answer: ${}^{l}/_{w} = \sqrt{2}/_{1}$



Have some of the students verify this fact with various sizes of metric paper. For more information, see "Metric Paper," in Activities for Junior High School and Middle School Mathematics.

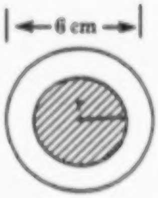
Problem: This activity will help students develop a sense of "reasonableness" with their algebra:

A cow and a rooster want to play on a teeter-totter. The rooster says its impossible, but the cow says that it will work and that he can prove it, since he's had a little algebra. The following argument is presented:

Let C = weight of cow
Let R = weight of rooster
then some W exists so that
$C = R + W$
multiply both sides by $C-R$.
$C(C-R) = (R+W)(C-R)$
so $C^2-CR = RC-R^2+WC-WR$
factor $C(C-R-W) = R(C-R-W)$
divide both sides by $(C-R-W)$ and $C = R$

$\therefore$ the weight of the cow equals the weight of the rooster and there won't be a problem with the rooster and the cow playing on the teeter-totter. Is the proof correct? Is the conclusion correct? Does the conclusion make sense? Where is the fault in the "proof"?

Problem: 1.2 This problem provides an opportunity to review area. Find r such that the areas of the washer and the hole are equal.



Area of the hole: πr^2
Area of the washer: $9\pi - \pi r^2$
Possible solutions:
$\pi r^2 = 9\pi - \pi r^2$
$2r^2 = 9$
$r = 3/\sqrt{2}$

or $r \approx 2.12$

For more **Problems** see: Problem Solving in Mathematics: Focus for the Future, pp. 65, 66, #2, 19; p. 69, #25.

Problem Solving Challenge for Mathematics, pp. 49, 50, #5, 20.

TECHNOLOGY INTEGRATION

For an example of using a spreadsheet approach to solving equations, see "Mathematical Modelling using Spreadsheets," delta-K, June 1989. Applewords and Microsoft Works are listed by Alberta Education as Basic Resources.

Software:
Equations II (Mindscape) provides structured remedial opportunities for solving equations of the form $Ax + B = Cx + D$.

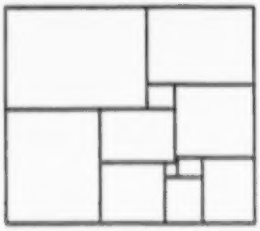
Equations III (Mindscape) provides structured practice in solving equations of the form $Ax + B(Cx + D) = Ex + F$.

ELECTIVE SUGGESTIONS

Extension: Let a, b, c, and d be non-zero integers between –10 and 10. What are the largest and smallest solutions possible for $ax + b = cx + d$? How does the answer to this question change if the bounds are $-n$ and n, or if they are 0 and n? (Answers: (20, –20), (2n, –2n), (n, –n)).

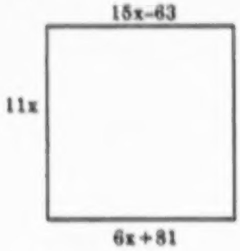
Reprinted with permission from "Reasons for Estimating," in Estimation and Mental Computation, copyright 1986 by the National Council of Teaching of Mathematics.

Problem: 1.1 Here is a rectangle that has been cut into 11 squares of various sizes. The smallest square measures 9 cm by 9 cm. Find the dimensions of the large rectangle.



Answer: 176 cm x 177 cm

Answer Hint: One possible solution is to define the measure of the sides of the square immediately to the right of the smallest square as x. All other squares can then be defined in terms of these two squares. Doing this, one arrives at a measure of the large rectangle as



$\therefore 15x - 63 = 6x + 81$
and $x = 16$

Activity 1.2 This activity extends studies done in Number Systems on square roots. It is a fact that the difference between two consecutive perfect squares is equal to the sum of their square roots. For example:

$$\begin{aligned} 121 - 100 &= \sqrt{121} + \sqrt{100} \\ &= 11 + 10 \\ &= 21 \end{aligned}$$

Have students investigate the patterns of difference between consecutive perfect squares and the sum of their square roots. Once the pattern indicated above has been established, have students investigate why this should be so by generalizing. How can we represent the general case of two consecutive perfect squares? If n is any number, then n^2 and $(n+1)^2$ is one way.

RESOURCE CORRELATION

M10: pp. 140–143, 150–161, 180–184
HM10: pp. 17–19, 70–71
MPP10: pp. 70–71
Mathématiques 10: pp. 70–71
HM10 TRM D/A Master 1-4

CONCEPT 2

Students will be expected to demonstrate an understanding that some situations and relationships can be represented through the use of inequalities.

Students will be expected to:

2.1 **solve, graph and verify the solution of any first degree inequality**
  - 2.1.1 verify the "reverse the sign" rule in the inequalities
  - 2.1.2 represent problems with inequalities and to solve the problems through the solution of the inequalities.

COMMENTS

Students have done some previous work with solving and graphing inequalities of the form $x + a \geq b$ and $cx \leq d$ (where c is positive and the direction of the inequalities varies). Some time has also been spent on verifying the solution to inequalities. There has been no work with multiplying or dividing an inequality by a negative number.

"Inequality" problems can be "equality" problems that are adjusted to become inequality problems.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: 2.1.1 Have students write several true inequalities involving pairs of rational numbers. To both sides of these inequalities apply the same operations. Check the results for truthfulness.

	+ 7	−9	× 5	× (−6)	÷ 2	÷ (−2)
6 < 10	13 < 17 True	−3 < 1 True	30 < 50 True	−36 < 60 False	3 < 5 True	−3 < −5 False
−3 < 6						
0 < 12						
−10 > −20						
6 > −3						

TECHNOLOGY INTEGRATION

Software:

Algebra Drill and Practise II (Conduit), options "Linear Inequalities" and "More Linear Inequalities" provide practice in solving inequalities.

RESOURCE CORRELATION

M10:	pp. 148–149
HM10:	pp. 24–25
MPP10:	pp. 22–25

Mathématiques 10: pp. 22–25
HM10 TRM D/A Master 1–8

CONCEPT 1

Students will be expected to demonstrate an understanding that points and lines can be represented on a Cartesian plane and that they have characteristics which can be defined and measured.

Students will be expected to:

- 1.1 use the formula for the distance between two points
  - 1.1.1 deduce the distance formula from the Pythagorean theorem
  - 1.1.2 solve problems that involve the use of the distance formula.

COMMENTS

Students have used the terminology used in graphing, have graphed points and lines, and have used the Pythagorean Theorem. A review of these may be necessary in discussing distance.

An initial discussion of distance occurs in working with number systems and the absolute value. Like the distance between two points on a number line, the distance on a Cartesian Plane is always positive.

In general, the distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_1)$ is given by $D = |x_2 - x_1|$ or $|x_1 - x_2|$

In general, the distance between $P_1(x_1, y_1)$ and $P_2(x_1, y_2)$ is given by $D = |y_2 - y_1|$ or $|y_1 - y_2|$

In general, the distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ or $D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ or $D = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

PROCESS/PROBLEM-SOLVING CONTEXT

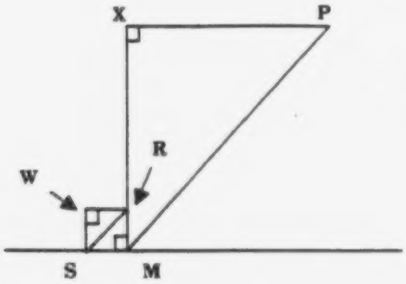
Activity: Have students first find the length of segments parallel to the x- and y-axes. This can be related to the work they did with the absolute value in number systems.

Activity: Once work with horizontal and vertical segments has been done, have students consider finding the length of an oblique line segment. Have students work on this in groups and have them "develop" a method for finding the length of any oblique line segment.

Problem: Find three points on a Cartesian Plane which define an equilateral triangle.

Activity: Have students first find the length of segments parallel to the x- and y-axes and then the length of an oblique segment by considering that segment as the hypotenuse of a right triangle. Encourage students to diagram the situations and finally to generalize and develop the formula.

Problem: In the figure, triangles WRS, MRS, and XPM are isosceles right triangles with legs of lengths 1 unit, 1 unit and 7 units, respectively. How many units from vertex W is vertex P?



Answer: 10 units

One useful strategy is to consider the diagram superimposed on the Cartesian plane, point M at (0, 0) and line SM lying along the x–axis.

TECHNOLOGY INTEGRATION

Software:
The Geometric preSupposer: Points and Lines (Sunburst) can be used as an exploratory activity related to measuring the distance between two points.

RESOURCE CORRELATION

M10: pp. 11, 190–193
HM10: pp. 164–165
MPP10: pp. 147–152
Mathématiques 10: pp. 147–152
HM10 TRM D/A Master 7–3

CONCEPT 1

Students will be expected to demonstrate an understanding that points and lines can be represented on a Cartesian plane and that they have characteristics which can be defined and measured.

Students will be expected to:

1.2 use the formula for the coordinates of the midpoint of a line segment
  1.2.1 verify the midpoint formula
  1.2.2 solve problems that involve the use of the midpoint formula.

COMMENTS

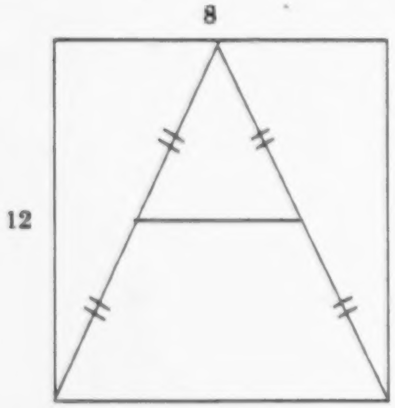
Students intuitively understand the concept of midpoint. Prior to introducing students to the formal and algebraic definition of a midpoint, have them identify the midpoints of horizontal and vertical line segments. From this, have students estimate the midpoint of an oblique line segment and generalize this finding in an algebraic form.

In general, the midpoint of the line segment with endpoints $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is $\left(\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}\right)$.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students find the midpoints of segments which are parallel to the x- and y-axes and generalize their procedures to segments whose endpoints are non-numeric. Discuss why the midpoint is the "average" of the endpoints. Extend and generalize this understanding to finding the midpoint of an oblique segment. Can these techniques be generalized to trisecting a segment?

Problem: An enormous A is to be painted with gold leaf on the square outside of the palace of the Sultan of Aruzza. The A is to fit exactly within a rectangular area 8 m × 12 m. Assuming that the bar across the middle of the A is between the midpoints of the sides, how many metres of gold leaf need to be purchased?



Answer: $2 \times 12.65 + 4 = 29.30$ m

Activity: To discover the algebraic meaning of midpoint, have students plot the following points: P(1, 6), Q(5, 2), R(6, 7), and S(12, 9). Find the point midway between points P and Q and label it T. What are the coordinates of point T? What is the relationship between the coordinates of point T and the coordinates of points P and Q? Then find the point midway between points R and S, and call it point E. What is the relationship between the coordinates of point E and the coordinates of points R and S? Is this the same relationship as the one between the coordinates of point T and the coordinates of points P and Q?

Problem: M10 TRB Teaching Notes, p. 2.

TECHNOLOGY INTEGRATION

See: MPP10, p. 163, "Computer Tip."

Software:
The Geometric preSupposer: Points and Lines (Sunburst) may be used in an informal way in activities related to midpoint.

RESOURCE CORRELATION

M10:	pp. 196–199
HM10:	pp. 166–167
MPP10:	pp. 153–155

Mathématiques 10: pp. 153–155
HM10 TRM D/A Master 7–4

CONCEPT 1

Students will be expected to demonstrate an understanding that points and lines can be represented on a Cartesian plane and that they have characteristics which can be defined and measured.

Students will be expected to:

1.3 use the formula for the slope of a line passing through two points
  - 1.3.1 solve problems that involve the use of the formula for the slope of the segment between two points
  - 1.3.2 solve problems that involve the relationship between the slopes of parallel and perpendicular lines.

COMMENTS

Students intuitively understand the concept of slope as they have used the term in other contexts, e.g., the slope of the roof, the slope of a ski hill, the grade on a highway. Building on this, students can make the translation from the real world to the Cartesian Plane.

Students should not be exposed to the algebraic formula for calculating slopes until they understand the concept of slope.

In general, the slope (m) of the line segment with endpoints $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} \quad \text{or} \quad m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$$

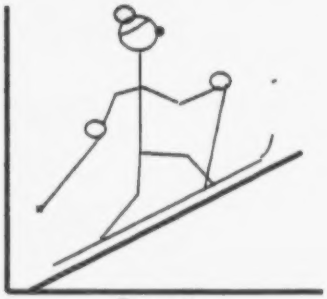
Students have some experience in discussing parallel and perpendicular lines, but have not discussed the relationships between their slopes. The slopes of parallel lines and parallel line segments are equal and the slopes of perpendicular lines are the negative reciprocal of one another. For instance, if $l_1 \parallel l_2$ then $ml_1 = ml_2$ or $m_1 = m_2$; and if $l_1 \perp l_2$ then $ml_1 = \frac{-1}{ml_2}$ or $m_1 = \frac{-1}{m_2}$

Thus $ml_1 \times ml_2 = -1$.

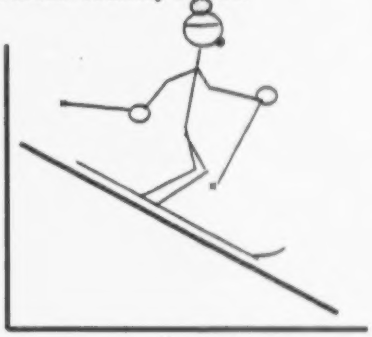
It is important for students to discover the relationships between the numerical and pictorial representation of a slope of a line segment. Students should discover these relationships and be able to express them in their own words. Discuss the effect of positive, negative, zero and undefined slopes on lines and line segments by having students drawing the lines and working together to identify the relationship between the graph of the line segment and its slope.

PROCESS/PROBLEM-SOLVING CONTEXT

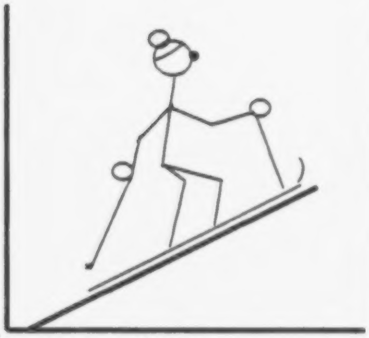
Activity: Relating slope to real life experience will aid in developing the meaningfulness of the concept. Give students "intuitive points of reference" to estimate slope. For example, consider the following sketches of cross–country skiers.



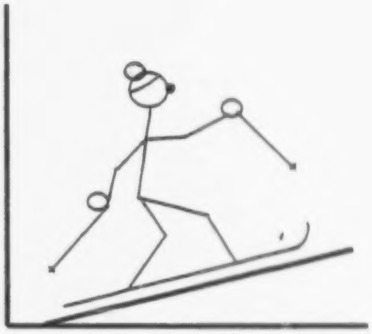
Person 1



Person 2



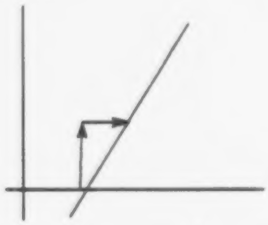
Person 3



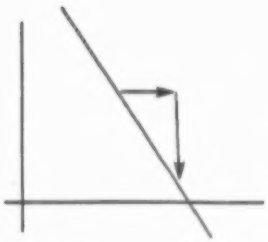
Person 4

Comparing person 1 and person 2, which one is doing work? Who is doing more work, person 3 or person 4? The steeper the hill, the greater the rate of work that is required. If we associate rate of work with a positive number, then going downhill is doing the opposite of work (person 2) and is associated with a negative number. A larger positive number is associated with person 3 than with person 4. Thus, slope becomes intuitively related to "steepness."

In the discussion of slope as $\frac{\text{rise}}{\text{run}}$, students may connect "rise" with "increase" – a positive change. They may find it helpful to draw horizontal and vertical arrows indicating the direction of change. For example:



$$\frac{\text{vertical change}}{\text{horizontal change}} = \frac{\uparrow}{\rightarrow} = \frac{+}{+}$$



$$\frac{\text{vertical change}}{\text{horizontal change}} = \frac{\downarrow}{\rightarrow} = \frac{-}{+}$$

Numerical estimates of the slope can also be quickly made using this approach.

Adapted with permission from "Concrete versus Abstract in Teaching Algebra," Mathematics Teacher, March 1988, copyright by the National Council of Teachers of Mathematics.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students develop a method of measuring the slope of a line. This can be achieved by having students work together and examining their own definition of slope and applying it to mathematics. Why do we measure rise/run instead of run/rise? (Answer: slope is associated with "steepness" which is related to the vertical). How is this measure of slope (rise/run) associated to the slope of a roof, the slope of a ski hill?

Activity: Have students draw line segments and note the similarities and differences in the steepness of the lines, e.g., one may be steeper than the other, one may be in a negative direction. Then have students calculate the values of the slope and discuss how the numerical representation relates to the physical representation. For instance the line segment with a negative slope rises to the left in all cases; the line segment with a fractional slope will not be as steep as a line segment with a whole number slope.

Activity: In discussing the slope of horizontal and vertical lines, have students guess the value of their slopes, prior to the calculation. Have students identify their reasons for making that particular guess. Then have students calculate the slope.

Problem: Castle Mountain is 2752 m high. How long would a road be to the top of the mountain with a constant 6% grade, if the altitude at the bottom is 1963 m ?
Answer: 13 174 m

Activity: Have students discuss the meaning of parallel and perpendicular lines and then speculate as to the probable relationships between the slopes. Draw pairs of perpendicular segments using a protractor or construction techniques, and find the slopes. Then generalize the relationship.

Activity: 1.3.2 Given pairs of parallel and perpendicular lines and line segments, have students calculate the slopes of the pairs of lines and generalize the relationships.

For a set of class activities which begin by considering slope as "steepness" and rise/run, and move onto exercises which develop and use the formula for the slope of a line passing through two points, see: "Skiing the Slopes," in Mathematics Teacher, December 1987.

TECHNOLOGY INTEGRATION

Use a graphing calculator or a computer to plot pairs of points.

Software:
Algebra Drill and Practice I (Mindscape), option "Slopes and Equations of Lines" provides practice in determining the slope of a line given two points.
Geometric preSupposer: Points and Lines (Sunburst). Measurement of line segments possible, determining ratios of the lengths of line segments possible. Construction of perpendicular and parallel lines possible. A grid may be superimposed. Suitable, therefore, for use in an exploratory way in activities related to slope.

RESOURCE CORRELATION

M10: pp. 200–211
HM10: pp. 160–163
MPP10: pp. 156–159, 175
Mathématiques 10: pp. 156–159, 175
HM10 TRM D/A Master 7-1, 7-2

CONCEPT 1

Students will be expected to demonstrate an understanding that points and lines can be represented on a Cartesian plane and that they have characteristics which can be defined and measured.

Students will be expected to:

1.4 determine whether points in a plane are collinear
  1.4.1 solve problems that involve the necessary and sufficient conditions for collinearity.

COMMENTS

Collinear points are those lying on the same line.

The discussion of a collinearity should follow the discussion of slope.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Given any three points, have students identify whether or not the points lie on the same line. From this, have students identify possible conditions that are necessary for the three points to be on the same line and ask students to explain why these conditions are viable. The discussion can be led in such a manner that students are examining the slopes between the three points and the distance relationships. If the slopes of the segments between three points are equal, are the points necessarily collinear? If the sum of the lengths of the two shorter segments between three points is the length of the larger segment, are the points necessarily collinear?

Given these two conditions plus three non-collinear points have students verify that the points are non-collinear.

TECHNOLOGY INTEGRATION

HM10, p. 170, "Computer Application"

Software:
The Geometric preSupposer: Points and Lines (Sunburst) may be used in an exploratory way in activities related to collinearity.

ELECTIVE SUGGESTIONS

Problem: Points P, Q, R and S are collinear with S between P and R and with R between S and Q. If $PS = 1/3 \cdot PR$, $PR = 3/4 \cdot PQ$, and $PQ = 40$, find SQ.

Answer: $SQ = 30$.

RESOURCE CORRELATION

M10:	pp. 218–222
HM10:	pp. 160–161, 198–199
MPP10:	pp. 160–163
Mathématiques 10:	pp. 160–163

CONCEPT 2

Students will be expected to demonstrate an understanding that the ordered pairs which satisfy a linear equation, $Ax + By + C = 0$, correspond to coordinates on the Cartesian plane and can be used to graph the equation on the Cartesian plane.

Students will be expected to:

2.1 find sets of ordered pairs that satisfy linear equations
2.2 graph linear equations by plotting sets of ordered pairs that satisfy the equation.

COMMENTS

Students have done some previous work in using sets of ordered pairs to graph linear relations. Hence, the concentration should be on the relationship between equations, sets of ordered pairs, and their graphs, that they are different ways to express the same relationship. Every situation considered should be illustrated with an equation, ordered pairs and a graph.

Variation problems provide a very good introduction into equations of a straight line and especially the y-intercept concept. Use the problems as a means to achieve the graphing objectives, not as a particular type of problem to be solved. The examples used should not only be taken from the algebraic realm, but also from the real world. Provide a variety of domains in which to graph the relations. Linear equations in x and y should be introduced only after work has been done with the graphs of direct and partial variation relationships.

The concepts of domain and range may be used but the terms should not be formally introduced.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students examine direct and partial variation problems. For example, consider a cyclist who travels at an average rate of 24 km/h. Write an equation and set up a table of values which describe the situation. With the table of values, graph the points suggested. Students can consider the following questions: Is it acceptable to join up the points with a line? Are there any "restrictions" on values which can be considered? Does the value of one of the variables depend on the other? Provide other situations which have different answers to the above questions. See "Slope as Speed," Mathematics Teacher, April 1988 for a set of activities that requires students to interpret various linear relations on time-distance graphs.

TECHNOLOGY INTEGRATION

Computers and graphing calculators can be used to graph the relations.

Software:
Graphing Linear Functions (Mindscape) provides practice in finding sets of ordered pairs that satisfy linear equations.

Spreadsheets may be used to determine sets of ordered pairs which satisfy linear equations. Software which has additional graphing capabilities can then graph the table of values. See "Mathematical Modeling Using Spreadsheets," delta-k, June 1989 for a discussion related to this topic. Appleworks and Microsoft Works are Alberta Education Basic Resources.

RESOURCE CORRELATION

M10: pp. 164–167, 212–213, 223–231
HM10: pp. 175–183, 196–199
MPP10: pp. 131–136, 266–273, 283–287
Mathématiques 10: pp. 131–136, 266–273, 283–287

CONCEPT 2

Students will be expected to demonstrate an understanding that the ordered pairs which satisfy a linear equation, $Ax + By + C = 0$, correspond to coordinates on the Cartesian plane and can be used to graph the equation on the Cartesian plane.

Students will be expected to:

2.3 find the x- and y-intercepts of linear equations
2.4 graph linear equations by drawing through the x- and y-intercepts.

COMMENTS

An x- and y-intercept is the directed distance from the origin to the point where the line cuts the x- or y-axis respectively.

Once the discussion of the x- and y-intercepts of oblique lines is complete, students should examine the x- and y-intercepts of horizontal and vertical lines.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students draw the graphs of several equations which have easily interpretable intercepts. What is always true about the intercepts? Can this be generalized to all linear equations? How can this fact be used to find the intercepts without drawing the graphs? That is, what is the relationship between the equation of the line and the x- or y-intercept?

Activity: Once students are familiar with the term intercept, have them find the x- and y-intercept from a linear equation and then draw the graph of the equation.

TECHNOLOGY INTEGRATION

Computers and graphing calculators can be used to graph the linear equations.

Software:
Master Grapher (Addison-Wesley), "Function Grapher" option. Linear equations can be entered and graphed by the program. The student can then use the vertical, horizontal line, or crosshairs options to detrmine x- and y-intercepts.

ELECTIVE SUGGESTIONS

HM10: TRM Lesson Notes, p. 77, "Mental Math"

RESOURCE CORRELATION

M10: pp. 245, 249
HM10: pp. 198–199
MPP10: pp. 176–177
Mathématiques 10: pp. 176–177

CONCEPT 2

Students will be expected to demonstrate an understanding that the ordered pairs which satisfy a linear equation, $Ax + By + C = 0$, correspond to coordinates on the Cartesian plane and can be used to graph the equation on the Cartesian plane.

Students will be expected to:

- 2.5 write linear equations in the slope-intercept form, $y = mx + b$
- 2.6 graph linear equations using the slope and the y-intercept or another point that satisfies the equation
  - 2.6.1 design and carry out an investigation involving the use of calculators or computers to determine the effects of the parameters m and b on the graphs of linear equations.

COMMENTS

It is the intent of sub-concept 2.5 above that students express the equation of a line in the slope-intercept form, that is, given an equation in the $Ax + By + C = 0$ form, students write it in the $y = mx + b$ form. Once the equation is in the $y = mx + b$ form, students then graph the equation by first plotting the y-intercept and then using the slope to find a second point; and finally examine the effects of m and b on the graph of the function.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students draw the graph of $y = x$. On the same axes, draw the graphs of $y = 2x$, $y = \frac{1}{2}x$, $y = x + 1$, $y = x - 3$, $y = -3x$, $y = -x + 2$. Have students describe the differences and similarities between each of the graphs and $y = x$. What is the effect if m gets larger?. . . gets smaller? What is happening with the b value? What happens if m is negative? Since this follows the discussion of slope, they may be obvious to students, it is important for students to be able to communicate these differences and similarities in preparation for their work in Math 20.

Activity: Have students discuss the slope of the segments between any two points on a line. Have them suggest and test a graphing methodology given a point on the line and the slope.

TECHNOLOGY INTEGRATION

Computers and graphing calculators can be used to graph the linear equations. The following program written in BASIC for the Apple computer will graph straight lines in the general form $y = mx + b$. Students input m and b. For an accompanying set of worksheet activities designed to permit students to explore the effects of m and b, see "Microcomputer Unit: Graphing Straight Lines" in Activities for Implementing Curricular Themes from the "Agenda for Action."

```
10   TEXT: HOME
20   VTAB 12
30   PRINT "THIS PROGRAM GRAPHS STRAIGHT LINES"
40   PRINT: PRINT "IN THE GENERAL FORM Y = M*X + B."
50   FOR N = 1 TO 5000: NEXT: REM PAUSE LOOP
60   HGR
65   HCOLOR = 3
70   PRINT: POKE 37,20: PRINT
80   PRINT: PRINT "GIVEN THE GENERAL FORM Y = M*X + B"
90   PRINT: PRINT "INPUT M,B";
100  INPUT M,B
110  PRINT: PRINT: PRINT
120  REM DRAW AXES
130  HPLOT 140,0 TO 140,159
150  HPLOT 0,80 TO 279,80
170  FOR H = 0 TO 270 STEP 10
180  HPLOT H,77 TO H,83
190  NEXT H
210  FOR K = 0 TO 160 STEP 10
220  HPLOT 137,K TO 143,K
230  NEXT K
240  HPLOT 271,65 TO 277,71
250  HPLOT 277,65 TO 271,71
260  HPLOT 147,0 TO 151,6
270  HPLOT 155,0 TO 147,12
280  REM  PLOT THE STRAIGHT LINE
290  FOR X =-14 TO 13.9 STEP .1
300  LET Y = M*X + B
310  IF Y > 8 THEN 340
320  IF Y <-7.9 THEN 340
330  HPLOT 140 + 10*X,80-10•Y
340  NEXT X
345  REM  PRINT LINEAR EQUATION
350  PRINT: POKE 37,20: PRINT
360  PRINT "THIS IS THE GRAPH OF Y = "M"*X + "B""
370  PRINT
380  PRINT "1 = ANOTHER GRAPH 0 = QUIT";
390  INPUT Z
400  PRINT: PRINT
410  IF Z = 1 THEN 60
420  TEXT: HOME
```

See the source article for an accompanying set of activities suited to investigating the effects of m and b in $y = mx + b$.

Reprinted with permission from Activities for Implementing Curricular Themes from the "Agenda for Action" copyright 1986 by the National Council of Teachers of Mathematics.

Software:

- Master Grapher (Addison-Wesley), option "Function Grapher," for graphing linear functions. Teacher demonstration or student use.
- Algebra Drill and Practice I (Conduit), option "Slopes and Equations of Lines," for practice in finding the slope given the equation of the lines as $Ax + By + C = 0$.
- Computer Graphing Experiments 1 (Addison-Wesley), options "Graphs of the form $y = mx$" and "Graphs of the form $y = mx + b$," and the accompanying student worksheets.

- Equation Math (MECC) "Exploring Equational Linear" option and accompanying student worksheets. Teacher demonstration or student use.
- Graphing Linear Functions (Mindscape) – "Harder Forms" option. This requires a student to first convert the given equation to $y = mx + b$ form, and then determine and plot ordered pairs which satisfy the equation.
- Green Globs and Graphing Equations (Sunburst). The "Equation Plotter" option is suited to teacher demonstration or student use.

RESOURCE CORRELATION

M10: pp. 218–222, 233–237, 246–249
HM10: pp. 200–201
MPP10: pp. 179–185
Mathématiques 10: pp. 179–185

CONCEPT 2

Students will be expected to demonstrate an understanding that the ordered pairs which satisfy a linear equation, $Ax + By + C = 0$, correspond to coordinates on the Cartesian plane and can be used to graph the equation on the Cartesian plane.

Students will be expected to:

2.7 identify and graph equations of lines parallel to the x- and y-axes.

COMMENTS

This expectation may be integrated throughout the study of coordinate geometry and graphing. Questions arise in the textbooks throughout the study of coordinate geometry.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students suggest what the graphs of $y = 2$ and $x = 3$ would be. Which would have a zero slope (0 rise over any run)? Which would have an undefined (infinite) slope (any rise over 0 run)? What would be the general form of the equations of lines parallel to the x- and y-axes?

Activity: Have students work together to graph a line that cannot be represented by an equation of the form $y = mx + b$.

Activity: M10 TRB Teaching Notes, p. 26, section 7-4.

Activity: MPP10, p. 184, "Problem Solving."

TECHNOLOGY INTEGRATION

Computers and graphing calculators can be used to graph the linear equations.

Software:

Equation Math (MECC), "Exploring Equations" option.

Green Globs and Graphing Equations (Sunburst) "Equation Plotter" option.

RESOURCE CORRELATION

M10: pp. 247-249
HM10: p.
MPP10: p.
Mathématiques 10: p.
MPP10 ASTRM: p. 12

CONCEPT 3

Students will be expected to demonstrate an understanding that lines in a plane can be uniquely determined.

Students will be expected to:

3.1 graph and write the equation of a line given the conditions which uniquely determine it
    3.1.1 solve problems that involve conditions which uniquely determine a line.

COMMENTS

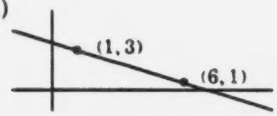
Students should work with graphing the lines given conditions before writing the equations. From here, students could write the equation of a line by reading the required information from the graph. Finally, students could be given the conditions, then graph the line and write the equation. This work should help to reinforce the graphing skills previously learned.

Students should sketch the graph of every condition not given in graphical form.

Students should be expected to write equations in the forms $y = mx + b$ (slope–intercept) and $Ax + By + C = 0$ (standard) and to transform the forms of the equations between the two.

Limiting Examples:
Determine the equations of the following lines:

a)



b) with a slope of –2 and a y–intercept of –6
c) through (5, –6) with a slope of 1/3
d) through (–4, 2) and (5, –8)
e) through (–3, –2) parallel (or perpendicular) to $2x + 3y = 9$

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students determine the necessary and sufficient conditions to define a line. When they have suggested the need for a point and a slope, determine the point and slope that have been given as conditions in the problem. Introduce the general point (x, y) as an auxiliary element that represents every point on the line. Can an equation be written that describes the conditions between the given point, the general point and the slope? Can the equation be simplified and does it represent the initial conditions?

TECHNOLOGY INTEGRATION

Computers and graphing calculators can be used to graph the linear equations.

Software:

- Master Grapher (Addison-Wesley), "Function Grapher" option. Once students have written the equation as a function of x, they may graph it using this option.
- Algebra Drill and Practice I (Conduit), "Slopes and Equations of Lines" option, provides practice in writing the equation of a line given the slope and a point, or two points.
- Computer Graphing Experiments 1 (Addison-Wesley). Using the "Linear Equations" option "Graphs of the form $y = mx + b$" and the accompanying worksheets, students may write and graph equations given various conditions.
- Equation Math (MECC), "Exploring Equations" and "Polygraph" options. Equation must be input in "y expression" or $x = b$ form. Listing of ordered pairs that satisfy equation available in "Polygraph."
- Graphing Linear Equations (Mindscape), "Harder Forms" option. Students are first required to convert the given equation to $y = mx + b$ form and then determine and plot ordered pairs that satisfy the equation.
- Green Globs and Graphing Equations (Sunburst), "Linear and Quadratic Graphs" option. In linear mode, this courseware provides students with the opportunity to develop their ability to interpret and write the equation for given straight line graphs.

RESOURCE CORRELATION

M10:	pp. 234–237, 240–244, 250–252
HM10:	pp. 202–203, 206–208
MPP10:	pp. 190–196, 202

Mathématiques 10: pp. 190–196, 202
HM10 TRM D/A Masters 9-2, 9-3, 9-4, 9-5

CONCEPT 1

Students will be expected to demonstrate an understanding that some mathematical circumstances can be represented algebraically by a system of two linear equations in two unknowns.

Students will be expected to:

1.1 solve simple systems of linear equations in two unknowns graphically

1.2 solve systems of linear equations in two unknowns algebraically

  1.2.1 solve problems that can be represented with a system of two equations in two unknowns.

COMMENTS

It is important to graph systems of equations to demonstrate to students that when solving a system that an intersection point will be the only point which satisfies (or solves) both equations. If the lines do not intersect, then there is no solution. Solving systems graphically may provide an inaccurate estimate of the solution, hence, it is convenient to use an algebraic method in order to obtain an exact solution.

Graphically: Reinforce for students that the coordinates of the points which make up a line are solutions for the equation of the line. Thus, the intersection point of two lines will be the only point which satisfies (solves) both equations. Impress upon students the importance of checking in both equations.

Addition/Subtraction: It is important that students understand why algebraic methods work. Remind them of the processes involved in solving equations, i.e., that performing the same operation on both sides of an equation creates an equivalent equation. Use this fact to demonstrate that the sum or difference of a system is satisfied by the same point as the system. This examination provides a reason why the addition/subtraction algorithm works.

Provide students with a method for keeping track of the equations and operations on them. The following is a suggestion:

$$\begin{array}{lrcl}
 & 3x + 4y &=& 29 \text{ ①} \\
 & 2x - 5y &=& -19 \text{ ②} \\
 & & & \\
\text{①} \times 5 & 15x + 20y &=& 145 \\
\text{②} \times 4 & 8x - 20y &=& -76 \\
\hline
\end{array}$$

Substitution: Remind students that the reason for solving systems is to find the coordinates of the point which satisfies both equations. At that point, the values of x and y for both equations are the same. For this reason, it is legitimate to solve for one of the variables and then to substitute the expression for that variable into the other equation.

Stress that the reason for learning different methods of solution is for efficiency. Little is to be gained by forcing students to use one method instead of another unless they can see an advantage.

Provide students with lots of practice in writing equations to represent situations. Do not attempt to categorize problems by type.

Many strategies can be used and should be considered by teachers and students as they attempt to set up systems of equations. Students should be encouraged to draw pictures and diagrams, set up tables and charts, and estimate, check and revise their estimates, among others. However, it is important that variables be clearly defined at some point within the process and that students be reminded that upon solution, the answer should be looked at with respect to the defined variables for reasonableness and to ensure that the entire problem is answered.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: 1.1 Have students draw the graphs of both equations in a given independent system. Discuss the importance of the point of intersection and then verify that it does satisfy both equations in the system.

Activity: 1.2 Have students sketch the graphs of the equations in a system on the same coordinate plane. Have them add and subtract the systems and graph the equations of the results. Have them multiply one or both of the equations, add or subtract the changed equations and graph. Why do all of the equations pass through the same point? Are there any restrictions on what can be done to the equations in a system and still maintain the graph through the same point? How could an equivalent system be created which had only lines parallel to the x- and y-axes?

Problem: 1.2 Aardvaark Auto Rentals charges $20/day plus $0.15/km to rent a new Thunderer. U-Drive Car Rentals charges $15/day plus $0.20/km. What must be considered before a decision is made as to which agency to use? What are the conditions under which each of the agencies would be chosen?

Problem: Find the area of the triangle determined by the x-axis, the line with equation $y = x$, and the line with equation $x + 3y = 12$.

Answer: The vertices of the triangle are (0,0), (3,3) and (12,0). The triangle has a base of 12 and an altitude of 3. The area is 18 square units.

For an engaging idea that will assist students in developing their skills in graphically and algebraically solving systems of linear equations, and which can be extended to involve students in problem solving, see "Secret Codes and Systems of Equations," Mathematics Teacher, April 1986.

For more **Problems** see: Problem Solving in Mathematics: Focus for The Future, Alberta Education, pp. 65, 66 #15, 22; p. 67, #8.

103

TECHNOLOGY INTEGRATION

Computers and graphing calculators can be used to graph the linear equations.

Short computer programs in the BASIC language, designed to solve two equations in two variables, are provided in M10 (p. 297), MPP10 (p. 234–235), and HM10 (p. 224). Have students work in pairs to input a program and use it to solve the problems or have students investigate all three programs and identify any similarities or differences. Do any of the differences in the programs produce significantly different results in solutions?

For an example of how spreadsheets can be used to help develop students' understanding of systems of linear equations, see "Integrating Spreadsheets into the Mathematics Classroom," Mathematics Teacher, November 1988.

Software:

- Master Grapher (Addison-Wesley), "Function Grapher" option.
- Algebra Drill and Practice II (Conduit), "Linear Systems in Two Variables" option.
- Computer Graphing Experiments I (Addison-Wesley), "Linear Equations – Simultaneous Equations" option.
- Equation Math (MECC), "Exploring Equations" option.
- Green Globs and Graphing Equations (Sunburst), "Equation Plotter" option.

ELECTIVE SUGGESTIONS

Problem: Suppose the government was considering the following two unemployment compensation plans:

Plan A: Pays 50 percent of the average weekly salary with a maximum benefit of $160 a week.

Plan B: Pays a base amount of $75 plus 20 percent of the worker's average weekly salary with a maximum benefit of $160 a week.

Have students consider: What questions come to mind in comparing the two programs?

Describe a mathematical model (of two equations in two unknowns) for comparing the two programs. Use it to answer some of the formulated questions.

Adapted with permission from "Wildlife, Unemployment and Insects: Mathematical Modeling in Elementary Algebra" in Applications in School Mathematics, copyright 1979, by the National Council of Teachers of mathematics.

RESOURCE CORRELATION

M10: pp. 258–270, 276–296
HM10: pp. 209–217
MPP10: pp. 206–209, 213–229, 236–252
Mathématiques 10: pp. 206–209, 213–229, 236–252
HM10 TRM D/A Masters 9–6, 9–7, 9–8, 9–9

CONCEPT 2

Students will be expected to demonstrate an understanding that lines in a plane can relate to each other in various ways.

Students will be expected to:

2.1 recognize the characteristics of systems of linear equations with graphs that are parallel, coincident or intersecting
  2.1.1 verify the conditions which are present in a system of equations that determine the relationships of the graphs.

COMMENTS

A **parallel system** has lines that are parallel, hence there is no solution.

An **intersecting system** has lines that intersect once, hence there is only one solution.

A **coincident system** has lines that are simultaneous, hence there are infinitely many solutions.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students graph systems of linear equations that are parallel, concident or intersecting. Have students identify and describe the similarities and differences between the equations describing the graphs. What are the characteristics of the equations which would indicate the kind of systems each was without having to graph or solve?

Problem: Problem Solving in Mathematics: Focus for the Future, p. 66, #16.

TECHNOLOGY INTEGRATION

Software:
Computers and graphing calculators can be used to graph the linear equations.

- Master Grapher (Addison-Wesley), "Function Grapher" option.
- Computer Graphing Experiments I (Addison-Wesley), "Linear Equations – Simultaneous Equations" option.
- Equation Math (MECC), "Exploring Equations" option.
- Green Globs and Graphing Equations (Sunburst), "Equation Plotter" option.

RESOURCE CORRELATION

M10:	pp. 271–273
HM10:	pp. 218–219
MPP10:	p. 212
Mathématiques 10:	p. 212

GENERAL COMMENTS

The following guidelines, prepared by the ASA/NCTM Joint Committee on Statistics and Probability, provide principles for consideration during the teaching of statistics.

1. Learning should be active, not passive, focused on asking questions about something in the students' environment and finding quantitative ways to answer the question.
2. Problems should be approached in more than one way with an emphasis on discussion and evaluation of these different methods.
3. Real data should be used whenever possible in any statistics lesson.
4. The emphasis in teaching statistics should be on good examples and building intuition.
5. Topics should be presented in ways designed to give students hands–on experience in working with data.
6. Student projects should be an integral part of any work in statistics.
7. The emphasis in all work with statistics should be on the analysis not on a single answer.
8. The progression should be from the concrete to the pictorial to the abstract.

Problems should be presented which can be used as vehicles to accomplish the objectives of the unit. The depth to which a class should be taken will be a function of the problems chosen and the ability of the class. Students should be actively involved in the examination of the problems and techniques for analysis should be introduced as needed by the class. Students should be encouraged to work with each other; discussion of possible interpretations and conclusions should play a major part in the delivery of the objectives. Consideration should be given to methods of evaluation other than a formal written examination.

CONCEPT 1

Students will be expected to demonstrate an understanding that populations have certain characteristics which can be determined by analyzing the characteristics of a suitable sample of that population.

Students will be expected to:

1.1 determine a sample that will adequately and accurately represent a population

1.2 evaluate samples for bias, appropriateness of sample type, amount of data and randomness

1.3 draw inferences about the population from which a sample was taken and express it using the language of probability

- 1.3.1 determine a valid sample, design and carry out a simple survey, and make a valid inference to the population based on the results.
- 1.3.2 defend or oppose a generalization that is made from a sample to a population.

COMMENTS

A sample is data that are chosen from, and hence believed to represent a population. Proper sampling is extremely important in data collection, in that improper sampling will affect the statistics and hence any inferences made from the sample to the population.

The following is a list of types of sampling. It is not important that students memorize each of the types but teachers should endeavour to discuss various types with students, in terms of the advantages and disadvantages of each. When students conduct surveys themselves, they should know the types of sampling which they use and be aware of the advantages and disadvantages which may result.

Convenience Sampling is acquiring data from a group which is convenient.

Self-selected Sampling involves people who voluntarily respond to a survey.

Judgment Sampling is where an expert selects a sample which he or she considers representative.

Probability Sampling includes a mechanism to ensure randomness.

Cluster Sampling is where a cluster of subjects within a particular area is randomly chosen to provide data.

Stratified Sampling is where a population is divided into identifiable groups from which samples are then randomly drawn.

Systematic Sampling is done by choosing every nth member of a population to form the sample.

Introduce and discuss the concept of bias. Have students evaluate analyses, question the outcome and come up with suggestions to produce more convincing results. Encourage them to be skeptical and prepared to question results and the methods used to produce them.

For problems in which a sample is examined, make a probability statement about the results and then try to check the accuracy of the original statement. For example, if a student did a survey of movie popularity and discovered that 60% of the students in the sample thought that Revenge of the Nerds 6 was the best movie that they had seen in the past 2 months, this could be stated as "The probability that a particular student thinks Revenge of the Nerds 6 is the best move seen in the past two months is 60%."

Discuss the meaning of the statement, that probability is the likelihood of the occurrence of an event. If they chose students at random, they could reasonably expect that 60% of them would favour that movie. Discuss as well that they could not reasonably expect 6 out of every group of 10 students to favour that same movie but that the percentage is an average taken over the entire school population.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students look at and criticize various sampling techniques, particularly those used by the media, government and polling companies. Give them problems and have them design sampling procedures which they think will give correct results and procedures which they think will bias the results in a certain way.

Example: If you wanted to find how many people really thought that a certain headache remedy was better than another, how would you structure the sample? How would you do it if you wanted to show that the remedy was definitely better?

Activity: Have students find newspaper stories which rely on some sort of statistic and discuss the confidence with which the results should be accepted. Discuss what factors could adversely affect the conclusion of the writer and what factors could positively affect the conclusion of the writer.

TECHNOLOGY INTEGRATION

Video:
Math Wise (ACCESS): "Predicting 02: Choosing a Sample"

ELECTIVE SUGGESTIONS

To help emphasize the idea of random selection, see HM10 TRM Lesson Notes, p. 109, Introductory Activity #1.

RESOURCE CORRELATION

M10: pp. 456–465
HM10: pp. 292–295, 314–315
MPP10: pp. 386–392
Mathématiques 10: pp. 386–392
MPP10 ASTRM: pp. 15–16

CONCEPT 2

Students will be expected to demonstrate an understanding that data collected from a sample must be organized, presented and analyzed in order that valid inferences will be drawn.

Students will be expected to:

2.1 identify a situation, formulate an hypothesis and plan what data need to be collected
2.2 organize data using stem-and-leaf plots and box plots
2.3 choose and calculate appropriate measures of central tendency for sets of data
2.4 represent data using appropriate graphs
2.5 do simple visual analyses on sets of data organized in stem-and-leaf plots, box plots and graphs
    2.5.1 derive and defend conclusions based on the visual analyses of sets of data
2.6 evaluate intuitively an analysis of data for the confidence with which results can be accepted.

COMMENTS

Students have had experience with statistics in junior high. Specifically, they have examined the use and misuse of statistics; biases in surveys; the representation of data in the form of pictographs, bar graphs, line graphs, circle graphs; the interpretation and analysis of arguments or conclusions based on statistical information; the mean, median, mode and range; have conducted a small survey and reported results; and have been introduced to the term probability.

This should not be taught as a set of six discrete skills, every attempt should be made to work through each expectation with a set of data.

Univariate Data

- Price of hamburgers (ice cream cones) in your area.
- Weight (height) of the linemen on your school's football team.
- Number of goals scored by defencemen in the NHL.
- Height of tallest buildings in Calgary (Edmonton, Canada).
- Weeks on the Top 40 for music (video, movie) hits.
- Number of rainy (frost free) days in a year in your area.
- Population of towns and villages in your county (municipal district).

Sources of information for problems such as these are the yearbooks and almanacs in your school library, Exploring Data published by Dale Seymour, sports record books, books on movies, records and entertainment, newspapers and magazines. Students may also have problems from other courses such as Social Studies that would lend themselves to examination within this unit.

The following is a list of some discrete human characteristics which may be investigated.

I Genetic Characteristics

1. Eye colour (specify)
2. Hair colour (specify)
3. Sex
4. Tongue type (curl)
5. Hair
6. Thumb type ("Hitchhiker's thumb")
7. Ear type (lobe)
8. Dimple (chin/cheek)
9. Freckles (none, few, lots)
10. Handedness
11. Eyedness (left or right dominance)
12. Finger relationship (relative lengths of ring and little fingers)
13. Handclasp (folded hands, interlocked fingers; which thumb is on top of the other?)

II Acquired Characteristics

1. Preferences
2. Ownership (e.g., of a guitar)
3. Number of certain possessions
4. Hours of sleep
5. Name (the beginning letter, or the number of letter/s)
6. Clothing sizes (e.g., shoes)
7. Birthday (use a perpetual calendar and determine day of the week of birth; hour of the day)
8. Age (e.g., year and month – 15.6 years old)

Reprinted with permission from "Applications in School Mathematics: Human Variability" in Applications in School Mathematics copyright 1979 by the National Council of Teachers of Mathematics.

See also "Projects" in Teaching Statistics and Probability for brief descriptions of a number of class projects.

Collection: Collection of data can be done by surveying a particular group, by researching a topic using library resources or by being given data in some form. If a survey is used, how the data are collected i.e., personnally, over the telephone, can be discussed. Are there any differences in the responses depending on the method of collection?

Organization and Presentation: Stem-and-leaf plots are a method used to organize data in which the data are not destroyed. The easiest way to set up a stem-and-leaf plot is to write the stems as all but the last digits in a data set while the leaves will be the last digit. For example, if the data were the following: 328, 335, 341, 326, 345, 365, 329, 357, 361, 345, 339, 367, the stems would be the numbers 32, 33, 34, 35, and 36. The stems are written in a column to the left of a vertical line. As the data are examined, the last digit (the leaf) is placed next to the appropriate stem to the right of the vertical line.

The stem-and-leaf plot for the above data would appear as follows:

32	8 6 9
33	5 9
34	1 5 5
35	7
36	5 1 7

It is useful to re-order the leaves in ascending order. This will help in other activities such as finding the median. The above stem-and-leaf plot would become the following:

32	6 8 9
33	5 9
34	1 5 5
35	7
36	1 5 7

Stem-and-leaf plots display a number of characteristics of a set of data, including the upper and lower values and the general shape of the distribution. It is also possible to see clusters and gaps as well as to immediately see the relative position of any item of importance.

It is sometimes useful to compare two sets of data on a single plot. For example, the following are marks from the same test for two different classes:

Class A
65, 56, 78, 98, 78, 67, 77, 75, 74, 64, 60, 51, 82, 80.

Class B
54, 76, 65, 87, 63, 62, 50, 58, 46, 69, 76, 79, 81, 72.

The back-to-back stem-and-leaf plot appears as follows:

6	4	
8 4 0	5	1 6
9 5 2 3	6	0 4 5 7
9 6 6 2	7	4 5 7 8 8
7 1	8	0 2
	9	8

From this plot it is possible to make an immediate comparison between the achievements of the two classes.

For a more detailed look at stem-and-leaf plots, see Exploring Data from the Quantitative Literacy Series published by Dale Seymour. See also: "Stem-and-Leaf Plots," in Activities for Implementing Curricular Themes from the 'Agenda for Action.'

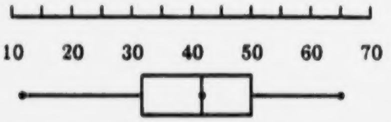
Box plots are useful because they highlight the majority of the data by placing a box around it and show the extremes by drawing lines to them. Box plots will be used to examine confidence intervals in a later course, but for now will be used to organize data and to compare two or more sets of data.

The method for setting up a box plot is as follows:

1. Find the median of the data.
2. Find the median of the upper half of the data. This is the upper quartile.
3. Find the median of the lower half of the data. This is the lower quartile.
4. Find the extremes of the data.
5. On or below a number line, mark the median, quartiles and extremes.
6. Draw a box between the two quartiles. Mark the median within the box. From the quartiles, draw "whiskers" to the extremes.

Example:

From a set of data, it has been established that the median is 42, the lower quartile is 31, the upper quartile is 50, and the extremes are 11 and 65. Draw a box plot to represent this.



Box plots can be very effective in displaying the differences between a number of sets of data, and, unlike stem-and-leaf, are not restricted to comparing two sets only. By placing the plots for a number of sets of data beneath each other, it is possible to see how the majority of the data compare and how the medians and extremes compare to each other.

For further information, see Exploring Data from the Quantitative Literacy Series published by Dale Seymour. See also: "Exploring Baseball Data," Mathematics Teacher, October 1987, and "Exploring Data with Box Plots," Mathematics Teacher, November 1988.

112

Students have some experience with pictographs, bar graphs, line graphs, and circle graphs. Histograms can be drawn by turning a stem-and-leaf plot sideways.

Analysis: Students have spent time calculating the measures of central tendency, hence attention should be paid to using these measures as a form of analysis.

The suggestions made for each organizational method mentioned previously include some suggestions about analysis. Encourage students to look at the appearance of the representation of the data and formulate conclusions. Expect them to defend their analyses and to be critical of them.

Discuss with students the use of an average as a single number used to describe an entire set of data. Students will know how to find "averages" (means). Emphasis should be placed on finding and using medians. Medians can be found directly from stem-and-leaf plots and used in box plots.

The use of the range as a measure of dispersion is of value particularly when extremes are plotted on a box plot. There is probably little need for other measures of dispersion to be introduced at this time. However, the concept can be discussed as students look at the representation of data in stem-and-leaf and box plots.

Provide students with an example such as the following to discuss, in order to provide them with an understanding of the different "averages" and the use of each.
A company is made up of 10 people and the president of the company tells the union that there is no need for a pay raise because the average salary in the company is $38 400/a. The manager disagreed and said that the average salary was only $30 000/a. The head of the secretaries union said that the average was only $20 000/a. Which of the three is correct? The salary list is given below. How do you think each person calculated their "average?" Which average do you think should be used in this situation? Why?

President	$150 000
Manager	$50 000
2 Salesmen	$40 000 ea
4 Secretaries	$20 000 ea
2 Clerks	$12 000 ea

Inferences: Statistical Inferences is the process of drawing judgments or making statements about a population, based on the results obtained from a sample.

Evaluation for Confidence: Introduce the term confidence to describe the expectation that the results can be viewed as true. At this level, an intuitive treatment is intended.

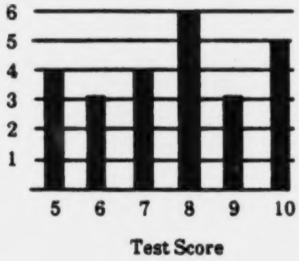
PROCESS/PROBLEM-SOLVING CONTEXT

Problems: 2.3 These problems provide an opportunity to review measures of central tendency and graph interpretation, where necessary.

(a) Five test scores were lost, but a summary of the five lost scores indicates that the mode was 90, the media, 85, and the mean, 83. If the grades were integers and could range from 0 to 100, what is the lowest possible grade from the missing set of scores?

Answer: 50.

(b) Find the average (mean) test score to the nearest integer.



Answer: $\frac{191}{25} \approx 8$

For more **Problems** see Problem Solving in Mathematics: Focus for the Future, p. 72, #3, 7

Activity: A possible follow-up situation to the students' own data collection and analysis activities again using class test data:

Class Scores Example

The data that follow represent the test scores of students in two classes. Each class has twenty students.

Class: A
82, 78, 57, 91, 29, 80, 85, 49, 82, 67,
99, 68, 83, 12, 87, 86, 38, 81, 58, 79

Class B:
72, 82, 74, 84, 81, 84, 76, 12, 72, 71,
70, 93, 13, 90, 80, 73, 91, 70, 99, 88

Have the students "explore" the data at first. They may find at first that there are no clear cut differences between the two classes. For example, the high and low scores in both cases is 99 and 12. The mean score for class A is 69.55 and 73.75 for class B. However, the median for class A is 79.5, while for class B it is 78.0. Is either of these a better indicator?

Have the students create a back-to-back stem-and-leaf plot.

Back-to-Back Stem-and-Leaf Plots for Scores of Class A and Class B

Class A		Class B
2	1	23
9	2	
8	3	
9	4	
87	5	
87	6	
98	7	00122346
76532210	8	012448
91	9	0139

Is there any sort of pattern beginning to emerge? Discuss.

Adapted with permission from "Realistic Examples in Elementary Statistics.", Mathematics Teacher, December 1987 copyright by the National Council of Teachers of Mathematics.

TECHNOLOGY INTEGRATION

Video:
Math Wise (ACCESS): "Locating /Interpreting 01 - Organizing Information."

RESOURCE CORRELATION

M10: pp. 434–447
HM10: pp. 290–291, 300–305, 310–314
MPP10: pp. 398–402, 411–413, 418–420
Mathématiques 10: pp. 398–402, 411–413, 418–420
ED: pp. 7–82
HM10 TRM D/A Masters 12–1, 12–4, 12–5, 12–6, 12–7
MPP10 ASTRM: pp. 17–21

CONCEPT 1

Students will be expected to demonstrate an understanding that similar triangles have equal angles and that the corresponding sides are proportional.

Students will be expected to:

- 1.1 recognize and write the similarity relationship between similar triangles
- 1.2 use the relationship between the lengths of the sides of similar triangles to calculate the lengths of unknown sides
  - 1.2.1 generalize the properties of similar triangles through the use of inductive processes
  - 1.2.2 solve problems involving the use of the properties of similar triangles.

COMMENTS

Students were introduced to the concept of "similarity" in grade seven. The study at that point included recognizing similar and congruent figures. Some review may be necessary prior to discussing similar triangles. The concept of similar triangles can be related to everyday life. For example, an ironing board uses similar triangles to stay level as it is raised. This can become the basis for an outside–of–classroom project.

Define similar triangles as triangles which have corresponding angles of equal measure.

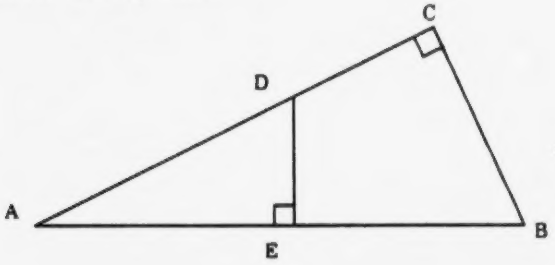
PROCESS/PROBLEM-SOLVING CONTEXT

Activity: In order to introduce similar triangles, give students the measures of three angles and ask them to draw any triangle with those three measures. Have them label the triangle and then collect the measures of the sides of the triangles — the sides should be proportionate. From here, have students construct their own pair of similar triangles — ensuring that the angles are equal and that the sides are proportionate. This then leads into a discussion on the way the similarity relationship is expressed.

Activity: Draw pairs of similar triangles. Measure the lengths of the sides. Find the three ratios between the lengths of the sides of each triangle and compare. Discuss why the ratios will be equal. Mention that similarity can be defined as "a transformation of simultude, i.e., one figure is a magnification of reduction of the other." Discuss whether or not the relationships between ratios of corresponding sides will continue for other pairs of similar polygons.

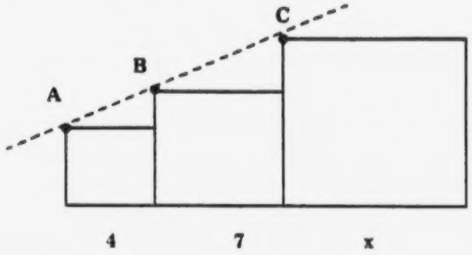
Problem: In the figure, $\measuredangle C$ is a right angle, $DE \perp AB$, $AE = 6$, $EB = 7$, and $BC = 5$. What is the area of EBCD?

Answer: 22.5 sq. units



116

Problem: If the three vertices A, B and C in the three adjacent squares are collinear, find the value of x.



Answer: $x = \frac{49}{4}$

Problem: See Problem Solving in Mathematics: Focus for the Future, p. 58, "The Telephone Pole Problem", solution #2; p. 70, #1.

TECHNOLOGY INTEGRATION

Software:
The Geometric preSupposer: Points and Lines and The Geometric Supposer: Triangles (Sunburst) may both be used in an exploratory way here.

ELECTIVE SUGGESTIONS

Extension of the area of EBCD problem on previous page.

Consider the following theorem: If two figures are similar with ratio of similitude r, then any two corresponding areas are in the ratio r^2.

How could this theorem have been applied to determine the area of EBCD?

RESOURCE CORRELATION

M10:	pp. 480–486
HM10:	pp. 330–331
MPP10:	pp. 438–444
Mathématiques 10:	pp. 438–444
MPP10 ASTRM:	pp. 13–14

CONCEPT 1

Students will be expected to demonstrate an understanding that similar triangles have equal angles and that the corresponding sides are proportional.

Students will be expected to:

- 1.2 use the relationship between the lengths of the sides of similar triangles to calculate the lengths of unknown sides
    - 1.2.3 develop the sine, cosine and tangent ratios in similar right triangles through the use of the properties of similar triangles.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have each student carefully draw one or two of the following 9 right triangles: 5°-85°, 10°-80°, 15°-75° ... 45°-45°. Measure the sides and find the sine, cosine, and tangent ratios. Use calculators to find ratios to two decimal places. Make a chart showing the ratios for each of the angles drawn. Discuss some of the characteristics of the list (for example, sine and tangent ratios increase as angles increase, cosine ratios decrease, $\tan 45^\circ = 1$, $\sin 45^\circ = \cos 45^\circ$, range of sin, cos, tan for $\theta \measuredangle 90^\circ$.)

TECHNOLOGY INTEGRATION

Software:
The Geometric preSupposer: Points and Lines and The Geometric Supposer: Triangles (Sunburst) may both be used in an exploratory way here.

RESOURCE CORRELATION

M10:	pp. 487, 494
HM10:	pp. 332, 335
MPP10:	pp. 445–447, 451
Mathématiques 10:	pp. 445–447, 451

CONCEPT 2

Students will be expected to demonstrate an understanding that the sine, cosine and tangent ratios of a given angle are particular ratios between pairs of corresponding sides of similar, right triangles that contain the given angle.

Students will be expected to:

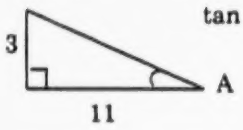
2.1 find the sine, cosine and tangent ratios, and the measures of the acute angles within right triangles, given the measures of any two sides
  2.1.1 use a calculator to find the sine, cosine and tangent ratios of any acute angle
  2.1.2 use a calculator to find the measure of an acute angle given the value of one of the trigonometric ratios of the angle
  2.1.3 solve problems in which an angle measure in a right triangle must be found.

COMMENTS

Students should be taught to use a scientific calculator to find the sine, cosine and tangent ratios. Angles should be whole degrees only.
Example:
$\tan 56 \doteq 1.4826$ or $\tan 56 \approx 1.4826$

Students should be taught to use a scientific calculator. Angles should be rounded off to the nearest degree. Students should be familiar with the following type of question.
Example:
$\sin\ \theta \doteq 0.4567$ or $\sin \theta \approx 0.4567$
$\theta \doteq 27°$ $\theta \approx 27°$

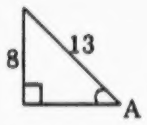
Students should use a calculator and be reminded that they are finding a specific ratio. They should always note which ratio is being found. Write the ratio to a maximum of four decimal places.



$\tan \measuredangle A = {}^{3}/_{11}$

$\doteq 0.2727$

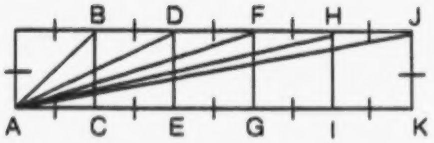
Examples:



Find the measure of $\measuredangle A$, to the nearest degree.

PROCESS/PROBLEM-SOLVING CONTEXT

Problem:



Find the measures of the three trigonometric ratios of ∢A and the measure of ∢A in triangles ABC, ADE, AFG, AHI and AJK. Can you generalize the ratios and predict the measure of ∢A in the tenth triangle?

Problem:
As tan θ increases, cos θ decreases. What is the measure of the angle where they are equal? What are the measures of the sides of a triangle in which $\tan\theta = \cos\theta$?

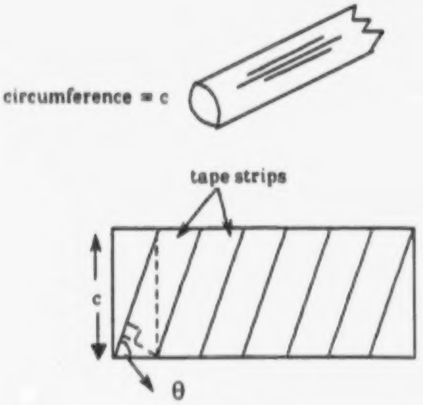
Activity: Ask students to develop processes whereby in any right triangle, given any two sides or one side and one non-right angle, they could find all of the other measures of the triangle. Remind them that they know the Pythagorean Theorem. Have them develop quick checks that their answers are reasonable, i.e., what must be the sum of the angles? What must always be true about the sum of the lengths of the two shorter sides in the triangle?

Activity: Have students draw right triangles randomly, measure the lengths of any two sides or one angle and the length of one side, then calculate the unknown measures. Check the calculations by measurement.

Activity: Have students develop a test whereby they can check an angle for "rightness" without using a protractor.

Problem:
A parking lot alongside a curling rink has room for 20 cars parked perpendicular to the building. In order to increase the width of the driveway, it was decided to have the cars park at a 75° angle to the building. Assuming that each parking space measures 3 m × 7 m, how many cars can be parked now?
Answer: 19

Problem: Consider a baseball bat handle slightly modified so that it is cylindrical in shape. It can be taped so that there is no overlap and no gaps, that is, no wood is showing. If the circumference of the handle is 8 cm and the width of the tape is 2.5 cm, find the angle, relative to the line of the handle, at which the tape should be wrapped for no overlap and no gaps to occur. See the diagram below:



Find θ.

Answer: 72° to nearest degree

Extension: If c = circumference of the handle and w = width of the tape, express cos θ and θ in terms of w and c.

Adapted with permission from Imaginative Ideas for the Teacher of Mathematics Grades 11–12: Ranucci's Reservoir, copyright 1988 by the National Council of Teachers of Mathematics.

ELECTIVE SUGGESTIONS

Problem: (Requires expressing angle measure to one decimal place): A wire 8 cm long is to be cut into three pieces that are to be used for the sides of a triangle. If all three pieces have integral lengths, what is the length of the shortest side? What is the measure of the angles?
Answer: 2 cm; 39°, 70.5°, 70.5°

Use this problem initially as a way of reminding students about conditions regarding the lengths of sides of triangles.

RESOURCE CORRELATION

M10:	pp. 487–507
HM10:	pp. 332–340
MPP10:	pp. 448–464
Mathématiques 10:	pp. 448–464

CONCEPT 2

Students will be expected to demonstrate an understanding that the sine, cosine and tangent ratios of a given angle are particular ratios between pairs of corresponding sides of similar, right triangles that contain the given angle.

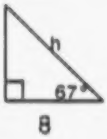
Students will be expected to:

2.2 find the measure of a side of a right triangle given the measure of a second angle and the length of a side

  2.2.1 solve problems in which the length of a side of a right triangle must be found.

COMMENTS

In ΔABC, $\measuredangle C = 90^\circ$, $a = 5$, $b = 8$. Find the measure of $\measuredangle B$, to the nearest degree.



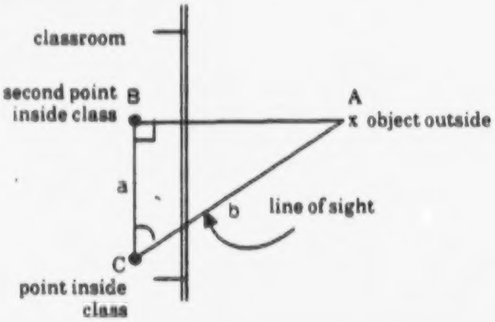
Find the length of h, to one decimal.

In ΔXYZ, $\measuredangle X = 90^\circ$, $\measuredangle Y = 43^\circ$, $x = 20$. Find the length of z.

Students should learn to solve problems which can be diagrammed using right triangles. They should understand and use terms such as inclination, angle of depression and angle of elevation. They should be taught the value of diagramming problem situations, and taught about the level of accuracy required.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: If you have a classroom with windows, have students solve the following problem:



Determine the length a, and the measure of $\measuredangle BCA$, and use these values to determine the distance from C to A.
(Adapted from M10 TRB, Teaching Notes, p. 45.)

Problem: In a 5-12-13 right triangle, what is the length of the bisector of the larger acute angle?
Answer: $\frac{5\sqrt{13}}{3} \approx 6.0$

TECHNOLOGY INTEGRATION

Spreadsheets may be used to detemine the relationships between the angles, lengths of sides and trigonometric ratios of a right triangle. Apple Works and Microsoft Works are listed by Alberta Education as Basic Resources.

RESOURCE CORRELATION

M10: pp. 487–507
HM10: pp. 332–340
MPP10: pp. 448–464
Mathématiques 10: pp. 448–464
HM10 TRM D/A Masters 13–1, 13–2

REVERSE TEXTUAL CORRELATION
ADDISON-WESLEY PUBLISHERS LTD.
MATHEMATICS 10

Page	Unit	Learner Expectation Number	TRM Page
xiv–xxiii	Number Systems	1–Review	
2–8	Number Systems	1–Review	
9–10	Number Systems	3.2	37–39
11	Coordinate Geometry and Graphing	1.1.2	76–77
14–17	Number Systems	1.1, 1.2, 1.3, 1.4	27–30
18–29	Number Systems	2.1, 2.2, 2.4, 3	31–39
26–27	Number Systems	3.1	37–39
30–41	Operations with Radicals		
46–48	Operations on Polynomials	1.1, 1.2	40–43
52–58	Operations on Polynomials	2.1, 2.2, 2.3	44–46
59–61	Scientific Notation		
65–67	Rational Exponents		
72–74	Operations on Polynomials	3.1	47–48
76–78	Operations on Polynomials	3.4	55–56
76–81	Operations on Polynomials	3.2	49–54
84–92	Operations on Polynomials	3.2, 3.3	49–54
93–96	Factoring Polynomials	1.1, 1.2	57–58
98–106	Factoring Polynomials	1.3, 1.4, 1.5	59–56
108	Factoring Polynomials	1.6	67
109–138	Rational Expressions		
140–143	Equations and Inequalities	1.1	70–74
144–147	Equations involving Rational Expressions		
148–149	Equations and Inequalities	2.1	75
150–161	Equations and Inequalities	1.1	70–74
156–157	Operations on Polynomials	1.2.1	40–43
164–167	Coordinate Geometry and Graphing	2.1, 2.2	85–86
170–173	Graphing Linear Inequalities		
174–177	Graphing Non-Linear Relations		
180–184	Factoring Polynomials	2.1, 2.2	68–69
190–193	Coordinate Geometry and Graphing	1.1	76–77
196–199	Coordinate Geometry and Graphing	1.2	76–77
200–211	Coordinate Geometry and Graphing	1.3	78–79
212-213	Coordinate Geometry and Graphing	2.1, 2.2	85–86
218–222	Coordinate Geometry and Graphing	1.4	84
223–231	Coordinate Geometry and Graphing	2.1, 2.2	85–86
233-237	Coordinate Geometry and Graphing	2.5, 2.6	88–90
234–237	Coordinate Geometry and Graphing	3.1	92–93
240–244	Coordinate Geometry and Graphing	3.1	92–93
245–249	Coordinate Geometry and Graphing	2.3, 2.4	87
246–249	Coordinate Geometry and Graphing	2.5, 2.6	88–90
247–249	Coordinate Geometry and Graphing	2.7	91
250–252	Coordinate Geometry and Graphing	3.1	92–93
258–270	Systems of Linear Equations	1.1, 1.2	94–96
271–273	Systems of Linear Equations	2.1	97
276–296	Systems of Linear Equations	1.2	94–96
301–340	Coordinates and Transformations		
341–432	Geometry and Reasoning		
434–447	Statistics	2	101-107
448–455	Statistics: Bivariate Data Analysis		
456–465	Statistics	1	98–100
466–475	Probability		
480–486	Trigonometry	1.1, 1.2, 1.2.2	108–109
487, 494	Trigonometry	1.2.3	110
487–507	Trigonometry	2.1	111–113
487–507	Trigonometry	2.2	114–115

REVERSE TEXTUAL CORRELATION
HOLT, RINEHART AND WINSTON OF CANADA LTD.
HOLTMATH 10

Page	Unit	Learner Expectation Number	TRM Page
1-7	Number Systems	1	27-30
9-11	Number Systems	1.1, 1.2, 1.3, 1.4	27-30
12-13	Rational Number Properties		
14-16	Number Systems	2.1, 2.2	31-36
14-16	Number Systems	3.1	37-39
17-19	Equations and Inequalities	1.1, 1.2	70-74
20-23	Percent and Proportion		
24-25	Number Systems	3.1	37-39
24-25	Equations and Inequalities	2.1	75
29-52	Problem Solving		
54-57	Operations on Polynomials	2.1, 2.2, 2.3	44-46
58-59	Scientific Notation		
60-67	Operations with Radicals		
68-69	Rational Exponents		
70-71	Equations and Inequalities	1.2	70-74
71	Number Systems	2.2	31-36
80-83	Operations on Polynomials	1.1, 1.2	40-43
84-85	Operations on Polynomials	3.1	47-48
86-92	Operations on Polynomials	3.2, 3.3	49-54
93	Operations on Polynomials	3.4	55-56
93-95	Factoring Polynomials	1.1, 1.2	57-58
96-102	Factoring Polynomials	1.3, 1.4, 1.5	59-66
103	Factoring Polynomials	1.6	67
103-105	Operations on Polynomials	3.4, 3.5	55-56
106-110	Factoring Polynomials	2.1, 2.2	68-69
113-128	Rational Expressions		
129-152	Geometry		
160-161	Coordinate Geometry and Graphing	1.4	84
160-163	Coordinate Geometry and Graphing	1.3	80-83
164-165	Coordinate Geometry and Graphing	1.1	76-77
166-167	Coordinate Geometry and Graphing	1.2	78-79
175-183	Coordinate Geometry and Graphing	2.1, 2.2	85-86
184-185	Inverse Variation		
187	Interpolation and Extrapolation		
196-197	Coordinate Geometry and Graphing	2.1, 2.2	85-86
198-199	Coordinate Geometry and Graphing	1.4	84
198-199	Coordinate Geometry and Graphing	2.3, 2.4	87
200-201	Coordinate Geometry and Graphing	2.5, 2.6	88-90
202-203	Coordinate Geometry and Graphing	3.1	92-93
204-205	Families of Lines		
206-208	Coordinate Geometry and Graphing	3.1	92-93
209-217	Systems of Linear Equations	1.1, 1.2	94-96
218-219	Systems of Linear Equations	2.1	97
220-223	Graphing Inequalities/Linear Programming		
231-262	Deductive Geometry		
263-288	Transformations		
290-291	Statistics	2.1, 2.4, 2.5	101-107
292-295	Statistics	1.1, 1.2, 1.3	98-100
296-299	Frequency Tables and Histograms		
300-305	Statistics	2.2, 2.3, 2.5, 2.6	101-107
306-309	Grouped Data/Data Distribution		
310-314	Statistics	2.2, 2.3, 2.5, 2.6	101-107
314-315	Statistics	1.1, 1.2, 1.3	98-100
316-322	Probability		
330-331	Trigonometry	1.1, 1.2.1, 1.2.2	108-109
332, 335	Trigonometry	1.2.3	110
332-340	Trigonometry	2.1	111-113
341-350	Matrices		
351-357	Volume and Surface Area		
358-359	Sketching in Three Dimensions		
360-361	Symmetry in Three Dimensions		
362-371	Vectors		
372	Computer Application in Trigonometry		
373-382	Statistics -Bivariate Data		

REVERSE TEXTUAL CORRELATION
NELSON CANADA MATHEMATICS PRINCIPLES & PROCESS 10
(ADVANCED LEVEL MATHEMATICS 10)

Page	Unit	Learner Expectation Number	TRM Page
10–16	Number Systems	1 – Review	
17	Number Systems	1.1, 1.2	27–30
10–21	Radicals		
22	Number Systems	2.1, 2.2, 2.4	31–36
22–25	Number Systems	3.1	37–39
22–25	Equations and Inequalities	2.1	70–74
26–33	Operations on Polynomials	2.1, 2.2, 2.3	44–46
34	Rational Exponents		
35–42	Operations on Radicals		
43–45	Number Systems	2.2	31–36
46–55	Problem Solving		49–54
58–61	Operations on Polynomials	3.2	40–43
61	Operations on Polynomials	1.2	55–56
62–65	Operations on Polynomials	3.4	47–48
66–69	Operations on Polynomials	3.1	70–74
70–71	Equations and Inequalities	1.1	49–54
72–75	Operations on Polynomials	3.2, 3.3	57–58
76–80	Factoring Polynomials	1.1, 1.2	59–66
81–91	Factoring Polynomials	1.3, 1.4, 1.5	67
92	Factoring Polynomials	1.6	68–69
93–96	Factoring Polynomials	2.1, 2.2	
99–120	Rational Expressions		
121–123	Inequalities		
126–130	Coordinate Plane/Plotting		
131–136	Coordinate Geometry & Graphing	2.1, 2.2	85–86
137–143	Graphing: Domain & Range/Linear & Non-Linear		
144–146	Graphing Inequalities		
147–152	Coordinate Geometry & Graphing	1.1	76–77
153–155	Coordinate Geometry & Graphing	1.2	78–79
156–159	Coordinate Geometry & Graphing	1.3	80–83
160–163	Coordinate Geometry & Graphing	1.4	84
165–174	Functions		
175	Coordinate Geometry & Graphing	1.3.1	80–83
176–177	Coordinate Geometry & Graphing	2.3, 2.4	87
179–185	Coordinate Geometry & Graphing	2.5, 2.6	88–90
186–189	Parallel and Perpendicular Lines		
190–196	Coordinate Geometry & Graphing	3.1	92–93
197–201	Families of Lines		
202	Coordinate Geometry & Graphing	3.1	92–93
206–209	Systems of Linear Equations	1.1	94–96
212	Systems of Linear Equations	2.1	97
213–229	Systems of Linear Equations	1.1, 1.2	94–96
230–233	More Complex Systems/Literal Coefficients		
236–237	Operations on Polynomials	1.2.1	40–43
236–252	Systems of Linear Equations	1.2.1	94–96
254–265	Systems of Linear Inequalities/Linear Programming		
266–273	Coordinate Geometry & Graphing	2.1, 2.2	85–86
274–279	Inverse Variation		
283–287	Coordinate Geometry & Graphing	2.1, 2.2	85–86
289–330	Geometry I		
331–360	Geometry II		
361–384	Three Dimensional Geometry		
385–392	Statistics	1.1, 1.2, 1.3	98–100
393–397	Frequency Distribution/Histograms		
398–402	Statistics	2.2, 2.3, 2.5	101–107
403–405	Grouped Data		
406–410	Dispersion/Deviation		
411–413	Statistics	2.2, 2.5	101–107
414–417	Bivariate Data		
418–420	Statistics	2	101–107
421–435	Probability		
438–444	Trigonometry	1.1, 1.2.1, 1.2.2	108–109
445–447	Trigonometry	1.2.3	110
448–464	Trigonometry	2.1	111–113
448 – 464	Trigonometry	2.2	114–115
467–510	Vectors and Matrices		
511–541	Transformations		

REVERSE TEXTUAL CORRELATION – ÉDITIONS BEAUCHEMIN LTÉE
MATHÉMATIQUES 10 (10ᵉ Année)

Page	Unité	Attentes	TRM Page
10-16	Les nombres	1 - Revue	
17	Les nombres	1.1, 1.2	27-30
10-21	Les radicaux		
22	Les nombres	2.1, 2.2, 2.4	31-36
22-25	Les nombres	3.1	37-39
22-25	Les équations et les inéquations	2.1	70-74
26-33	Les opérations des polynômes	2.1, 2.2, 2.3	44-46
34	Les exposants et les nombres rationnels		
35-42	Les opérations sur les radicaux		
43-45	Les nombres	2.2	31-36
46-55	La résolution de problèmes		49-54
58-61	Les opérations des polynômes	3.2	40-43
61	Les opérations des polynômes	1.2	55-56
62-65	Les opérations des polynômes	3.4	47-48
66-69	Les opérations des polynômes	3.1	70-74
70-71	Les équations et les inéquations	1.1	49-54
72-75	Les opérations des polynômes	3.2, 3.3	57-58
76-80	La factorisation des polynômes	1.1, 1.2	59-66
81-91	La factorisation des polynômes	1.3, 1.4, 1.5	67
92	La factorisation des polynômes	1.6	68-69
93-96	La factorisation des polynômes	2.1, 2.2	
99-120	Les expressions rationnelles		
121-123	Les inéquations		
126-130	Les coordonées cartésiannes		
131-136	Les coordonées géométriques	2.1, 2.2	85-86
137-143	Les graphiques: la domaine et la région linéaire et non-linéaire		
144-146	Les graphiques: les inéquations et les régions		
147-152	Les coordonées géométriques	1.1	76-77
153-155	Les coordonées géométriques	1.2	78-79
156-159	Les coordonées géométriques	1.3	80-83
160-163	Les coordonées géométriques	1.4	84
165-174	Les fonctions		
175	Les coordonées géométriques	1.3.1	80-83
176-177	Les coordonées géométriques	2.3, 2.4	87
179-185	Les coordonées géométriques	2.5, 2.6	88-90
186-189	Les lignes parallèles et perpendiculaires		
190-196	Les coordonées géométriques	3.1	92-93
197-201	Les familles de droite		
202	Les coordonées géométriques	3.1	92-93
206-209	Les systèmes d'équations linéaires	1.1	94-96
212	Les systèmes d'équations linéaires	2.1	97
213-229	Les systèmes d'équations linéaires	1.1, 1.2	94-96
230-233	Des systèmes plus complex des coefficients littéraux		
236-237	Les opérations des polynômes	1.2.1	40-43
236-252	Les systèmes d'équations linéaires	1.2.1	94-96
254-265	Les systèmes d'inéquations la programmation linéaire		
266-273	Les coordonées géométriques	2.1, 2.2	85-86
274-279	Les variations inverses sur graphique		
283-287	Les coordonées géométriques	2.1, 2.2	85-86
289-330	Geométrie I		
331-360	Geométrie II		
361-384	La géométrie et trois dimensions		
385-392	Notions de statistique	1.1, 1.2, 1.3	98-100
393-397	La distribution le frequences et les histogrammes		
398-402	Notions de statistiques	2.2, 2.3, 2.5	101-107
403-405	Les données bivariées		
406-410	Les measures de dispersion et de variation		
411-413	Notions de statistiques	2.2, 2.5	101-107
414-417	Les données bivariées		
418-420	Notions de statistiques	2	101-107
421-435	La probabilité		
438-444	La trigonométrie	1.1, 1.2.1, 1.2.2	108-109
445-447	La trigonométrie	1.2.3	110
448-464	La trigonométrie	2.1	111-113
448-464	La trigonométrie	2.2	114-115
467-510	Les vecteurs et les matrices		
511-541	Les transformations		

127

MATH 13

Program Emphasis

Required Content

The major part of the content of each high school mathematics course consists of topics required of all students who take the course. The required content comprises 80% of the course and contains the concepts, skills and attitudes that all students are expected to acquire. As well, the required portion of all courses includes specific expectations in regard to problem solving and the use of technology.

Elective Material

Each course will include a compulsory component comprising 20% of the course, made up of elective material which will be consistent with the content and expectations of their required component. The purpose of the elective is to provide for enrichment, remediation, or innovative experimental presentations or activities. It is not intended to provide acceleration or advanced placement. However, horizontal enrichment and extension is appropriate and students should have access to elective material that serves their individual needs and interests.

Suggested Program Emphasis (in %) for Required Portion of Mathematics 13

Unit	Suggested Program Emphasis (in %)
Number Systems	10
Polynomials and Factoring	28
Coordinate Geometry and Graphing	18
Relations	12
Statistics	14
Geometry	18

Number Skills
Program Emphasis - 10%

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding of the four arithmetic operations on rational numbers, carried out in conjunction with the operations of rounding and estimation.	Students will be expected to perform calculations on rational numbers, using mental calculation, calculators and paper-and-pencil techniques, including multiple operations involving the rules for order of operations.	Students will be expected to use calculators in calculations involving rational numbers. Students will be expected to solve problems involving calculations with rational numbers.	pp. 133–137
	Students will be expected to perform calculations using percents.	Students will be expected to use calculators in calculations that involve the use of percents. Students will be expected to solve problems that involve the use of percent calculations.	pp. 138–142
	Students will be expected to calculate numerical squares and square roots.	Students will be expected to use electronic calculators to find numerical squares and square roots.	p. 143
Students will be expected to demonstrate an understanding of the laws and properties pertaining to calculations involving expressions with exponents.	Students will be expected to perform calculations involving powers with integral bases and integral exponents.	Students will be expected to verify the properties of zero and negative exponents through the use of numeric examples. Students will be expected to use calculators to evaluate the powers with integral bases and integral exponents.	pp. 144–145

Polynomials and Factoring
Program Emphasis -28%

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that polynomials can be used to describe some mathematical situations in which values vary.	Students will be expected to identify the following as they occur in the study of polynomials: term, variable, factor, monomial, binomial, trinomial, polynomial, numerical coefficient, degree, exponent, base, power. Students will be expected to evaluate a polynomial for given values of the variables. Students will be expected to use polynomials to describe mathematical situations.		pp. 146–149
Students will be expected to demonstrate an understanding of the laws and properties pertaining to calculations involving expressions with exponents.	Students will be expected to use the multiplication law of exponents for powers with literal bases and whole number exponents, $a^m \times a^n = a^{m+n}$.	Students will be expected to verify the multiplication law of exponents through the use of examples.	pp. 150–152
	Students will be expected to use the power law for exponents, $(a^m)^n = a^{mn}$.	Students will be expected to verify the power law for exponents through the use of examples.	
	Students will be expected to use the division law of exponents for powers with literal bases and whole number exponents, - $a^m/a^n = a^{m-n}$.	Students will be expected to verify the division law for exponents through the use of examples.	
	Students will be expected to use zero and negative exponents.	Students will be expected to verify the laws of zero and negative exponents through the use of examples.	
Students will be expected to demonstrate an understanding that calculations and operations on polynomials depend on the application of the laws of operations for numbers and powers.	Students will be expected to add and subtract polynomials.		pp. 153–155
	Students will be expected to multiply polynomials by monomials. Students will be expected to multiply binomials by binomials. Students will be expected to recognize the expansion of the square of a binomial and the product of conjugate binomials as special cases.		pp. 156–161
	Students will be expected to divide polynomials by monomials.		pp. 162–163

Polynomials and Factoring
Program Emphasis -28% (continued)

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that polynomials can be written as a product of their factors.	Students will be expected to factor polynomials that have a monomial common factor.		pp. 164–167
	Students will be expected to factor trinomials of the form $ax^2 + bx + c$; where $a = 1$ or is a common factor.		
	Students will be expected to recognize and factor perfect square trinomials recognizing them as a special case.		
	Students will be expected to recognize and factor difference of square polynomials, recognizing them as a special case.		
	Students will be expected to factor polynomials that require a combination of methods in order to be fully factored.	Students will be expected to solve problems that involve factoring polynomials	

Coordinate Geometry and Graphing
Program Emphasis – 18%

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that points and lines can be represented on a Cartesian plane and that they have characteristics which can be defined and measured.	Students will be expected to use the formula for the slope of a line passing through two given points.	Students will be expected to solve the problems that involve the use of the slope formula.	pp. 168–172
		Students will be expected to verify the relationships between the slopes of parallel and perpendicular lines by diagraming pairs of lines on a Cartesian plane.	
		Students will be expected to solve problems that involve the relationships between the slopes of parallel and perpendicular lines.	
	Students will be expected identify the characteristics of lines that have positive, negative, zero and undefined slopes.		
	Students will be expected to use the Pythagorean relation to find the distance between two points on the Cartesian plane.	Students will be expected to solve problems that involve finding the distance between two points on the Cartesian plane.	pp. 173–174
	Students will be expected to find the midpoint of a segment.	Students will be expected to solve problems that involve finding the midpoint of a line segment.	p. 175

Coordinate Geometry and Graphing
Program Emphasis - 18% (continued)

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that ordered pairs which satisfy linear equations correspond to coordinates on the Cartesian plane and can be used to graph the equation on the Cartesiah plane.	Students will be expected to find sets of ordered pairs that satisfy a linear equation and use them to sketch the graph of the equation.		p. 176
	Students will be expected to find the x- and y-intercepts of linear equations.		pp. 177-178
	Students will be expected to graph linear equations by drawing a graph through the x- and y-intercepts.		
	Students will be expected to write linear equations in the slope-intercept form, $y = mx + b$.		pp. 179-181
	Students will be expected to graph linear equations using the slope and the y-intercept.	Students will be expected to investigate the effects of the parameters m and b in $y = mx + b$. Students will be expected to use calculators or computers to draw the graphs of any linear equations.	
	Students will be expected to recognize and graph the equations of lines that are parallel to the x- and y-axes.		p. 182

133

Relations
Program Emphasis - 12%

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that phenomena can be related and that these relations can be represented by graphs and tables of values.	Students will be expected to sketch graphs that describe physical phenomena or everyday occurrences including the identification of the dependent and independent variables and appropriately labeled axes. Students will be expected to interpret graphs drawn to describe the relationships between real life variables. Students will be expected to interpolate and extrapolate to find information from given graphs and tables of data.	Students will be expected to solve problems involving physical phenomena and everyday occurrences by modelling them graphically.	pp. 183–187
Students will be expected to demonstrate an understanding that some pairs of variables relate to each other directly or inversely, according to a relationship that can be written as an equation.	Students will be expected to recognize the relations in which the variables vary directly. Students will be expected to recognize relations in which the variables vary inversely.	Students will be expected to solve direct variation problems. Students will be expected to solve partial variation problems. Students will be expected to solve inverse variation problems.	pp. 188–189

Statistics
Program Emphasis – 14%

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that populations have certain characteristics which can be determined by analysing the characteristics of a suitable sample of that population.	Students will be expected to determine a sample that will adequately and accurately represent a population.		pp. 190–193
	Students will be expected to evaluate samples for bias, appropriateness of sample type, amount of data and randomness.		
	Students will be expected to draw inferences about the population from which the sample was taken and express it using the language of probability.	Students will be expected to determine a valid sample, design and carry out a simple survey, and make a valid inference about the population based on the results.	
		Students will be expected to defend or oppose a generalization that is made about a population from a sample .	
Students will be expected to demonstrate an understanding that data collected from a sample must be organized, presented and analysed in order that valid inferences can be made.	Students will be expected to identify a situation, formulate an hypothesis and plan what data needs to be collected.		pp. 194–201
	Students will be expected to organize data using stem-and-leaf plots and box plots.		
	Students will be expected to choose and calculate appropriate measures of central tendency for sets of data.		
	Students will be expected to represent data using appropriate graphs.		
	Students will be expected to do simple visual analyses on sets of data organized in stem-and-leaf plots, box plots and graphs.	Students will be expected to derive and defend conclusions based on the visual analyses of sets of data.	
	Students will be expected to evaluate intuitively an analysis of data for the confidence with which the results can be accepted.		

Geometry
Program Emphasis – 18%

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that angles can relate to each other, in ways that can be determined by identifying particular characteristics of the segments that form them. .	Students will be expected to use the following terms: parallel, perpendicular, transversal, corresponding angles, alternate angles, interior angles.		pp. 202–203
	Students will be expected to use the relationship between the vertically opposite angles to calculate the measures of angles.	Students will be expected to verify that vertically opposite angles are equal.	
	Students will be expected to use the relationship amongst angles made by parallel lines and a transversal.	Students will be expected to verify the relationships amongst angles made by cutting parallel lines with a transversal.	
Students will be expected to demonstrate an understanding that triangles can be uniquely determined by appropriate combinations of sides and angles.	Students will be expected to construct a triangle, given two sides and the included angle (SAS), three sides (SSS) and two angles and a side (ASA).	Students will be expected to verify that any of the three conditions SAS, SSS or ASA uniquely determine a triangle.	pp. 204–205
Students will be expected to demonstrate an understanding that similar triangles have the same shape but may differ in size, that angles are equal and that the ratios between corresponding sides are equal and that congruent triangles are similar triangles of the same size.	Students will be expected to use the relationship between the lengths of the sides of similar triangles to calculate the lengths of unknown sides.	Students will be expected to verify that the ratios between corresponding sides of similar triangles are equal.	pp. 206–208
		Students will be expected to solve problems by using the relationships between the sides of similar triangles.	
	Students will be expected to identify the postulate by which a pair of triangles can be shown to be congruent.		

Attitudes

Within the study of the content of this course, it is expected that *students will be encouraged to*

- be confident in their mathematical knowledge and in their ability to acquire new knowledge;
- demonstrate persistence, resolve, flexibility and ingenuity in finding the solution to problems;
- develop an attitude of intellectual curiosity and openness to new ideas, insights and change in the pursuit of mathematical knowledge;
- exhibit an attitude of curiosity and spontaneity and appreciate creativity and innovation in representing situations mathematically;
- value and be aware of the importance of communication skills in mathematics;
- appreciate and develop a positive attitude toward the usefulness of computational competence, mathematical processes and problem solving skills which are used in the decision making and modeling processes in our society;
- value the contributions of mathematics to our culture and civilization.

Problem Solving

Problem solving is to be integrated into all areas of the high school mathematics program because a major purpose of studying mathematics is to learn to solve problems. A student who has successfully completed this course will understand that thinking and problem-solving skills are required in order to explain the world, and that mathematics can provide some of the tools for problem solution. Students will understand that problems can be solved in many ways, and that problems can be routine, requiring only the application of a known procedure or algoarithm, or non-routine, requiring the development of a process or the conducting of an investigation to solve them.

Students will be expected to use a variety of procedures to help them understand mathematical problems. In particular, they will be expected to

- read the problem thoroughly
- identify and clarify key components
- restate the problem using familiar terms
- evaluate the given information as to whether it is insufficient or extraneous
- interpret pictures, charts and graphs
- determine any hidden assumptions
- ask relevant questions
- identify given, needed and wanted information
- diagram or model the problem situation
- use suitable notation
- determine valid inferences.

Students will be expected to develop a variety of strategies for use in the solution of mathematical problems. In particular, they will be expected to

- conduct an investigation
- use estimation and approximation
- develop equations or use formulae
- use flow charts
- make lists and charts
- look for patterns
- work backward

- break the problem into smaller parts
- look for a simpler or related problem
- make diagrams or models
- use manipulatives
- choose and sequence a series of mathematical operations
- sketch the graph of a problem situation
- establish procedures to gather and organize data.

Students will be expected to develop a variety of skills which can be used to carry out the plan for the solution of a problem. In particular, they will be expected to

- apply selected strategies.
- present ideas clearly
- document the solution process
- utilise appropriate group behaviours
- use calculators and computers
- evaluate problem-solving strategies for effectiveness
- alter or abandon non-productive strategies
- alter or abandon non-productive strategies
- search for additional information
- ask questions
- be open to inspirations, intuitions and "bright ideas".

Students will be expected to employ a variety of skills to help them look back over the solution of a problem. In particular, they will be expected to

- determine the reasonableness of an answer
- explain the solution in oral or written form
- consider the possibility of additional solutions
- search for other strategies and processes of solution
- create and solve similar problems
- note the characteristics that will be identifiable in similar problems
- make a generalization.

CONCEPT 1

Students will be expected to demonstrate an understanding of the four arithmetic operations on rational numbers, carried out in conjunction with the operations of rounding and estimation.

Students will be expected to:

1.1 **perform calculations on rational numbers, using mental calculation, calculators and paper-and-pencil techniques, including multiple operations involving the rules for order of operations**
  - 1.1.1 **use calculators in calculations involving rational numbers**
  - 1.1.2 **solve problems involving calculations with rational numbers.**

COMMENTS

Students have done some previous work with rational numbers. In junior high, they perform the operations of addition, subtraction, multiplication, and division on positive and negative rational numbers; apply the rules of order of operations to evaluate expressions that involve rational numbers; convert rational numbers from $^a/_b$ to decimal form ($b < 10$ or b is a power of 10). Students should be familiar with integers and rational numbers and should have a good understanding of the processes used to calculate with them. Emphasis should be on using calculation techniques appropriate to the questions, whether mental, paper–and–pencil or electronic. It is important to contrast exact calculation and approximate calculation where each is appropriate. For example, adding $^2/_3 + ^5/_8$ by pencil–and–paper is an exact calculation, while doing the same calculation on a standard calculator will yield an approximation. Encourage students to use their calculators and spend time teaching them how to enter and work with arithmetic expressions correctly.

Emphasize the order of operations through such calculator work as:
Evaluate $0.2 \times 0.5 + 6.4 \div 0.2$.

Note: Sets and set notation are not part of the junior high mathematics program. Make sure students are familiar with terminology associated with set theory.

Estimation and Mental Calculation:

The 1986 NCTM Yearbook, Estimation and Mental Computation, focuses on these topics.

Many of the articles are appropriate for Grade 10 mathematics. Those cited below are particularly appropriate as teacher resources.

"Reasons for Estimating" and "Mental Calculation: Anachronism or Basic Skill?" provide excellent rationales for actively encouraging the development of estimation and mental computation skills:

The following two references discuss the need for (i) the development of an awareness for, and an appreciation of, estimation; (ii) the development of number sense; (iii) the development of number concepts; and (iv) the development of estimation strategies. They also provide teaching suggestions.

"Teaching Computational Estimation: Establishing an Estimation Mind-set"
"Teaching Computational Estimation: Concepts and Strategies"

"Evaluating Computational Estimation" takes up the question of how to evaluate students' ability to estimate. Testing guidelines are presented.

The NCTM book Activities for Implementing Curricular Themes from the 'Agenda for Action' contains a variety of estimation-related activities. Some of these include percent estimations, and are listed on page 225.

"Calculators and Estimation" contains activities designed to help develop skills in estimating factors and divisors.

See also "Estimating with 'Nice' Fractions," Mathematics Teacher, November 1986 for activities designed to help students become better estimators when working with fractions.

PROCESS/PROBLEM-SOLVING CONTEXT

Problems:

1. Use a calculator to find an approximation (to 1 decimal place) for the sum of $1\,{}^{3}/_{7} + 2\,{}^{11}/_{15}$.
2. Is $6\,{}^{3}/_{4} - 3\,{}^{5}/_{6} + 2\,{}^{5}/_{9}$ greater than 5?
3. Write the following in order from smallest to largest: ${}^{5}/_{8}$, $0.\dot{6}$, ${}^{2}/_{3}$, 0.67.
4. Have students develop a procedure to use a calculator to do calculations involving fractions.

   Example: To add ${}^{5}/_{27} + {}^{25}/_{36}$ use the following steps:

   a) Multiply the denominators together and use as the denominator of your answer. (Why?)

   b) Multiply the numerators in each fraction by the denominator in the other and add the two products. What will the following calculation give?
   $5 \times 36 + 25 \times 27$

   c) Both of the original denominators can be divided by 9. How can we use this fact to write our sum in lowest terms?

Problem: Dave computed an answer to be 22.5. However, in the last step of the computation he multiplied by 0.3 instead of dividing by 0.3. What is the correct answer?

Answer: 250.

This problem provides an opportunity for students to use the "work backward" problem-solving strategy.

Problem: If $200 \leq a \leq 400$ and $600 \leq b \leq 1200$, then the largest value of the quotient $^{b}/_{a}$ is

(A) 3/2 (B) 3 (C) 6 (D) 300 (E) 600

Answer: 6.

Problem: See Problem Solving Challenge for Mathematics, pp. 10–11, "Problem Two."

Example Estimation and Mental Calculation Activities and Problems:

(1) Context has been shown to be important in successful estimation. Pose estimation situations such as the following to students.
 (a) Computation (estimate only): $1^{1}/_{2} \times 1.67$
 (b) Application (estimate only): I need $1^{1}/_{2}$ m of material. It costs $1.67 per metre. About how much will it cost?

(2) Encourage mental calculation and estimation by giving students problems such as:
Give an estimate of the cost of 603 copies of a magazine that costs $1.98 per copy.

(3) "About how many times will your heart beat this month?"
Answer: About 3 200 000 times.
If the students do not know what the average heart rate is, have them research the problem.

Note: This type of estimation problem is sometimes referred to as a Fermi-type problem.

(4) Show the method used to solve this problem mentally:
$(1986 + 1986) \times 50$

One solution: $(1986 + 1986) \times 50 = 1986 \times 100 = 198\,600$
"There are two 1986s. Two times 50 is 100. Add two zeros to 1986. The answer is 198 600."
Emphasize "showing the method" by asking students to describe what they did orally or in writing.

TECHNOLOGY INTEGRATION

See Problem Solving in Mathematics: Focus for the Future, p. 40, for a computer program that will add fractions, and that may be adapted to perform other operations.

For a set of activities that makes use of the calculator to explore patterns in the products of special numbers see, "More Calculator Capers," Mathematics Teacher, September 1983. For example, one pattern examined is that created by the factors:

$$\begin{gathered} aa \times ab \\ aaa \times aab \\ aaaa \times aaab \ldots \end{gathered}$$

when a, b are non–zero digits and $b = a-1$. Another pattern explored, and used to predict products where possible is:

$$\begin{gathered} ab \times cd \\ aab \times ccd \ldots \end{gathered}$$

where a and c are non–zero digits that are multiples of 3 and b and d are any non–zero digits.

ELECTIVE SUGGESTIONS

Problem: Continue the pattern:

$1^3 = 1^2 - 0^2$
$2^3 = 3^2 - 1^2$
$3^3 = 6^2 - 3^2$
$4^3 =$ ____________
$5^3 =$ ____________

Answer: $4^3 = 10^2 - 6^2$
$5^3 = 15^2 - 10^2$

Discuss various approaches to determining the general pattern.

Problem: Your father gives half the money in his pocket to your mother, a fourth of what is left to your brother, and a third of what is left to your sister; he then splits the remainder with you. If you get $2, how much did your father start with?

Answer: $16.

Encourage the problem–solving strategy of working backward.

Problem: For which positive integers a, b and c will $^1/_a + {}^1/_b + {}^1/_c = 1$?

Answer: 2, 3, 6.

Activity: Consider the following class activity as a means of developing students' word number skills.

Create a deck of index cards, at least one card for each student in the class. On the front of each card write a number. On the back of each card, write instructions that involve computations with the present number that lead to a number on another card in the deck.

For example, three consecutive cards might be:
7 – Who has twice this number less 3?
11 – Who has this number squared less 100?
21 – Who has the square root of the sum of this number and 15?

Distribute all the cards. Designate a student to read his or her number and the instruction on the back. A second student, holding the card with the correct response to the first instruction, should follow. The final card should contain an instruction that leads back to the first number read, thus completing the cycle.

In addition to encouraging the development of skills in translating from words to mathematical expressions, this activity also encourages mental and calculator computation.

To extend this activity, have students create another set of cards. (Include rational numbers in $^a/_b$ form or in decimal form.)

Adapted with permission from, "I Have a Number. . ." Mathematics Teacher, October 1988, copyright by the National Council of Teachers of Mathematics.

RESOURCE CORRELATION

MMW2:	pp.2–13, 16–24, 26–29, 38–40
MM2:	pp.6–22, 31–33, 35, 74–83, 124
MPMM2:	pp. 2–13, 16–24, 26–29, 38–40

CONCEPT 1

Students will be expected to demonstrate an understanding of the four arithmetic operations on rational numbers carried out in conjunction with the operations of rounding and estimation.

Students will be expected to:

1.2 perform calculations using percents
  1.2.1 use calculators in calculations that involve the use of percents
  1.2.2 solve problems that involve the use of percent calculations.

COMMENTS

Students have done some previous work with percent. It is important to reinforce the decimal equivalent of a percent so that they feel at ease with any calculator. Using the decimal equivalent in calculations provides a reinforcement of order of operations and the understanding that percent represents "a part of. . ."

PROCESS/PROBLEM-SOLVING CONTEXT

Activity:

(a) Have students examine sale flyers to determine if the sale price is a true saving. The following procedure is suggested:
  1) Find a number of items on sale in the flyer
  2) Calculate the sale price of the goods according to
    (a) percent discount
    (b) amounts deducted.
  3) Decide if the item on sale is a bargain or not. Some comparison shopping may be necessary here.
  4) Explain your reason for saying why the sale item is a bargain or not.

(b) Have students create their own percentage "sales" problems. If computers with word processing and graphics capabilities are available, have small groups of students create their own "sales flyers." Some intentional errors may be included. These may then be checked by other groups. For further ideas, see "Using Percent Problems to Promote Critical Thinking," Mathematics Teacher, January 1988.

Activity: Have students keep a record of how they spend their money over the period of one week. Through class discussion, develop a standard set of expense categories. At the start of the week, ask students to estimate what percent of their money they will spend in each category. At the end of the week, have students calculate the actual percent of their money spent in each classification. A discussion on budgeting could follow.

Problem: Use a calculator to solve the following problem.

A 37.5% discount was offered on a $750 stereo. However, a 7% sales tax was added. What was the total amount of money that would have to be spent to purchase the stereo?

Answer: $501.56

Problem: An item is discounted 25% for a sale, another 10% because of the size of the purchase, and 3% for paying cash. These discounts are taken one after the other. The customer paid $195.89. What was the original purchase price?

Answer: $299.18.

Problem: Tom bought a $30 jacket at a 20 percent discount and a $20 shirt at a 30 percent discount. What is the single percentage discount he received on his total purchase?

Answer: 24%.

Problems: See Problem Solving Challenge for Mathematics, p. 22, #7.15; p. 30, #8.15, p. 40, #8; and p. 50, #15.

Problem: Assuming you made $40 000 least year, would you rather receive a 15 percent raise this year, followed by no raise next year, or would you prefer a 7.5 percent raise each year?

Answer: First is the best.

First year: 7.5% raise:	$40 000 (1.075) =	$43 000
Second year: 7.5% raise	$43 000 (1.075) =	$46 225
Total		$89 225
First year, 15% raise:	$40 000 (1.15) =	$46 000
Second year, 0% raise:	$46 000 (1.00) =	$46 000
Total		$92 000

A difference of $2775.

Reprinted with permission from "7.5 + 7.5 ≠ 15," Mathematics Teacher, February 1981, copyright by The National Council of Teachers of Mathematics.

Problem: Ninety–nine girls and one boy are in a mathematics class. How many girls must leave the room so that the percentage of girls becomes 98 percent?

Answer: 50.

See "Percent and the Hand Calculator," and "Developing Estimation Strategies" in Activities for Implementing Curricular Themes from the 'Agenda for Action,' for activities related to percent estimation, and the use of the calculator. See "Estimation and Mental Arithmetic with Percent" in Estimation and Mental Computation, for a number of teaching strategies.

TECHNOLOGY INTEGRATION

Software:
Algebra Drill and Practice I (Conduit) "Percentages" option provides practice in calculations using percent (except "find the base" type) and in percent increase/decrease problems.
Percents (Media Materials) provides practice and remedial opportunities.
SRA Computer Drill and Instruction, Mathematics: Level D (rev. ed.) – "Ratio and Percent" option. Select percent–related lessons. Suitable for drill.

Video:
MathWays: "The Percent" (Kinetic). Focus is on understanding the concept of percent, and determining what a given percent of a number or quantity represents. Includes some conversion of fraction and decimal notation to percent.
MathWise: "Comparing 02 – Percent" (ACCESS) shows applications of percents in everyday life situations.

ELECTIVE SUGGESTIONS

Problem: A bicycle, originally priced under $100, was on sale at a 25% discount. When the original price, a whole number of dollars, was discounted, the discounted price was also a whole number of dollars. What is the largest possible number of dollars in the original price of the bicycle?

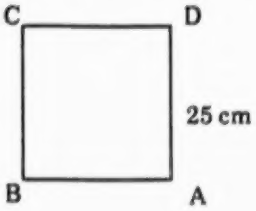
Answer: $96.

This problem helps strengthen students' skills with percent. It also requires them to draw upon other number knowledge, such as the concept of factor, and the inequality relation.

Problem: The value of a machine declines by 10 percent for each year of use. If the original cost of the machine was $5000, after a minimum of how many full years will its value be less than 50 percent of its original value?

Answer: 7 years. $(0.9)^7 < 0.5$
This problem can be solved by a process of iteration. Suggest students set up an appropriate table.

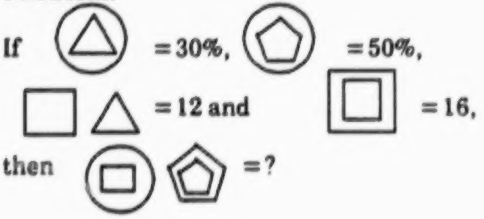
Problem: What are the dimensions of the square EFGH you must cut from the square ABCD so that the area of square ABCD is reduced by 25%?



Answer: 12.5 cm by 12.5 cm.

The value of this problem is its connections to a number of facets of mathematics: percent, a review of area, geometry, squares and square root, calculator use and problem solving.

Problem:



Answer: 10.

A Manipulative Approach to Percent

A manipulative transparency model can be used to solve a limited number of percent problems. On a sheet of clear acetate divide a 5 cm by 5 cm square into 100 smaller congruent squares with an indelible marker. This is the "percent" sheet. On a separate worksheet of activities, construct 5 cm by 5 cm squares. Subdivide each of these into the number of parts equal to the "whole" of the respective percent problems. For example, in the problem 12 = ____% of 20, the large square should be subdivided into 20 congruent parts (rectangles); in the problem 80% of 25 = ____, create 25 congruent rectangles.

To solve 12 = ____% of 20, the student shades in 12 of the 20 parts. The percent transparency is then superimposed, and the number of one percent squares covering the shaded portion is counted, showing the solution of 60 percent.

To solve 80% of 25 = ____, the student shades in 80 of the 100 squares on the percent sheet using an erasable marker. The sheet is then superimposed on the 5 cm by 5 cm square that has been subdivided into 25 congruent parts. The number of rectangles covered by the shading is counted, producing an answer of 20.

This model cannot be used to solve problems of the type 30% of____ = 15. A second manipulative model can be used to solve all three "types" of percent problems.

Adapted with permission from "Manipulating Percentages," in Mathematics Teacher, January 1986, copyright by the National Council of Teachers of Mathematics.

Activity: The second model requires a metre stick and a piece of 2.5 cm wide braided sewing elastic sufficiently long that when fully stretched it will be at least 1 m. Stretch the elastic to its maximum length, and mark if off in units of 1 cm with an indelible black marker. Highlight multiples of 10 percent. Finally, trim off the excess elastic.

The 100% mark on the elastic represents the "whole" in the general form of the problem "% of whole = part." The percentage value on the elastic will be aligned with the "part." The elastic with its one hundred units is used to calibrate a particular "whole" (as indicated on the metre stick) into a like number of one hundred equal parts. Thus, the model corresponds to the concepts involved in percentage problems.

a) Problems of the form 60% of 50 = _______
Align 0 cm on the metre stick with 0% on the elastic. Stretch the tape so that 100% is aligned with 50 on the stick. Read the metre stick marking (30) opposite the 60% mark on the elastic.

b) Problems of the form ___% of 80 = 20?
Align 0 cm with 0% and 80 with 100%. Read on the elastic the percent value (25%) corresponding to 20 on the metre stick.

c) Problems of the form 30% of ___ = 24
Align 0 cm with 0% and stretch the elastic until 30% coincides with 25 on the metre stick. Read on the metre stick the value of the "whole" that corresponds to 100%, that is, 80.

Notice that the elastic and metre stick model correspond very closely to the more symbolic (and abstract) proportions method of solving percentage problems. As students become familiar with the use of this concrete approach, have them also describe the proportions being represented.

Both models clearly have their limits. It is best to work within the range for which the models were intended. Answers will also often be inexact. This should be acknowledged: encourage students to estimate.

RESOURCE CORRELATION

MMW2:	pp. 30–31, 184–198
MM2:	pp. 28–30, 33–34
MPMM2:	pp. 30–31, 184–198

CONCEPT 1

Students will be expected to demonstrate an understanding of the four arithmetic operations on rational numbers carried out in conjunction with the operations of rounding and estimation.

Students will be expected to:

1.3 calculate numerical squares and square roots
  1.3.1 use electronic calculators to find numerical squares and square roots.

COMMENTS

Students should know the perfect squares and their square roots to 100. Time should be spent doing mental estimates of roots of non-perfect squares to 100. The square root of any number over 100 should be found using a calculator.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Using paper marked out in squares, have students draw squares with dimensions of 1 by 1, 2 by 2, ... 10 by 10 and find the area of each. What would be the area of a square with dimensions of 7.5 by 7.5? How could we draw a square that had an area of 75? What should be the length of each side?

Problem: What can the last digit of a perfect square be? The last two digits? Could 836 254 121 be a perfect square?

Activity: First, estimate $\sqrt{363}$ mentally. Write down your estimation.
Second, using your calculator but WITHOUT using the √ key, determine $\sqrt{363}$ correct to the nearest whole number. Describe on paper what you did.

Estimating square roots is not only a practical skill but also serves to fix in mind the meaning of "square root" itself.

Adapted with permission from " A Do-It-Yourself Estimation Workshop," in Estimation and Mental Computation copyright 1986, by the National Council of Teachers of Mathematics.

TECHNOLOGY INTEGRATION

Students should use their calculators to find the square of and square root of any number. Emphasis should be on the reasonableness of answers and students estimating their answer before calculation.

ELECTIVE SUGGESTIONS

Estimate $\sqrt{69}$ to 1 decimal place and give a rationale for your answer.

RESOURCE CORRELATION

MMW2: pp. 56–59
MM2: pp. 50–53, 64
MPMM2: pp. 56–59

CONCEPT 2

Students will be expected to demonstrate an understanding of the laws and properties pertaining to calculations involving expressions with exponents.

Students will be expected to:

- 2.1 perform calculations involving powers with integral bases and integral exponents
  - 2.1.1 verify the properties of zero and negative exponents through the use of numeric examples
  - 2.1.2 use calculators to evaluate powers with integral bases and integral exponents.

COMMENTS

Stress the use of calculators. Develop with students the concept of the relationship between a power with a negative exponent and its reciprocal with a positive exponent. Teach students to use the $^1/_x$ key on the calculator as well as the y^x key with positive and negative exponents.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students try an exercise such as the following:
By expansion
$2^5 = 2\times2\times2\times2\times2 = 32$ and
$2^5 \div 2^5 = \frac{2\times2\times2\times2\times2}{2\times2\times2\times2\times2} = \frac{32}{32} = 1$ ①
but by the division rule for powers $2^5 \div 2^5 = 2^{?-?} = 2^?$. ② Therefore, from ① and ② we can conclude that $2^0 = 1$.

Use your calculator to try some other numbers raised to an exponent of 0. How would you finish this sentence?
A power with an exponent of zero ______________________________.
____

By expansion, $2^3 = 2\times2\times2 = 8$ and $2^5 = 2\times2\times2\times2\times2$ and
$\frac{2^3}{2^5} = \frac{2\times2\times2}{2\times2\times2\times2\times2} = \frac{1}{2\times2} = \frac{1}{2^2}$ ①

But by the division rule for powers, $2^3 \div 2^5 = 2^{3-5} = 2^?$ ②

Therefore, from ① and ②, $\frac{1}{2^2} = 2^{-2}$
How would you finish the following sentence? A power with a negative exponent ______________________________
____

Activity: Another way to have students generalize the laws for zero and negative exponents is by having them build a table such as the following:

$2^5 = 32$
$2^4 = 16$
$2^3 = 8$
$2^2 = 4$
$2^1 = 2$

By continuing to follow this pattern, what would 2^0 equal? 2^{-1}?

Build a table like this with a base of 3 and 5. Can you come up with a generalization for powers with an exponent of 0? for powers with a negative exponent?

Problem: How can the $^1/_x$ key on a calculator be used to find the decimal equivalent of $^1/_8$, $^3/_8$, $^1/_{12}$? How can the $^1/_x$ on a calculator be used to find the fractional equivalent of 0.125, 0.25? Can the $^1/_x$ key be used to help find the fractional equivalent of 1.25? of 0.2?
Have students experiment with this and come up with their own answers.

Problem: What is half of 2^{40}?

Answer: Although students may determine the answer in a variety of ways, ensure that they understand 2^{39} as one form of expressing the solution.

Problem: Which is larger, 2^{30} or 3^{20}?

Answer: $2^{30} < 3^{20}$.

Students can easily determine the answer with a calculator. Suggest they apply the power law first and then check the result with the calculator.

Problem: See Problem Solving Challenge for Mathematics, p. 41, #20.

ELECTIVE SUGGESTIONS

MMW2: p. 65, "Side Trip"

Activity: Compare the solution of the above "What is half of 2^{40}?" problem to the solution of the "ninety-nine girls and one boy" percent problem on page 140. Note how a decrease by a factor of 2 effected a small change (40 to 39) in the first case, and a large change (50 girls, or more than half) in the second case. Why?

RESOURCE CORRELATION

MMW2:	pp. 32–35
MM2:	pp. 23–27, 98–100
MPMM2:	pp. 32–35

CONCEPT 1

Students will be expected to demonstrate an understanding that polynomials can be used to describe some mathematical situations in which values vary.

Students will be expected to:

1.1 identify the following as they occur in the study of polynomials: term, variable, factor, monomial, binomial, trinomial, polynomial, numerical coefficient, degree, exponent, base, power

1.2 evaluate a polynomial for given values of the variables

1.3 use polynomials to describe mathematical situations.

COMMENTS

Students may have been previously introduced to the terms related to polynomials. This is an extension of the Grade 9 program. Students have experience solving equations in one variable and have evaluated polynomials in the sense of "checking" an answer after solving an equation.

It is important to emphasize what a polynomial is: a function that describes a situation in terms of one or more variables. A polynomial is a general rule that can then be evaluated for values of the variables as conditions and parameters change. Formulae such as the formula for the perimeter of a rectangle, $2L + 2W$, is an example of a polynomial. $2L + 2W$ describes the perimeter of any rectangle; substituting values for L and W we can describe the perimeter of a particular rectangle and hence describe a specific case.

It is appropriate to mention to students that there will be some values of the variable that will make the polynomial equal zero. These special values are called zeros. Limiting example: Evaluate $2a^2 + 3b$ for $a = 5, b = -2$.

Introduction to the vocabulary associated with polynomials should be done within the appropriate context. See, the Mathematics Dictionary 4th. Edition, written by James & James for accurate definitions.

Students have a difficult time understanding the concept of "variable"; it is important to spend some time discussing it.

A Discussion of Variable:

The meaning of variable is variable; using the term differently in different contexts can make it hard for students to understand. As teachers, we should be sensitive to this multiple usage.

Complete the following activity yourself, and then have the students do it. Discuss the list generated with the class.

Complete the following sentence:
A variable is a______________________________.

Use the *one* English word that, in your opinion, best captures the meaning of the term *variable*. (If you generate a number of possible candidates, please list them in order of preference.)

Mere synonyms are not acceptable; try to get to the heart of the concept. Some of the words in the list may be better than others, but there is no *one right* word for "variable."

Here are three suggestions related to teaching the concept of variable.

1. Variables are tools for expressing mathematical generalizations. It should help, therefore, if students had the habit of verbalizing such generalizations before they were asked to formalize those generalizations using the language of mathematics. It might be useful to make a habit of asking students to summarize some of their observations about arithmetic in their own words, in short exercises. ("What happens when you add three times a number to two times that number?" or "Can you say whether the sum of two odd numbers will necessarily be even or odd?") By asking students to observe patterns to summarize them verbally, we may help make the transition from arithmetic to algebra.

2. The dynamic aspects of the variable concept should be stressed whenever it is appropriate and feasible. At the simplest level, students can be led to observe that the cost (variable) that appears on a gasoline pump is a (dynamic, linear) function of the amount of gasoline that emerges from the pump. Other dynamic relationships include time-dependent phenomena, for example, the temperature in a room at time t, the distance an object falls in t seconds, and so on.

3. Much of the algebra curriculum consists of solutions of equations independent of the context from which they were drawn. Many applications of algebra, for example, the standard word problems, are so neatly packaged, and taught in separate units, that students miss the point - that the language of algebra is a powerful means of capturing the mathematical essence of a wide variety of situations and not just the ones they've been taught to solve. We recommend that students be exposed to a wide range of problems for which algebra, especially in its role of a tool for mathematical generalization can be useful.

Reprinted with permission from "On the Meaning of Variable," Mathematics Teacher, September 1988, copyright by the National Council of Teachers of Mathematics.

For another examination of the concept of variable, see "What are These things called Variables?", Mathematics Teacher, October 1983 and "Children's Difficulties in Beginning Algebra," The Ideas of Algebra, K-12.

For a discussion (with illustrative examples) of fostering greater understanding of algebraic concepts through the use of diagrams, see "Developing Algebraic Representation Using Diagrams," in The Ideas of Algebra, K-12.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: To establish an understanding of "variable," students should draw from their own experience. This will lead into a discussion of how mathematics can be used to describe events that occur in their lives. For instance, use the discussion of $P = 2L + 2W$ to discuss how L and W are variables because this formula always describes the perimeter of a rectangle. Have students evaluate this polynomial for certain rectangles, given the dimensions of a rectangle.

Activity: Have students write and evaluate expressions that describe recognizable situations such as the number of books in a library, etc. and have students identify the parts that "vary."

Problem: Maple Ridge High School had a drama night at which they charged $4 for an adult ticket and $2 for a student ticket. Write an expression to describe the income generated.

Activity: A further discussion of variable: Write an expression that describes the number of tires of moving vehicles touching the pavement in your community when:

a) only the 4–wheeled motorized vehicles are currently being driven
b) only the 2–wheeled motorized vehicles are currently being driven
c) the combined number of 4– and 2–wheeled motorized vehicles are currently being driven
d) only the passenger buses are currently being driven.

Things that could be discussed: Does "currently being driven" include those vehicles stopped at an intersection? What difficulties are there in defining variables? Is precision necessary?

TECHNOLOGY INTEGRATION

Have students use their calculators when evaluating polynomials with large numbers. Mental computation and estimation should be reinforced. For instance, have students estimate their answer before actual calculation.

See Problem Solving Challenge for Mathematics, p. 46 #18.

MMW2, p.21, "Evaluating Expressions"

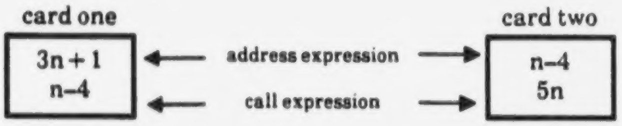
Evaluate polynomials using a spreadsheet. This reinforces the concept of variable. Since the spreadsheet will quickly calculate the value of an expression, values of the variable can be easily changed. Students will have an opportunity to predict what will happen to the value of a polynomial, if the variable increases, decreases, etc. See for example, "Integrating Spreadsheets into the Mathematics Classroom," Mathematics Teacher, November 1988. AppleWorks and Microsoft Works are Alberta Education Basic Resources that have spreadsheet capabilities.

ELECTIVE SUGGESTIONS

Holtmath 10, pp. 156–157, 276–277

Activity: Designed to help develop elementary algebra skills, and listening skills.

The following activity is similar to the "I have a Number. . ." activity described for Polynomials and Factoring, Concept 2. Divide the class into two groups of students, and with a deck of index cards create a set of "algebraic expression" cards for each group. On the front of each card write an address expression and a call expression. For example:



Each set of cards forms a closed loop. That is, the card that follows card one above will have n–4 as its address expression. Its call expression is 5n, therefore the card following will have 5n as its address expression. The call expression of the last card in the set is also the address expression of the first card.

A student starts by announcing, in English, his or her address and call expressions. Using the above example, the first student would say, "I have one more than three times a number. Who has four less than a number?" The student with the second card shown would follow in a similar manner. Thus, each student has to (a) translate a sentence that is heard into an algebraic expression, and (b) translate an algebraic expression into a spoken sentence.

Variations include:

(1) A mixture of algebraic expressions and English sentences for the call and address expressions.
(2) Multiple responses on some cards from which a student may choose. Thus some students may be called on more than once. Set rules to avoid "inner loops."
(3) Encourage students to create equivalent English sentences for the same algebraic expression. For example, the call expression on card two could also be described as "Who has 10 times a number divided by two?"
(4) Create more complex expressions, such as (x–2)(x + 1).

Adapted with permission from, "I have. . .Who has . . .?" Mathematics Teacher, October 1980, copyright by the National Council of Teachers of Mathematics.

RESOURCE CORRELATION

MMW2:	pp. 14–15, 168–169 (Note: This book introduces the terms throughout the discussion of polynomials.).
MM2:	pp. 87–88, 92–94, 112–113, 117
MPMM2:	pp. 14–15, 168–169

CONCEPT 2

Students will be expected to demonstrate an understanding of the laws and properties pertaining to calculations involving expressions with exponents.

Students will be expected to:

- 2.1 use the multiplication law of exponents for powers with literal bases and whole number exponents, $a^m \times a^n = a^{m+n}$
  - 2.1.1 verify the multiplication law of exponents through the use of examples
- 2.2 use the power law for exponents, $(a^m)^n = a^{mn}$
  - 2.2.1 verify the power law for exponents through the use of examples
- 2.3 use the division law of exponents for powers with literal bases and whole number exponents, $a^m/a^n = a^{m-n}$
  - 2.3.1 verify the division law for exponents through the use of examples
- 2.4 use zero and negative exponents
  - 2.4.1 verify the laws for zero and negative exponents through the use of examples.

COMMENTS

In junior high and in Math 13 Number Skills, students have discussed the exponent laws in relation to numeric bases only. In leading to a discussion of literal bases, you may wish to review the skills with numeric bases.

"To verify" requires the student to substantiate the validity of an operation, solution, formula or theorem through the use of examples that may or may not be generalized.

Emphasis should be on the process, not on difficult or complex calculations. Do not go beyond the level of difficulty in the textbooks.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Power Law
Emphasize that $(a^3)^4$ means a^3 used as a factor 4 times. Have students write an explanation of why the multiplication rule for exponents works. Using zero and negative exponents in operations such as this should be used to show that the operations do not change when they are used. Zero and negative exponents play an important role in the evaluation of expressions.

Problem: Power Law

Evaluate $\left(\left(\left(\left(2^{-2}\right)^{2}\right)^{-2}\right)^{2}\right)^{-2}$

Answer: 2^{-32}

Problem: Division Law
This problem will also reinforce the use of order of operations.

Simplify the following:
$\underline{x^{16}}$
$\underline{x^{8}}$
$\underline{x^{4}}$
$\underline{x^{2}}$
$\underline{x}$
Answer: x.

Activity : Zero Exponents
This is particularly helpful when explaining why $x^0 = 1$.

Have students try an exercise such as the following:
By expansion $x^6 = x \cdot x \cdot x \cdot x \cdot x \cdot x$

Activity : Zero Exponents (continued)

so $x^6 \div x^6 = \frac{x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x \cdot x} =$ ______ ①

according to the division law

so $x^6 \div x^6 = x^{?-?} = x^?$ ②

therefore, from ① and ② we can conclude

______ = ______

Try this law out for various values of x then finish this sentence: Are there any limitations?

Any power with an exponent of zero ____________________

____

Activity: Negative Exponents

By expansion,

$$x^6 \div x^8 = \frac{x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x} = \text{\_\_\_\_\_\_\_\_} \quad ③$$

but according to the division law,

$x^6 \div x^8 = x^{?-?} = x^?$ ④

therefore, from ③ and ④ we can conclude that

__________ = __________

Try this out on your calculator for $x = 2, 5, 10$ and exponents of $-1, -2, -3$.

Finish this sentence:

A power with a negative exponent ____________________

____

See Problem Solving in Mathematics: Focus for the Future, p. 65, #5, 6.

TECHNOLOGY INTEGRATION

Software:

Algebra Drill and Practice II (Conduit). "Integral Exponents" option provides practice in simplifying exponential expressions involving the four laws and positive, zero and negative exponents.

ELECTIVE SUGGESTIONS

Problem: Evaluate $x^0 - 0^x (x \neq 0)$.

Answer: 1.

Problem: Arrange the following in increasing numerical order:

$a = 2^{55}$ $b = 3^{44}$ $c = 5^{33}$ $d = 6^{22}$

Answer: $a < d < b < c$ $\{(2^5)^{11} < (6^2)^{11} < (3^4)^{11} < (5^3)^{11}\}$

Have the students use their calculators to determine the order. Then suggest that this can be answered without using the calculator by making use of the laws of exponents. For example, what are some ways to write 2^{55} and 3^{44} by using these laws? What is "common" about the exponents? Work to arrive at $a=(2^5)^{11}$, $b=(3^4)^{11}$, $c=(5^3)^{11}$, $d=(6^2)^{11}$.

The calculation in the bracket can be performed mentally or with pencil and paper.

RESOURCE CORRELATION

MMW2:	pp. 46–47
MM2:	pp. 95–100
MPMM2:	pp. 46–47

CONCEPT 3

Students will be expected to demonstrate an understanding that calculations and operations on polynomials depend on the application of the laws of operations for numbers and powers.

Students will be expected to:

3.1 add and subtract polynomials.

COMMENTS

Addition and subtraction of polynomials are no longer included in the junior high curriculum. Many students will not have experienced polynomials other than solving equations. If students have done some work with addition and subtraction of polynomials, then time should be spent on visualizing a polynomial and how addition/subtraction is a generalization of the addition/substraction properties that were learned in earlier grades. Time should be spent on the connection between previous work and this work so the polynomials are not treated as completely different from number operations.

Emphasis should be on understanding not on long or difficult computations with complex polynomials. Questions should be no more difficult than those involving two trinomials of maximum degree 3 and two variables. Teach and encourage students to check their calculations by substitution.

Limiting example:

$(3x^3 - 2x^2 + 5) + (4x^2 - 3x + 7)$

$2a^2b + 4a^2 - 3b - (a^2b - 3a^2 - 7b)$

Manipulatives: The introductory study of polynomials provides an ideal opportunity for teachers to provide instruction geared to the students' cognitive levels. Students in this course will be at a concrete stage of development with respect to polynomials and can benefit from thoughtful development of concepts using concrete manipulatives. However, the aim is not just to acquire facility with manipulatives, but rather to use them to progress from the concrete, through a transitional (pictorial) stage and finally to a formal understanding of the abstract concepts and processes involved in the study of polynomials.

The purpose of manipulatives is to provide students with a concrete example of an abstract concept of process. Work with manipulatives does not replace the development of a mathematical algorithm, but development of the algorithm should follow introduction of the concept using manipulatives. There is no use in a teacher demonstrating the concept to a class using manipulatives or in allowing students to work with manipulatives after presentation of a formal algorithm.

Students at a concrete level must be allowed to internalize the concrete representation of the concept and must be given the opportunity to work with manipulatives on their own for these methods to be effective.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Present students with a situation such as the following as an introduction to the addition and subtraction of polynomials.

For the first game of the Calgary '88s season, 4200 seats were sold in the blue section, while for the second game 3500 seats were sold.

1) Write an expression to describe the income generated by the seats in the blues that were sold for the first two games.

2) Write an expression for the total income generated by the seats in the blue section for the first two games.

Suppose 1600 seats in the red section were sold for game one and 2300 seats in the reds were sold for game two.

3) Write an expression of the income generated in the first game by seats in the blues and the reds.

4) Write an expression of the income generated in the second game by seats in the blues and reds.

5) Write an expression for the total income generated for the first two games.

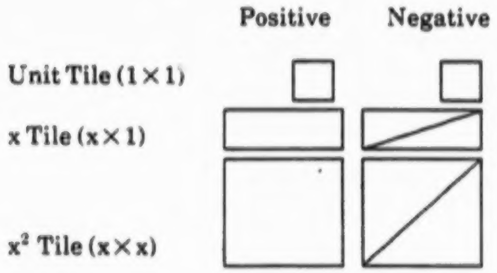
Suppose seats in the blue section cost $12 and seats in the reds cost $10.

6) What would the total income be for the first two games?

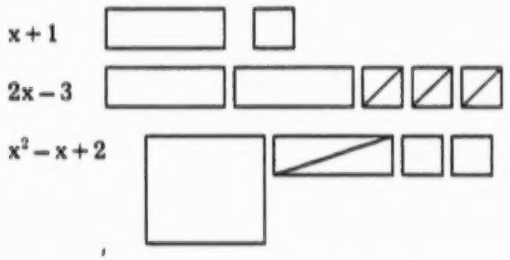
Answers: where b is the price of a seat in the blue section and r is the price of a seat in the red section.

1) $4200b + 3500b$
2) $7700b$
3) $4200b + 1600r$
4) $3500b + 2300r$
5) $7700b + 3900r$
6) $7700 \times 12 + 3900 \times 10 = \$131\,400$

Activity: Have students cut, from construction paper, algebraic tiles representing x^2, x and 1. (See template in Appendix A.) Mark the tiles on one side with a diagonal to represent negative. The tiles will appear as follows:



Using these tiles, polynomials can be represented as follows:



Have students practise representing various polynomials using the tiles.

Manipulatives can be used effectively to teach addition and subtraction of polynomials.

RESOURCE CORRELATION

MMW2:	pp. P2–P5, 44–45
MM2:	pp. 89–91
MPMM2:	pp. 44–45

CONCEPT 3

Students will be expected to demonstrate an understanding that calculations and operations on polynomials depend on the application of the laws of operations for numbers and powers.

Students will be expected to:

3.2 multiply polynomials by monomials
3.3 multiply binomials by binomials
3.4 recognize the expansion of the square of a binomial and the product of conjugate binomials as special cases.

COMMENTS

This concept is no longer included in the junior high curriculum. Many students will not have experienced polynomials other than solving equations. If students have done some work with the multiplication of polynomials, then time should be spent on visualizing a polynomial and how multiplication is a generalization of the multiplication properties that were learned in earlier grades. Time should be spent on the connection between previous work and this work so that polynomials are not treated as completely different from number operations. Remind students of the distributive law. Emphasis should be on the process, not on long or complex calculations. Do not go beyond the difficulty level of the exercises in the textbooks. Recognizing the square of a binomial [$(a + b)^2 = a^2 + 2ab = b^2$] and the product of conjugate binomials [$(a + b)(a-b) = a^2 - b^2$] should be done within the multiplication of binomials and recognized as special cases only after the development of multiplication.

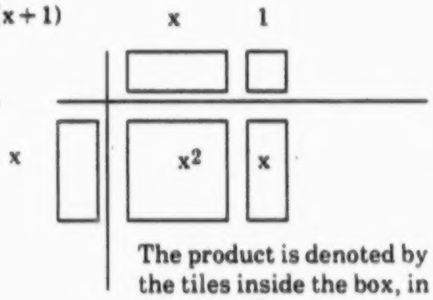
The use of manipulatives in the multiplication of binomials can be very useful. It allows a student to concretely express multiplication as finding an area.

Multiplication should be limited to two binomials. The product should have a maximum of 2 variables and degree 2.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: When multiplying using manipulatives, set up a table and compare the multiplication to finding the area of a rectangle. Remember to multiply all terms on the side by all terms on the top.

1) $(x)(x+1)$

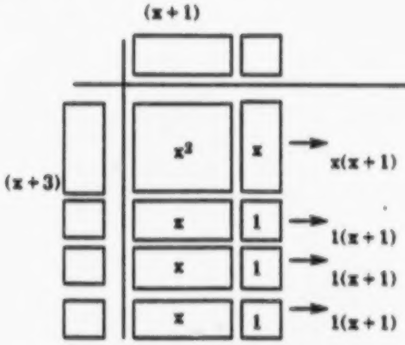


The product is denoted by the tiles inside the box, in this case $x^2 + x$.

Have students write an explanation of why the multiplication rule for exponents works.

Activity:

2) $(x+3)(x+1)$

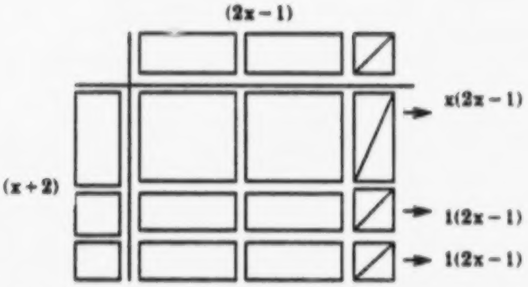


The tiles inside the box show that the product is x^2+4x+3. Notice that the product is a rectangle. This is important when we factor.

$(x+2)(2x-1)$

Activity:

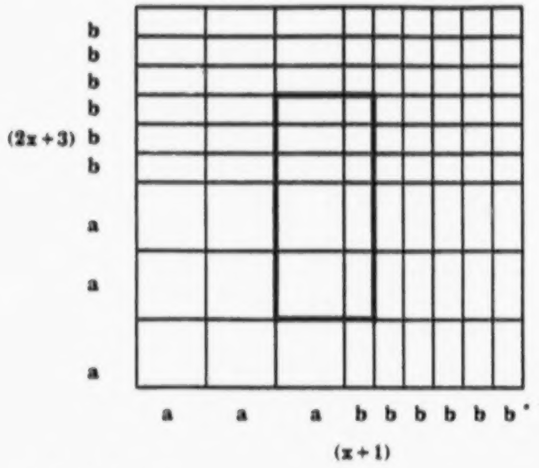
3) $(x+2)(2x+1)$



The tiles inside the box show that the product is $2x^2+4x-x-2=2x^2+3x-2$.

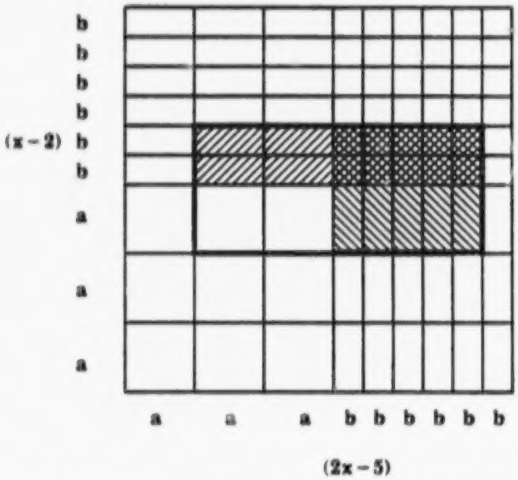
Activity: Another possible approach to teaching multiplication of binomials is to use binomial grids. The idea is to provide students with a sheet upon which is a number of grids, formed by geometrically interpreting the multiplication of two binomials. (See template in Appendix A.) Particular multiplications can be accomplished by outlining a rectangular region on the grid with sides that are the lengths of the given binomials. The area of the rectangle will be the product of the two binomials.

Activity:
For example, the product of (2x + 3)(x + 1) would be found as follows:



Product is $2x^2 + 5x + 3$

To find products that involve negatives, mark the areas derived from negative terms in some way. Those areas that are marked once only will be interpreted as negative in the final product. Those that are marked twice will be interpreted as positive. For example, the product of (x − 2)(2x−5) would be found as follows:

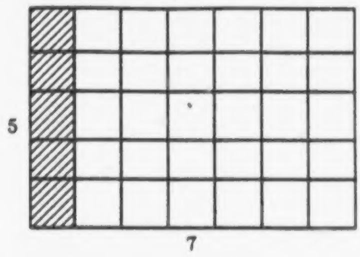


Product is $2x^2 - 9x + 10$

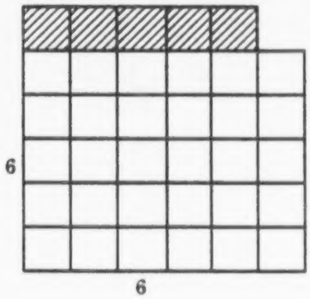
Adapted with permission from "The Binomial Grid," Mathematics Teacher, May, 1986.

Activity: Special Cases
Have the students do several multiplications involving squaring binomials and multiplying conjugate binomials. Discuss the results and generalize.

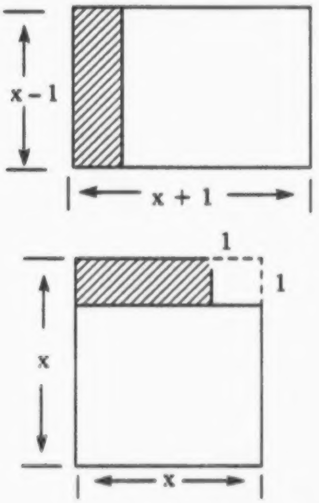
Here is one way of pictorially introducing the special case $(x-1)(x+1)=x^2-1$. Start with an example, for instance, 5 × 7, represented as a rectangle:



The 5 by 7 rectangle has thirty-five squares. If the five squares on the left are shifted to the top of the figure, the large figure can be described as "almost" a square, containing one less unit square than the thirty-six in a 6 by 6 square.



This geometric representation can be generalized to the case where, for all real numbers $x > 1$, the following holds:



$$(x-1)(x+1) = x^2 - 1$$

Adapted with permission from "Visual Thinking and Number Relationships," Mathematics Teacher, April 1988, copyright by the National Council of Teachers of Mathematics.

Activity: Extension of 3.4
Have the students explore the pattern in the product of two numbers whose sum is 20.

Construct a table such as the following:

9×11	$(10-1)(10+1)$	$10^2 - 1^2$	$100 - 1$	99
8×12	$(10-2)(10+2)$		$100 - 4$	
		$10^2 - 3^2$		91
	$(10-4)(10+4)$			
5×15				
1×19				

How can a knowledge of such products be applied to the mental calculation of 18 × 22 and 33 × 27, for example?

Adapted with permission from "Benchmark: Number Patterns and the Development of Expert Mental Calculation," in Estimation and Mental Calculation copyright 1986 by the National Council of Teachers of Mathematics.

Problems: See Problem Solving in Mathematics: Focus for the Future, p. 63 #22.

ELECTIVE SUGGESTIONS

Activity: It has been observed that $39 \times 62 = 93 \times 26$. That is, when you reverse the order of the digits in 39 and 62, the product of the reversed numbers is the same as the product of the original numbers. Do any other pairs of two–digit numbers have this property?

Let the students guess and test, and then suggest that they can generalize their examination by expressing each two–digit number as a binomial, making use of place value.

Answer: Yes, there are other two–digit numbers may have this property. The key is that the product of the 10s digit (3×6) equals the product of the units digits (9×2). This can be derived algebraically.

Adapted with permission from "On the Meaning of Variable," in Mathematics Teacher, September 1988 copyright by The National Council of Teachers of Mathematics.

Problem: Which two consecutive numbers have squares that differ by 75?

Answer: 37, 38

This problem can be solved through guess–and–check using numbers. Encourage students to think about algebraic ways of describing two consecutive numbers, and their squares. If n and n + 1 (or n and n–1) are the forms chosen, how can we apply our knowledge of binomial multiplication to the problem? Can we find an expression for the difference between the squares of these two numbers? Does this expression apply only to the case where the difference is 75?

A slightly different approach is to have the students construct a table of the differences between the squares of two consecutive numbers, and have them determine a general expression for the difference. Apply this to the problem of a difference of 75; of 1379. Is the difference always odd? Why or why not?

Stress the value of algebra in generalizing.

RESOURCE CORRELATION		
	MMW2:	pp. P6, P8–P9, 46, 48–49, 100–103
	MM2:	pp. 106–111, 118–120, 125–128
	MPMM2:	pp. 46, 48–49, 100–103

CONCEPT 3

Students will be expected to demonstrate an understanding that calculations and operations on polynomials depend on the application of the laws of operations for numbers and powers.

Students will be expected to:

3.5 divide polynomials by monomials.

COMMENTS

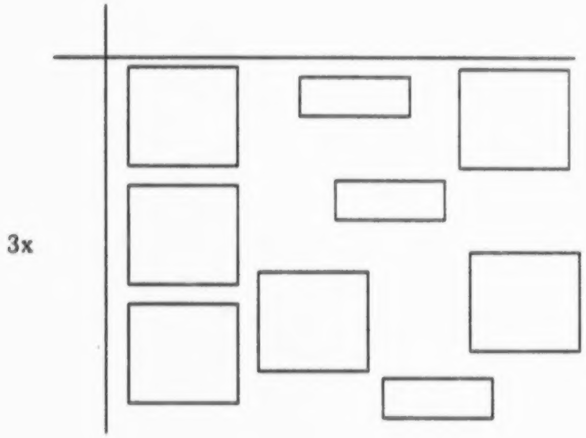
Students can be taught to write their results in the form of $P = D \times Q + R$ the remainder.

The introduction to this should begin with dividing a polynomial by a whole number (e.g., $(3x^2 + 6x + 9) \div 3$ and $(2x^2 + 4) \div 2$) approach is concrete in the sense that students can use the algebra tiles to separate the polynomial into three groups. This is particularly useful as students have not yet discussed factoring.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: A concrete approach to introducing the division of a polynomial by a monomial is to begin dividing a binomial by a monomial with algebra tiles: $(6x^2 + 3x) \div 3x$

6–x^2 tiles and 3–x tiles must be arranged in a rectangle where 3x is one dimension:

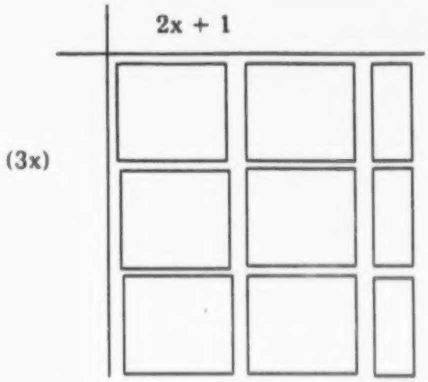


See Algebra Tiles for the Overhead Projector, pp. 25–27, for a more detailed description.

RESOURCE CORRELATION

MMW2:	pp. 46, 112–113
MM2:	pp. 114–116, 120
MPMM2:	pp. 46, 112–113

Arranging the remaining tiles; the rectangle looks like:



$\therefore (6x^2 + 3x) \div (3x) = (2x + 1)$

See Algebra Tiles for the Overhead Projector, pp. 25–27, for a more detailed description.

RESOURCE CORRELATION

MMW2:	pp. 46, 112–113
MM2:	pp. 114–116, 120
MPMM2:	pp. 46, 112–113

CONCEPT 4

Students will be expected to demonstrate an understanding that polynomials can be written as a product of their factors.

Students will be expected to :

4.1 factor polynomials that have a monomial common factor.
4.2 factor trinomials of the form $ax^2 + bx + c$, where $a = 1$ or is a common factor
4.3 recognize and factor square trinomials recognizing them as a special case
4.4 recognize and factor difference of squares polynomials, recognizing them as a special case
4.5 factor polynomials that require a combination of methods in order to be fully factored
    4.5.1 solve problems that involve factoring of polynomials

COMMENTS

This concept is no longer included in the junior high curriculum, and this will be new to students.

Remind students of factoring numbers into the product of prime factors. Extend this to factoring monomials. Remind students of how to find the GCF of numbers and introduce "common factoring" as a way of writing a polynomial as the product of the greatest common factor of its terms and a dividend.

Use and assign questions that are of the same difficulty level students are liable to encounter in further study. Remind them that it is possible to check their work by reversing the procedure.

Some students may find the concrete manipulatives developed previously of use in developing understanding of this concept.

Have students do several multiplications involving monomials and polynomials and then try to establish a procedure to "undo" the multiplication. Have students write the rules they develop in their own words.

Remind students that factoring is a way to write polynomials in a manner that may be more understandable or appropriate. Encourage them to examine polynomials for common factors and general forms.

Provide exercises involving polynomials that students might reasonably see at some future time.

PROCESS/PROBLEM-SOLVING CONTEXT

The process of factoring is the "undoing"of multiplication. The procedure is to use the manipulative tiles to form a rectangle. The dimensions of the rectangle will be the factors of the polynomial.

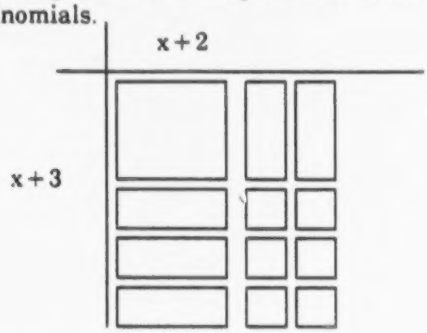
Remind students that $-1+1=0$ and that

□ + ⧄ = 0 as well. This concept will

be used to factor trinomials where the constant term is negative.

1) Factor x^2+5x+6

Arrange the tiles within a table like the one used for multiplication. There is only one way in which they can be arranged into a rectangle. Refer to the multiplication of binomials.

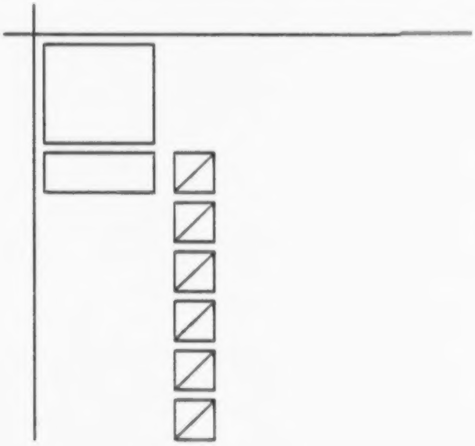


$\therefore\ x^2+5x+6=(x+3)(x+2)$

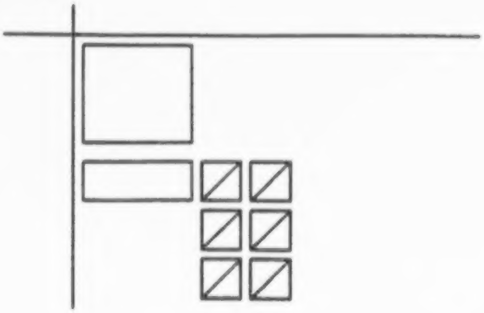
2) Factor x^2+x-6

One x^2 tile, one x tile and six negative unit tiles must be formed into a single rectangle. This is impossible to form. However, adding the same number of positive and negative x tiles will allow a rectangle to be formed.

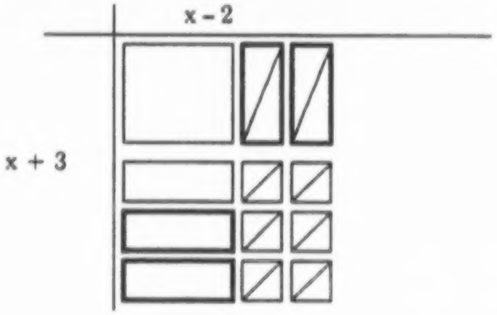
Form the units tiles into a rectangle in the lower right-hand corner. There are only two alternatives for this, either a 6 by 1 rectangle:



or a 3 by 2 rectangle:



Only the 3 by 2 rectangle will leave the same number of spaces two to be filled by "x" tiles along both the top and the side. By placing two positive x tiles along the side, (next to the original x tile), and then providing two negative x tiles along the top, the overall polynomial doesn't change but the rectangle is completed.



For more information, see "Finding Factors Physically," Mathematics Teacher, May, 1982, or in Activities for Implementing Curricular Themes from the 'Agenda for Action'.

Activity: Have students do several multiplications using manipulatives involving the squaring of binomials and the product of conjugate binomials. Discuss the results and generalize. Discuss the generalized products and extend the understanding to factoring perfect square trinomials and difference of square binomials.

Problem: Take any three-digit number and write it down twice to make a six-digit number. Show that this number is exactly divisible by 7 or 11 or 13. Why?

Have the students verify this with some examples. Indicate that a major strength of algebra is that it permits us to generalize. Remind students of place value, and indicate that such six-digit numbers as described in the problem can be written as abcabc. What is the place value of each of these literal digits? What is this general number in expanded form?

$$100\,000a + 10\,000b + 1000c + 100a + 10b + c$$

Are there any like terms?

$$100\,100a + 10\,010b + 1001c$$

Is there a common factor?

$$(1001)(100a + 10b + c)$$

What is the relationship between 7, 9, 11 and 1001?

(7, 9, 11 are all divisors of 1001)

Problems: See Problem Solving in Mathematics: Focus for the Future, p. 63, #24, 25.

TECHNOLOGY INTEGRATION

Software:

Algebra Drill and Practice II (Conduit) - "Factoring Bi- and Trinomials" option. Binomial common factor sub-option is suitable for practice/remedial activities.

Factoring Algebraic Expressions (Mindscape) - "Common Factor" and "Trinomials of the form $x^2 + bx + c = 0$" options provide practice/remedial possibilities.

Solving Quadratic Equations (Mindscape) - "Easier (A = 1)" option provides very structured practice in solving trinomials of the form $x^2 + bx + c = 0$.

ELECTIVE SUGGESTIONS

Problem: Express the number 96 as the difference of two squares in four different ways.

Answer: $10^2 - 2^2$
$11^2 - 5^2$
$14^2 - 10^2$
$25^2 - 23^2$

This can be answered in a guess-and-check manner. Suggest that the students look for patterns. One pattern that can be suggested is to consider the pairs of factors of 96. How can $a^2 - b^2 = (a + b)(a - b)$ be related to these pairs? Does it work for all such pairs of factors of 96?

The factor pairs of 96 are (12, 8), (16, 6), (24, 4), 32, 3) and (48, 2). Four of these pairs of numbers can be expressed as the sum and difference of two other positive integers. For example, (12, 8) = (10 + 2, 10 - 2). Is there a way to determine these a and b values? Which pair of factors cannot be treated in this way ($a, b \in I$)?

RESOURCE CORRELATION

MMW2:	pp. P7, P10-P11, 104-108
MM2:	pp. 121-123, 129-134
MPMM2:	pp. 104-108

CONCEPT 1

Students will be expected to demonstrate an understanding that points and lines can be represented on a Cartesian plane and have characteristics that can be defined and measured.

Students will be expected to:

- 1.1 use the formula for the slope of a line passing through two given points.
  - 1.1.1 solve problems that involve the use of the slope formula
  - 1.1.2 verify the relationships between the slopes of parallel and perpendicular lines by diagraming pairs of lines on a Cartesian plane
  - 1.1.3 solve problems that involve the relationships between the slopes of parallel and perpendicular lines
- 1.2 identify the characteristics of lines that have positive, negative, zero and undefined slopes.

COMMENTS

Students intuitively understand the concept of slope as they have used the term in other contexts e.g., the slope of the roof, the slope of a ski hill, the grade on a highway. Building on this, students can make the translation from the real world to the Cartesian Plane. The concept of slope could be introduced as a way of comparing the inclination of physical objects and geometric phenomena.

Students should not be exposed to the algebraic formula for calculating slopes until they understand the concept of slope. That is, concentration should be on understanding that slope is a ratio between the rise and the run of a segment.

In general, the slope (m) of the line segment with endpoints P_1 (x_1, y_1) and P_2 $(x_2, y_2$ is:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$$

Students have some experience in discussing parallel and perpendicular lines, but have not discussed the relationships between their slopes. The slopes of parallel lines and parallel line segments are equal and the slopes of perpendicular lines are the negative reciprocal of one another. For instance, if $l_1 \parallel l_2$ then, $ml_1 = ml_2$ or $m_1 = m_2$; and if $l_1 \perp l_2$ then $ml_1 = {}^{-1}/_{ml_2}$, or $m_1 = {}^{-1}/_{m_2}$. This can also be expressed as $(m_1)(m_2) = -1$.

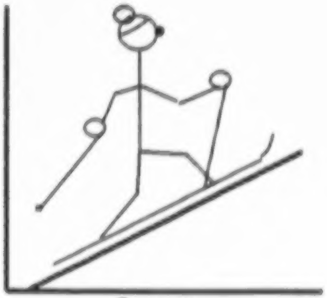
It is important for students to discover the relationships between the numerical and pictorial representation of a slope of a line segment. Students should discover these relationships and be able to express them in their own words. Discuss positive, negative, zero and undefined slopes.

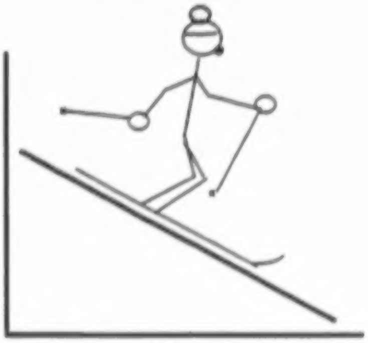
Various terminologies that students may have experienced, that describe slope, include inclination, pitch, gradient and elevation.

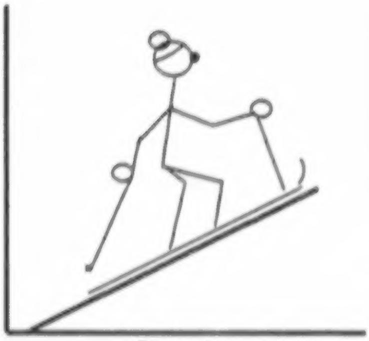
In addition to the following suggestions, a geoboard could be used to find the slopes of line segments, to illustrate parallel and perpendicular lines, and to illustrate lines with zero and undefined slopes.

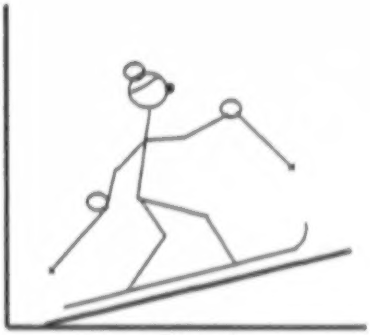
PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Relating slope to real life experience will aid in developing the meaningfulness of the concept. Give students "intuitive points of reference" to estimate slope. For example, consider the following sketches of cross-country skiers.


Person 1

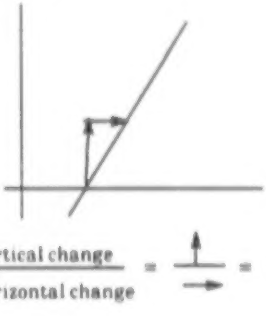

Person 2


Person 3

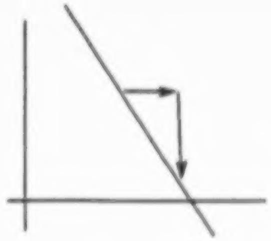

Person 4

Comparing person 1 and person 2, which one is doing work? Who is doing more work, person 3 or person 4? The steeper the hill, the greater the rate of work that is required. If we associate rate of work with a positive number, then going downhill is doing the opposite of work (person 2) and is associated with a negative number. A larger positive number is associated with person 3 than with person 4. Thus, slope becomes intuitively related to "steepness."

In the discussion of slope as $\frac{\text{rise}}{\text{run}}$, students may connect "rise" with "increase" – a positive change. They may find it helpful to draw horizontal and vertical arrows indicating the direction of change. For example:



$$\frac{\text{vertical change}}{\text{horizontal change}} = \frac{\uparrow}{\rightarrow} = \frac{+}{+}$$



$$\frac{\text{vertical change}}{\text{horizontal change}} = \frac{\downarrow}{\rightarrow} = \frac{-}{+}$$

Numerical estimates of the slope can also be quickly made using this approach.

Adapted with permission from "Concrete Versus Abstract in Teaching Algebra," Mathematics Teacher, March 1988, copyright by the National Council of Teachers of Mathematics.

(a) For a set of class activities that begins by considering slope as "steepness," and rise/run, and moves on to exercises that develop and use the formula for the slope of a line passing through two points, see "Skiing the Slopes," in Mathematics Teacher, December 1987.

(b) For a set of activities that follows well on previous direct variation studies, focuses on slope as speed and requires students to interpret various linear relations on distance-time graphs, see "Slope as Speed," in Mathematics Teacher, April 1988.

Activity: Have students devise ways of measuring the inclination of roads, roofs, etc. have students apply their "ways" to finding the slope of a line segment. In other words, have students "model" their situations on graph paper and then extend their procedures for finding slope. Then discuss how these "procedures" can be used to find the slopes of any oblique segments.

Activity: Have students develop a method of measuring the slope of a line. This can be achieved by having students work together and examining their own definition of slope and applying it to mathematics. Why do we measure rise/run instead of run/rise? (Answer: slope is associated with "steepness" which is related to the vertical). How is this measure of slope (rise/run) associated to the slope of a roof, the slope of a ski hill?

Activity: Have students draw line segments and note the similarities and differences in the steepness of the line, i.e., one may be steeper than the other, one may be in a negative direction. Then have students calculate the values of the slope and discuss how the numerical representation relates to the physical representation. For instance the line segments with a negative slope rise to the left in all cases; the line segment with a fractional slope will not be as steep as a line segment with a whole number slope.

Activity: In discussing the slope of horizontal and vertical lines, have students guess what their slopes would be, before calculation. Have students identify their reasons for making that particular guess. Then have students calculate the slope.

Activity: Have students discuss the meaning of parallel and perpendicular lines and then speculate as to the probable relationships between the slopes. Have students draw pairs of perpendicular and parallel segments using protractors, construction techniques or by tracing (opposite sides of a ruler, adjacent sides of a book, etc.). Find the slopes of the pairs of lines and generalize the relationships.

Problem: Draw the segment AB where A(-1, -5) and B(8, 1). What is the slope of AB? Note that C(2, -3) and D(5, -1) are on AB. What is the slope of CD? AC? DB? What generalization can be made about the slope of a segment and its parts? Draw any other segment, find its slope and then verify that the slope of any part of the segment is the same as the slope of the segment.

TECHNOLOGY INTEGRATION

Use a graphing calculator or a computer to plot pairs of points.

Software:

Algebra Drill and Practice I (Conduit) – "Slopes and Equations of Lines" option. Provides practice in determining the slope of a line given two points on a Cartesian plane.

Geometric preSupposer: Points and Lines (Sunburst) Measurement of length of line segments possible, determining ratios of the lengths of line segments possible. Construction of perpendicular and parallel lines possible. A grid may be superimposed. Suitable therefore as exploratory elective software in the topic of slope.

Examine segments that have positive, negative, zero and undefined slopes. Have students find the slope of each and then generalize by completing an exercise such as the following:

Segments that rise from left to right____________________
____

All segments that have a negative slope________________
____

The slope of a floor in a house is____________________
____

The slope of a vertical cliff is______________________
____

Have students identify "lines" in the classroom that illustrate the properties of zero and undefined slopes. For example, the floor is horizontal, therefore its slope is zero, the wall is vertical, therefore its slope is undefined.

RESOURCE CORRELATION

MMW2:	pp. 72–77
MM2:	pp. 175–182, 186–189
MPMM2:	pp. 72–77

CONCEPT 1

Students will be expected to demonstrate an understanding that points and lines can be represented on a Cartesian plane and have characteristics that can be defined and measured.

Students will be expected to:

1.3 use the Pythagorean relation to find the distance between two points on the Cartesian plane.

  1.3.1 solve problems that involve finding the distance between two points on the Cartesian plane.

COMMENTS

Students have used the terminology used in graphing, have graphed points and lines, and have used the Pythagorean Theorem. A review of these may be necessary in discussing distance.

It is not expected that students will simply memorize the distance formula. The use of the Pythagorean relation to find the distance between two points or the length of a segment is sufficient. Approximate irrational distances as decimals.

In general the distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by: $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ or $D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students first find the length of segments parallel to the x- and y- axes.

Activity: Once work with horizontal and vertical segments has been done, have students consider finding the length of an oblique line segment. The oblique line segment becomes the hypotenuse of a right triangle. Have students work on this in groups and have them "develop" a method for finding the length of any oblique line segment. Test the procedure by having other students try to use it exactly as written. This is an algorithm.

Problem: Find three points on a Cartesian Plane that define an equilateral triangle.

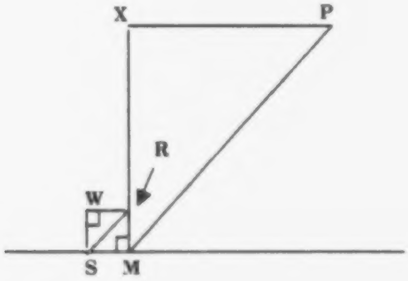
Problem: This problem provides an opportunity to discuss appropriate graphing scales as well as finding the distance between two points. A ship in distress is located at (300, 120). Rescue ship "A" is located at (120, 300), and rescue ship "B" is at (600, 150). Which rescue ship is closer to the ship in distress? How much closer? Round to the nearest 10.

TECHNOLOGY INTEGRATION

Software:
Geometric preSupposer: Points and Lines (Sunburst)
Measurement of length of line segments and distance between two points possible. A grid may be superimposed. Suitable therefore, as exploratory elective software.

ELECTIVE SUGGESTIONS

Problem: In the figure, triangles WRS, MRS and XPM are isosceles right triangles with legs of lengths 1 unit, 1 unit and 7 units, respectively. How many units from vertex W is vertex P?



Answer: 10 units.

One possible strategy: Redraw the problem on the Cartesian plane, placing point M at the origin and line SM along the x-axis.

RESOURCE CORRELATION

MMW2:	pp. 78–79
MM2:	pp. 163–166
MPMM2:	pp. 78–79

CONCEPT 1

Students will be expected to demonstrate an understanding that points and lines can be represented on a Cartesian plane and have characteristics that can be defined and measured.

Students will be expected to:

1.4 the midpoint of a segment
    1.4.1 solve problems that involve finding the midpoint of a line segment.

COMMENTS

Students intuitively understand the concept of midpoint. Before introducing students to the formal and algebraic definition of a midpoint, have them identify "midpoints" of horizontal and vertical line segments. From this, have students estimate the midpoint of an oblique line segment and generalize this finding in an algebraic form.

It is not intended that students only memorize the midpoint formula. In general, the midpoint of the line segment with end points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is:

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: To discover the algebraic meaning of midpoint, have students plot the following points: P(1, 6), Q(5, 2), R(6, 7), and S(12, 9). Find the point midway between point P and Q and label it T. What are the coordinates of point T? What is the relationship between the coordinates of point T and the coordinates of points P and Q? Then find the point midway between points R and S? Is this the same relationship as the one between the coordinates of point T and the coordinates of points P and Q?

Problem: M is the midpoint of line segment AB. C and D are on line AB so that AC = CM and MD = DB. What percent of AM is AD?

Answer: 150%.

TECHNOLOGY INTEGRATION

Software:
Geometric preSupposer: Points and Lines (Sunburst)
Measurement of length of line segments, and the construction of bisectors of lines possible. A grid may be superimposed. Suitable therefore as exploratory elective software.

RESOURCE CORRELATION

MMW2:
MM2:
MPMM2:

CONCEPT 2

Students will be expected to demonstrate an understanding that the ordered pairs which satisfy linear equations correspond to coordinates on the Cartesian plane and can be used to graph the equation on the Cartesian plane.

Students will be expected to:

2.1 find sets of ordered pairs that satisfy a linear equation and use them to sketch the graph of the equation.

COMMENTS

Students have done some previous work in using sets of ordered pairs to graph linear relations. Hence, the concentration should be on the relationship between equations, sets of ordered pairs, and their graphs; that they are different ways to express the same relationship. Every situation considered should be illustrated with an equation, ordered pairs and a graph.

Use variation problems as a means to achieving the graphing objectives, not as a particular type of problem to be solved. The examples used should not only be taken from the algebraic realm, but also from the real world. Provide a variety of domains in which to graph the relations. Linear equations in x and y should be introduced only after work has been done with the graph's direct and partial variation relationships.

TECHNOLOGY INTEGRATION

Computers and graphing calculators can be used to graph the relations.

Software:

Equation Math (MECC) – "Polygraph" option. Once function has been input and graphed, can get list of ordered pairs that satisfy equation.

Graphing Linear Functions (Mindscape) –"Easier Forms" option. Provides practice. Equations in the form $y = mx + b$ are presented, and the student must determine three sets of ordered pairs that satisfy the equation. The students then graph the ordered pairs to form the straight line.

SRA Computer Drill and Instruction, Mathematics: Level D – " Pre-Algebra" option. Lesson 22 suitable for drill/remedial: Given linear equation of $y = mx + b$ form, complete table of ordered pairs, then graph using cursor.

Spreadsheets may be used to determine sets of ordered pairs that satisfy linear equations. Software that has additional graphing capabilities can then graph the table of values. See "Mathematical Modeling Using Spreadsheets," delta-k, June 1989 for a discussion related to this topic. Appleworks and Microsoft Works are Alberta Education Basic Resources.

RESOURCE CORRELATION

MMW2:	pp. 136–137
MM2:	pp. 148–150
MPMM2:	pp. 136–137

CONCEPT 2

Students will be expected to demonstrate an understanding that the ordered pairs which satisfy linear equations correspond to coordinates on the Cartesian plane and can be used to graph the equation on the Cartesian plane.

Students will be expected to:

2.2 find the x- and y-intercepts of the graphs of linear equations
2.3 graph linear equations by drawing a graph through the x- and y-intercepts.

COMMENTS

An x- and y-intercept is the distance from the origin to the point where the line cuts the x- or y-axis respectively. At the x-intercept, $y = 0$ and at the y-intercept, $x = 0$. Reinforce to students that every point on a line is a solution of the equation that describes that line.

Once the discussion of the x- and y-intercepts of oblique lines is complete, students should examine the x- and y-intercepts of horizontal and vertical lines.

Graphing lines using the x- and y- intercepts provides students with an alternate way to graph linear equations. Since few equations have integral intercepts, care should be exercised in the equations given if the graphs must be exact. However, if the graphs do not have to be exact, this technique can be used effectively for any equation and, using intercepts that are fractions will reinforce number skills.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students draw the graphs of several equations that have easily interpretable intercepts. What is always true about the intercepts? Can this be generalized to all linear equations? How can this be used to find the intercepts without drawing the graphs? That is, what is the relationship between the equation of the line and the x- or y-intercept?

Activity: Have students draw several lines through the point (2,1). Point out and name the intercepts. Discuss what happens to the value of the x (or y) coordinate as the lines get close to the y (or x) axis. Generalize.

Activity: Once students are familiar with the term intercept, have them find the x- or y- intercept from a linear equation and then draw the graph of the equation.

TECHNOLOGY INTEGRATION

Computers and graphing calculators can be used to graph the linear equations

Software:
Master Grapher (Addison-Wesley) - "Function Grapher" option. Linear equations can be entered and graphed by the program. The student can then use vertical line, horizontal line or crosshairs option to determine x- and y-intercepts.

RESOURCE CORRELATION

MMW2:	pp. 148-149
MM2:	pp. 183-185
MPMM2:	pp. 148-149

CONCEPT 2

Students will be expected to demonstrate an understanding that the ordered pairs which satisfy linear equations correspond to coordinates on the Cartesian plane and can be used to graph the equation on the Cartesian plane.

Students will be expected to:

2.4 write linear equations from the slope-intercept form, $y = mx + b$

2.5 graph linear equations using the slope and y-intercept

  2.5.1 investigate the effects of the parameters m and b in $y = mx + b$

  2.5.2 use calculators or computers to draw the graphs of any linear equations.

COMMENTS

It is the intent of 2.4 that students could express the equation of a line in the slope-intercept form, that is, given that an equation in the $Ax + By + C = 0$ form, students would write it in the $y = mx + b$ form. Once the equation is in the $y = mx + b$ form, students then graph the equation by first plotting the y-intercept and then using the slope to find a second point; and then examine the effects of m and b on the graph of the function.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students draw the graph of $y = x$. On the same axes, draw the graphs of $y = 2x$, $y = {}^1/_2x$, $y = x + 1$, $y = x - 3$, $y = -3x$, $y = -x + 2$. Have students describe the differences and similarities between each of the graphs and $y = x$. What is the effect if m gets larger?...gets smaller? What is happening with the b value? What happens if m is negative? Since this follows the discussion of slope, they may be obvious to students, but it is important for students to be able to communicate these differences and similarities in preparation for their work in Math 23.

Activity: Have students discuss the slope of the segments between any two points on a line. Have them suggest and test a graphing methodology given a point on the line and the slope.

Activity: Discuss with students the criteria needed to uniquely define a line. Have them develop a system, given any point and a slope, to graph a line. Have them extend and test this procedure for graphing an equation written in slope-intercept form.

Problem: Point A is a (2,0) and AB has a slope of -1/2. Find a possible coordinate for point B.

Note: For a discussion of proportional reasoning, the algebraic representation of proportionality ($y = mx$), and the graphical interpretation of proportionality, see "Proportionality and the Development of Prealgebra Understandings," The Ideas of Algebra, K-12.

For a set of activities that (a) provides practise in plotting points in the Cartesian plane, (b) helps develop inductive skills in the form of pattern recognition and (c) promotes the development of skills in translation from English to symbolic form (expressions of the form $y = mx + b$), see "Finding Patterns in Linear Relations," Mathematics Teacher, May 1983.

TECHNOLOGY INTEGRATION

Computers and graphing calculators can be used to graph the linear equations. The following program may be input by students and used to graph straight lines. It is written in BASIC for the Apple II computer.

```
10    TEXT: HOME
20    VTAB 12
30    PRINT "THIS PROGRAM GRAPHS STRAIGHT LINES"
40    PRINT: PRINT "IN THE GENERAL FORM Y = M*X + B."
50    FOR N = 1 TO 5000: NEXT: REM PAUSE LOOP
60    HGR
65    HCOLOR = 3
70    PRINT: POKE 37,20: PRINT
80    PRINT: PRINT "GIVEN THE GENERAL FORM Y = M*X + B"
90    PRINT: PRINT "INPUT M,B";
100   INPUT M,B
110   PRINT: PRINT: PRINT
120   REM DRAW AXES
130   HPLOT 140,0 TO 140,159
150   HPLOT 0,80 TO 279,80
170   FOR H = 0 TO 270 STEP 10
180   HPLOT H,77 TO H,83
190   NEXT H
210   FOR K = 0 TO 160 STEP 10
220   HPLOT 137,K TO 143,K
230   NEXT K
240   HPLOT 271,65 TO 277,71
250   HPLOT 277,65 TO 271,71
260   HPLOT 147,0 TO 151,6
270   HPLOT 155,0 TO 147,12
280   REM  PLOT THE STRAIGHT LINE
290   FOR X = -14 TO 13.9 STEP .1
300   LET Y = M*X + B
310   IF Y > 8 THEN 340
320   IF Y < -7.9 THEN 340
330   HPLOT 140 + 10*X,80-10*Y
340   NEXT X
345   REM  PRINT LINEAR EQUATION
350   PRINT: POKE 37,20: PRINT
360   PRINT "THIS IS THE GRAPH OF Y = "M"*X + "B""
370   PRINT
380   PRINT "1 = ANOTHER GRAPH 0 = QUIT";
390   INPUT Z
400   PRINT: PRINT
410   IF Z = 1 THEN 60
420   TEXT: HOME
```

See the source article for an accompanying set of activities suited to investigating the effects of m and b in y = mx + b.

Reprinted with permission from "Microcomputer Unit: Graphing Straight Lines" in Activities for Implementing Curricular Themes from the 'Agenda for Action' copyright 1986 by the National Council of Teachers of Mathematics.

Software:

Master Grapher (Addison-Wesley) - "Function Grapher" option. A utility program suitable for exploring the effects of m and b. Uses function notation.

Algebra Drill and Practice I (Conduit) - "Slopes and Equations of Lines" option. Equations of the form $Ax + By + C = 0$ are given and student is to rewrite in $y = mx + b$ form. Student inputs m, and program draws graph and identifies y-intercept. Student may compare own graph of the equation with that displayed.

Computer Graphing Experiments I (Addison-Wesley) - "Linear Equations" option, "Graphs of the form $y = mx$" and "Graphs of the form $y = mx + b$" sub-options. Suitable for exploring the effects of m and b. Guided student worksheets accompany the software.

Equation Mqth (MECC), "Exploring Equations- Linear" option. Program suitable for exploring the effects of m and b. Student worksheets for investigating slope included. Hard copy option available. Specific parameters can be quickly incremented/decremented by cursor key action.

Graphic Linear Equations (Mindscape) - "Easier forms" option. Student is given equation in $y = mx + b$ form and must determine and graph three sets of ordered pairs that satisfy the equation.

Green Globs and Graphing Equations (Sunburst) - "Equation Plotter" option. A versatile utility graphing program suitable for exploring the effects of m and b.

RESOURCE CORRELATION

MMW2: pp. 146 -149
MM2: pp. 190 -195
MPMM2: pp. 146-149

CONCEPT 2

Students will be expected to demonstrate an understanding that the ordered pairs which satisfy linear equations correspond to coordinates on the Cartesian plane and can be used to graph the equation on the Cartesian plane.

Students will be expected to:

2.6 recognize and graph the equations of lines that are parallel to the x- and y-axes.

COMMENTS

This expectation may be integrated throughout the study of coordinate geometry and graphing. Questions arise in the textbooks throughout the study of coordinate geometry. Remind students of the meaning of zero and undefined slopes.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students suggest what the graphs of $y = 2$ and $x = 3$ would be. Which would have a zero slope (0 rise over any run)? Which would have an undefined (infinite slope (any rise over 0 run)? What would be the general form of the equations of lines parallel to the x- and y- axes?

Activity: Have students work together to graph a line that cannot be represented by an equation of the form $y = mx + b$.

Computers and graphing calculators can be used to graph the linear equations.

TECHNOLOGY INTEGRATION

Software:

Equation Math (MECC) - "Exploring Equations - Linear" option will accept equations of the form $x = b$, $y = b$.

Green Globs and Graphing Equations (Sunburst) - "Equation Plotter" option is a utility program suitable for graphing lines parallel to the x- and y-axes.

RESOURCE CORRELATION

MMW2:	pp. 74, 149
MM2:	p. 196
MPMM2:	pp. 74, 149

CONCEPT 1

Students will be expected to demonstrate an understanding that phenomena can be related and that these relations can be represented by graphs and tables of values.

Students will be expected to :

1.1 sketch graphs that describe physical phenomena or everyday occurrences including the identification of the dependent and independent variables and appropriately labeled axes
  1.1.1 solve problems involving physical phenomena and everyday occurrences by modelling them graphically
1.2 read and interpret graphs drawn to describe the relationships between real life variables
1.3 interpolate and extrapolate to find information from given graphs and tables of data.

COMMENTS

Grade 7:
In Grade 7 the emphasis should be placed on choosing appropriate scales and on investigating the effect of using different scales (e.g., distortion when different scales are used on each of the axes; accuracy of plots when large or small scales are used). Plotting a picture or design in all four quadrants is not only an interesting activity, but can effectively demonstrate the effect that varying scales can have on the outcome. It is important to continue to emphasize the notion of pairs of numbers and their relationships. (In this case the pairs of numbers will not have a functional relationship but, rather, serve to describe a location on a plane.)

Grade 8:
Review the concepts of a mathematical relation. Note the similarities and differences of several different relationships (e.g., height and age of students; skill (in sport, playing an instrument, etc.) and amount of practise; the total cost of a multiple purchase of a single item(e.g., pairs of socks); the cost of a taxi ride and distance). The concept of function should begin to emerge; i.e., with some of the relationships a rule can be used to describe the relationship when one of the values is given; and there can be only one possible value for the second element when the first is given; and there can be only one possible value for the second element when the first is given. In other relationships, while the value of one element depends on the other, there is no hard and fast rule to describe the relationship. (e.g., skill depends on practice).

Use several practical examples of (linear) relations. Have students determine the function rule that determines the relationship (e.g., perimeter of a circle depends on the radius; in this case the function is a formula; multiple purchase of a single item) and then generate the ordered pair that describes the relationship for any given value.

Grade 9:
Interpretive skills must be developed at an intuitive level. Whenever graphs are discussed, the concepts of increase, decrease, intercepts, maximum, minimum, rate of change and slope should also be discussed as appropriate to the graph.

Except for the word "function," the use of the terminology must be de–emphasized. Students should be encouraged to draw or interpret graphs that describe many different types of situations. (Some may not even have, or need numerical information.)

Examples:

1) Sketch a graph that would describe (over time) the temperature of a glass of water just after an ice cube has been dropped into it.
2) Sketch a graph that would describe the speed of a race car on a given track. (Sketch the track on the board or an overhead with several curves, corners, etc.)

From: Junior High Mathematics, Teacher Resource Manual, Alberta Education, 1988.

Remind students that variables are symbols used to represent numbers that may vary according to a particular interpretation.

Emphasis should not be placed on quantifying the variable or the graphs unless such quantification aids understanding. Emphasis should be placed on how the variables relate to each other and students should be able to describe what is occurring within their graphs.

Many phenomena can be represented by graphs. Students should be given the opportunity to examine and discuss many such examples and should be taught to analyze them in order to conceptualize a situation. Examples should include graphs with and without numeric values.

The skills of accurate interpolation and extrapolation are extremely useful. Make sure that students examine their answers for appropriateness. Once again stress that the graph represents all solutions to a problem and that individual points represent particular situations.

Examples do not all have to be linear but some caution should be exercised in the presentation of non–linear examples.

Note: The terminology, Domain and Range, should not be included. However, discussing the meaning of Domain and Range should be included.

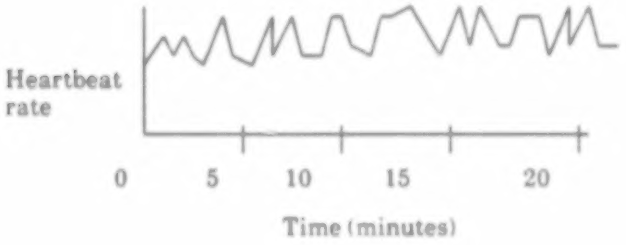
PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students suggest situations where two phenomena are related. For example, the age of a car and its value, the month of the year and the maximum temperature, etc. Discuss how strong the relationships are. Can the examples be used with phrases such as "caused by," "depends on" etc.? Have students graph the values to see if they can make any statements about the relationships.

Problems:

- A glass of water is at room temperature. An ice cube is dropped into it. Draw a graph that describes the temperature of the water over time.
- A swimmer is in a race that covers two lengths of a pool. Draw a graph that describes her speed over time.
- Draw a graph that describes how hungry you are according to the time of day.
- Draw a graph that describes the noise level of the seventh game of the Stanley Cup final, from the singing of the national anthem to the end of the game.
- You fill an electric kettle with cold water and plug in the kettle. Sketch the graph of the water temperature versus the length of time the kettle is plugged in.
- On a sunny day, the length of the shadow of a telephone pole depends on the time of day. Sketch a graph of the length of the shadow versus the time of day.

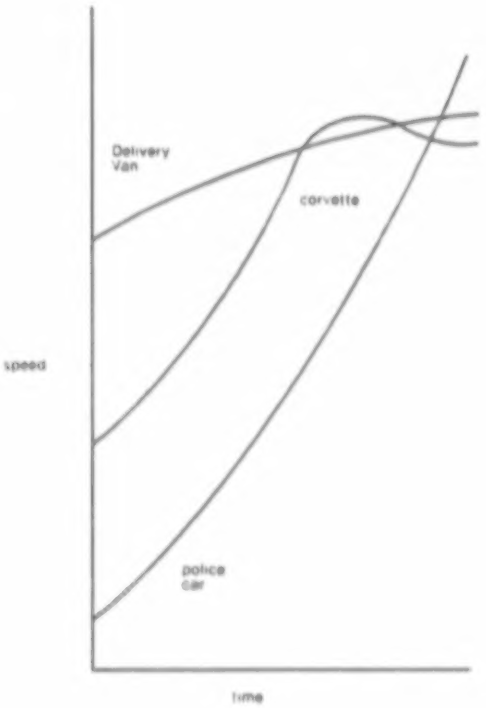
Problem: The following graph describes the heart rate of a hockey player during the third period of the final game of a championship series.



Can you tell what this player did during the period? When did he play? How much did he play? Was this player in good shape? What could have occurred around the three-minute mark? the 12-minute mark? Do you think that this player was on the winning team? Why?

Activity: Look at the following graph. It describes the actions of three vehicles, beginning at a busy intersection. Describe what is happening. Who is accelerating/decelerating? When?

Activity: (cont'd)

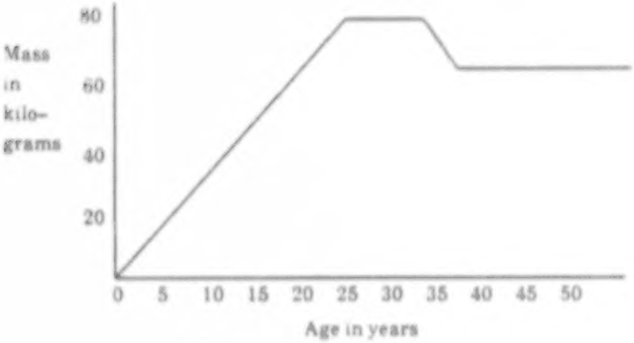


For further information, see The Language of Graphs pp. 19–47.

Activity: Make up stories to explain the graphs.

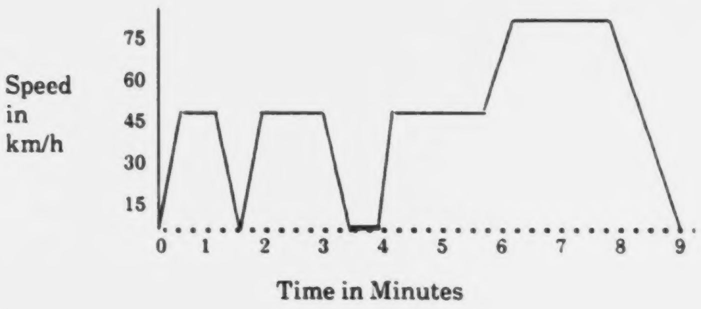
The graph illustrates a person's mass from birth to 50 years of age. Make up a story to explain the graph. Include in your story—

1. whether the person is a man or a woman,
2. about how much that person's mass was at birth,
3. what the person's maximum mass was, and
4. whether the person dieted sometime and, if yes, when.



A man is driving from home to work. The graph shows his speed at any time during his nine-minute trip. Make up a story about his trip to explain the graph. In your story tell—

1. approximately how far he lives from where he works,
2. what happened one and one-half minutes after he started,
3. what happened between three and one-half minutes after he started and four minutes after he started, and
4. something observed about speed limits.



Discuss in detail what the students have written. Are there misinterpretations that need to be clarified?

Reprinted with permission from "Make up a Story to Explain the Graph," Mathematics Teacher, January 1984, copyright by The National Council of Teachers of Mathematics.

Problem: At Peace River Trust, the monthly payments on a loan of $1000 spread over two years are $46.49. The monthly payments on loans of $3000 and $4500 are $139.47 and $209.21 respectively. If Tom wants to borrow $7500 for a used Firebird and Barbara needs $2800 for a dirt bike, what should they expect their payments to be?

The monthly payments on a loan of $1000 change according to the length of time the loan is for. For example, a loan of $5000 over one year has monthly payments of $441.35. Over two years the monthly payments are $232.45 and over five years, the payments are $108.10.

Approximately what would the payments be over four years? over 30 months?

RESOURCE CORRELATION

MMW2:	pp. 128–137
MM2:	pp. 143–156
MPMM2:	pp. 128–137

CONCEPT 2

Students will be expected to demonstrate an understanding that some pairs of variables relate to each other directly or inversely according to a relationship that can be written as an equation.

Students will be expected to :

- 2.1 recognize relations in which the variables vary directly
  - 2.1.1 solve direct variation problems
  - 2.1.2 solve partial variation problems
- 2.2 recognize relations in which the variables vary inversely
  - 2.2.1 solve inverse variation problems.

COMMENTS

This will be a new area for students; no work is done in junior high school.

Students intuitively understand variation problems. Initial emphasis should be on examining the relationships between the variables, graphing the problems and using interpolation and extrapolation techniques rather than the formal development of algebraic processes to deal with the problems. After students are throughly familiar with the notion of variation and the idea that variations are relations that can be graphed and examined by their graphs, the development of equations to represent variation problems should be done.

Little time should be spent on exercises that involve only numbers (e.g., x and y vary directly. When $x = 5$, $y = 3$. Find x when $y = 9$) and questions involving special types of the direct and inverse variation (direct squared, etc.) should probably be avoided unless they are being examined graphically.

It is advisable to examine the graphs of situations involving variation by making tables, drawings and interpreting graphs before formally introducing types of variation problems.

Note: When discussing partial variation, **do not** include solving a system of equations in two unknowns.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: A suggestion for introducing the discussion of direct, partial and inverse relations is to provide the students with a series of graphs representing all three relations, and have the students discuss in small groups the similarities and differences between the graphs.

For more Problem Ideas see:
Mathematics 10 (Addison-Wesley), pp. 228–231
Holtmath 10 (Holt), pp. 182–185
Mathematics Principles and Process (Nelson), pp. 274–279, 283–287
See also "Using Linear Functions" in Mathematics Teacher, October 1988 for a set of activities designed to develop understanding of partial variation through graphing and graph interpretation in an "everyday life" context.

INTEGRATION TECHNOLOGY

For an example of how spreadsheets can be used to help develop students' interpretation of relationships between two variables, see "Integrating Spreadsheets into the Mathematics Classroom," Mathematics Teacher, November 1988. Appleworks and Microsoft Works are Alberta Education Basic Resources.

ELECTIVE SUGGESTIONS

Problem: The following example would work well as a project with students working in groups of two or three.

Suppose that you have become the manager of a motel, and you wish to investigate the possibility of raising room rates. You examine what other motels are charging, and from that you produce the following table, relating the price per night and the average number of rooms that are occupied nightly.

P: price per night	37	45	53
N: Number of rooms occupied	50	40	30

What type of variation best describes the situation? What might be an appropriate rate assuming 45 rooms were occupied? 80 rooms? 55 rooms ? 10 rooms? Do you think it is reasonable to interpolate and extrapolate in this way? Identify the things that you think are most likely contributing to the change in room rates.

Adapted with permission from Computing and Mathematics: The Impact on Secondary School Curricula, p. 94, copyright 1984 by the University of Maryland.

Problem: What happens to the time of a trip of given length if the average speed is increased 25 percent?

Answer: Decreased 20%.

Suggest to students that they draw upon their knowledge and understanding of slope and graph the situation represented by the problem.

Problem: (a) A train travelling at 100 km an hour takes three seconds to enter a tunnel and an additional 30 seconds to pass completely through it. What is the length of the train?

(b) What is the length of the tunnel?

Answers: (a) 1/12 km; (b) 5/6 km

RESOURCE CORRELATION

MMW2:	pp. 252–255 (Note: no inverse variation)
MM2:	pp. 220–224, 228 – 239
MPMM2:	pp. 252–255 (Note: no inverse variation)

COMMENTS

Maintenance of skills should be done as needed when introducing new material. It should not be done as a separate section at the beginning of a unit.

The following guidelines, prepared by the ASA/NCTM Joint Committee on Statistics and Probability, provide principles for consideration during the teaching of statistics.

1. Learning should be active, not passive, and focused on asking questions about something in the students' environment and finding quantitative ways to answer the question.
2. Problems should be approached in more than one way with an emphasis on discussion and evaluation of these different methods.
3. Real data should be used whenever possible in any statistics lesson.
4. The emphasis in teaching statistics should be on good examples and building intuition.
5. Topics should be presented in ways designed to give students hands on experience in working with data.
6. Student projects should be an integral part of any work in statistics.
7. The emphasis in all work with statistics should be on the analysis not on a single answer.
8. The progression should be from the concrete to the pictorial to the abstract.

Problems should be presented that can be used as vehicles to accomplish the objectives of the unit. The depth to which a class should be taken will be a function of the problems chosen and the ability of the class. Students should be actively involved in the examination of the problems and techniques for analysis should be introduced as needed by the class. Students should be encouraged to work with each other and discussion of possible interpretations and conclusions should play a major part in the delivery of the objectives. Consideration should be given to methods of evaluation other than a formal written examination.

CONCEPT 1

Students will be expected to demonstrate an understanding that populations have certain characteristics which can be determined by analyzing the characteristics of a suitable sample of that population.

Students will be expected to:

1.1 determine a sample that will adequately and accurately represent a population
1.2 evaluate samples for bias, appropriateness of sample type, amount of data and randomness
1.3 draw inferences about the population from which a sample was taken and express it using the language of probability
    1.3.1 determine a valid sample, design and carry out a simple survey, and make a valid inference about the population based on the results.
    1.3.2 defend or oppose a generalization that is made about a population from a sample.

COMMENTS

A sample is data that are chosen from, and hence believed to represent a population. Proper sampling is extremely important in data collection, in that improper sampling will effect the statistics and hence any inferences made from the sample to the population.

The following is a list of types of sampling. It is not important that students memorize each of the types but teachers should endeavour to discuss various types with students, in terms of the advantages and disadvantages of each. When students conduct surveys themselves, they should know the types of sampling they use and be aware of the advantages and disadvantages that may result.

Convenience Sampling is acquiring data from a group that is convenient.

Self-selected Sampling involves people who voluntarily respond to a survey.

Judgement Sampling is where an expert selects a sample he or she considers representative.

Probability Sampling includes a mechanism to ensure randomness.

Cluster Sampling is where a cluster of subjects within a particular area is randomly chosen to provide data.

Stratified Sampling is where a population is divided into identifiable groups from which samples are then randomly drawn.

Systematic Sampling is done by choosing every nth member of a population to form the sample.

Introduce and discuss the concept of bias. Have students evaluate analyses, question the outcome and come up with suggestions to produce more convincing results. Encourage them to be skeptical and prepared to question results and the methods used to produce them.

For problems in which a sample is examined, make a probability statement about the results and then try to check the accuracy of the original statement. For example, if a student did a survey of movie popularity and discovered that 60% of the students in the sample thought that Revenge of the Nerds 6 was the best movie they had seen in the past two months, this could be stated as "The probability that a particular student thinks Revenge of the Nerds six is the best move seen in the past two months is 60%."

Discuss the meaning of the statement , that probability is the likelihood of the occurrence of an event. If they chose students at random, they could reasonably expect that 60% of them would favour that movie. Discuss as well that they could not reasonably expect six out of every group of 10 students to favour that same movie but that the percentage is an average taken over the entire school population.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students look at and criticize various sampling techniques, particularly those used by the media, government and polling companies. Give them problems and have them design sampling procedures they think will give correct results and procedures they think will bias the results in a certain way.

Example:
If you wanted to find how many people really thought that a certain headache remedy was better than another, how would you structure the sample? How would you do it if you wanted to show that the remedy was definitely better?

Examine the following statements for possible bias:
"Of all the people who died of lung cancer in 1977, 70% had smoked cigarettes during their lives."

"Four out of five dentists interviewed recommend sugarless gum for their patients who chew gum."

Activity: Have students find newspaper stories that rely on some sort of statistic and discuss the confidence with which the results should be accepted. Discuss what factors could adversely affect the conclusion of the writer and what factors could positively affect the conclusion of the writer.

TECHNOLOGY INTEGRATION

Video:
Math Wise (ACCESS): "Predicting 02 –Choosing a Sample." Provides everyday life examples of situations where carefully choosing a sample of people for surveying is important.

To help emphasize the idea of random selection, see HM10 TRM Lesson Notes, p. 109, Introductory Activity #1.

RESOURCE CORRELATION

MMW2:	pp. 82–85, 94–95
MM2:	pp. 245–247
MPMM2:	pp. 82–85, 94–95

CONCEPT 2

Students will be expected to demonstrate an understanding that data collected from a sample must be organized, presented and analyzed in order that valid inferences will be drawn.

Students will be expected to:

2.1 identify a situation, formulate an hypothesis and plan what data need to be collected
2.2 organize data using stem-and-leaf plots and box plots
2.3 choose and calculate appropriate measures of central tendency for sets of data
2.4 represent data using appropriate graphs
2.5 do simple visual analyses on sets of data organized in stem-and-leaf plots, box plots and graphs
    2.5.1 derive and defend conclusions based on the visual analyses of sets of data
2.6 evaluate intuitively an analysis of data for the confidence with which results can be accepted.

COMMENTS

Students have had experience with statistics in junior high. Specifically, they have examined the use and misuse of statistics; biases in surveys; the representation of data in the form of pictographs, bar graphs, line graphs, circle graphs; the interpretation and analysis of arguments or conclusions based on statistical information; the mean, median, mode and range; have conducted a small survey and reported results; and have been introduced to the term probability.
This should not be taught as a set of six discrete skills, every attempt should be made to work through each expectation with a set of data.

Univariate Data

Price of hamburgers (ice cream cones) in your area.
Weight (height) of the linemen on your school's football team.
Number of goals scored by defencemen in the NHL.
Height of tallest buildings in Calgary (Edmonton, Canada).
Weeks on the Top 40 for music (video, movie) hits.
Number of rainy (frost free) days in a year in your area.
Population of towns and villages in your county (municipal district).
Sources of information for problems such as these are the yearbooks and almanacs in your school library, Exploring Data published by Dale Seymour, sports record books, books on movies, records and entertainment, newspapers and magazines. Students may also have problems from other courses such as Social Studies that would lend themselves to examination within this unit.

Collection

Collection of data can be done by surveying a particular group, by researching a topic using library resources or by being given data in some form. If a survey is used, how the data are collected i.e., personnally, over the telephone, can be discussed. Are there any differences in the responses depending on the method of collection?

The following is a list of some discrete human characteristics that may be investigated.

I Genetic Characteristics

1. Eye colour (specify)
2. Hair colour (specify)
3. Sex
4. Tongue type (curl)
5. Hair
6. Thumb type ("Hitchhiker's thumb")
7. Ear type (lobe)
8. Dimple (chin/cheek)
9. Freckles (none, few, lots)
10. Handedness
11. Eyedness (left or right dominance)
12. Finger relationship (relative lengths of ring and little fingers)
13. Handclasp (folded hands, interlocked fingers; which thumb is on top of the other?)

II Acquired Characteristics

1. Preferences
2. Ownership (e.g., of a guitar)
3. Number of certain possessions
4. Hours of sleep
5. Name (the beginning letter, or the number of letter/s)
6. Clothing sizes (e.g., shoes)
7. Birthday (use a perpetual calendar and determine day of the week of birth; hour of the day)
8. Age (e.g., year and month – 15.6 years old)

Reprinted with permission from "Applications in School Mathematics: Human Variability" in Applications in School Mathematics copyright 1979 by the National Council of Teachers of Mathematics.

See also "Projects" in Teaching Statistics and Probability for brief descriptions of a number of possible class projects.

Organization and Presentation: Stem–and–leaf plots are a method used to organize data in which the data are not destroyed. The easiest way to set up a stem–and–leaf plot is to write the stems as all but the last digits in a data set while the leaves will be the last digit. For example, if the data were the following: 328, 335, 341, 326, 345, 365, 329, 357, 361, 345, 339, 367, the stems would be the numbers 32, 33, 34, 35 and 36. The stems are written in a column to the left of a vertical line. As the data are examined, the last digit (the leaf) is placed next to the appropriate stem to the right of the vertical line.

The stem-and-leaf plot for the above data would appear as follows:

32	8 6 9
33	5 9
34	1 5 5
35	7
36	5 1 7

It is useful to re-order the leaves in ascending order. This will help in other activities such as finding the median. The above stem-and-leaf plot would become the following:

32	6 8 9
33	5 9
34	1 5 5
35	7
36	1 5 7

Stem-and-leaf plots display a number of characteristics of a set of data, including the upper and lower values and the general shape of the distribution. It is also possible to see clusters and gaps as well as to immediately see the relative position of any item of importance.

It is sometimes useful to compare two sets of data on a single plot. For example, the following are marks from the same test for two different classes:

Class A
65, 56, 78, 98, 78, 67, 77, 75, 74, 64, 60, 51, 82, 80.

Class B
54, 76, 65, 87, 63, 62, 50, 58, 46, 69, 76, 79, 81, 72.

The back-to-back stem-and-leaf plot appears as follows:

6	4	
8 4 0	5	1 6
9 5 2 3	6	0 4 5 7
9 6 6 2	7	4 5 7 8 8
7 1	8	0 2
	9	8

From this plot it is possible to make an immediate comparison between the achievements of the two classes.

For a more detailed look at stem-and-leaf plots, see Exploring Data from the Quantitative Literacy Series published by Dale Seymour. See also: "Stem-and-Leaf Plots," in Activities for Implementing Curricular Themes from the 'Agenda for Action.'

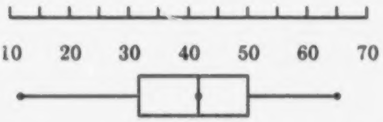
Box plots are useful because they highlight the majority of the data by placing a box around it and show the extremes by drawing lines to them. Box plots will be used to examine confidence intervals in a later course, but for now will be used to organize data and to compare two or more sets of data.

The method for setting up a box plot is as follows:

1. Find the median of the data.
2. Find the median of the upper half of the data. This is the upper quartile.
3. Find the median of the lower half of the data. This is the lower quartile.
4. Find the extremes of the data.
5. On or below a number line, mark the median, quartiles and extremes.
6. Draw a box between the two quartiles. Mark the median within the box. From the quartiles, draw "whiskers" to the extremes.

Example:

From a set of data, it has been established that the median is 42, the lower quartile is 31, the upper quartile is 50 and the extremes are 11 and 65. Draw a box plot to represent this.



Box plots can be very effective in displaying the differences between a number of sets of data, and, unlike stem-and-leaf, are not restricted to comparing two sets only. By placing the plots for a number of sets of data beneath each other, it is possible to see how the majority of the data compare and how the medians and extremes compare to each other.

For further information, see Exploring Data from the Quantitative Literacy Series published by Dale Seymour. See also: "Exploring Baseball Data," Mathematics Teacher, October 1987, and "Exploring Data with Box Plots," Mathematics Teacher, November 1988.

Students have some experience with pictographs, bar graphs, line graphs and circle graphs. Histograms can be drawn by turning a stem-and-leaf plot sideways.

Analysis: Students have spent time calculating the measures of central tendency, hence attention should be paid to using these measures as a form of analysis.

The suggestions made for each organizational method mentioned previously include some suggestions about analysis. Encourage students to look at the appearance of the representation of the data and formulate conclusions. Expect them to defend their analyses and to be critical of them.

Discuss with students the use of an average as a single number used to describe an entire set of data. Students will know how to find "averages" (means). Emphasis should be placed on finding and using medians. Medians can be found directly from stem–and–leaf plots and used in box plots.

The use of the range as a measure of dispersion is of value particularly when extremes are plotted on a box plot. There is probably little need for other measures of dispersion to be introduced at this time. However, the concept can be discussed as students look at the representation of data in stem–and–leaf and box plots.

Provide students with an example such as the following to discuss, to provide them with an understanding of the different "averages" and the use of each.
A company is made up of 10 people and the president of the company tells the union that there is no need for a pay raise because the average salary in the company is $38 400/a. The manager disagreed and said that the average salary was only $30 000/a. The head of the secretaries' union said that the average was only $20 000/a. Which of the three is correct? The salary list is given below. How do you think each person calculated their "average?" Which average do you think should be used in this situation? Why?

President	$150 000
Manager	$50 000
2 Salesmen	$40 000 ea
4 Secretaries	$20 000 ea
2 Clerks	$12 000 ea

Inferences: Statistical Inferences is the process of drawing judgments or making statements about a population, based on the results obtained from a sample.

Evaluation for Confidence: Introduce the term confidence to describe the expectation that the results can be viewed as true. At this level, an intuitive treatment is intended.

204

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: A sample hypothesis: There are more ______ (make of vehicle) in the community of __________ than __________, __________ and __________ (three more makes of similar vehicles). Design a strategy for determining the validity of the hypothesis.

The following activity may serve to help strengthen the skills and understanding the students developed during their class project.

Activity: Class Scores Example. The data that follow represent the test scores of students in two classes. Each class has 20 students.

Class: A
82, 78, 57, 91, 29, 80, 85, 49, 82, 67,
99, 68, 83, 12, 87, 86, 38, 81, 58, 79

Class B:
72, 82, 74, 84, 81, 84, 76, 12, 72, 71,
70, 93, 13, 90, 80, 73, 91, 70, 99, 88

Have the students "explore" the data at first. They may find at first that there are no clear cut differences between the two classes. For example, the high and low scores in both cases are 99 and 12. The mean score for class A is 69.55 and 73.75 for class B. However, the median for class A is 79.5, while for class B it is 78.0. Is either of these a better indicator?

Have the students create a back-to-back stem-and-leaf plot, such as that shown.

Back-to-Back Stem-and-Leaf Plots for Scores of Class A and Class B

Class A		Class B
2	1	23
9	2	
8	3	
9	4	
87	5	
87	6	
98	7	00122346
76532210	8	012448
91	9	0139

Is there any sort of pattern beginning to emerge, based on these two plots? Discuss.

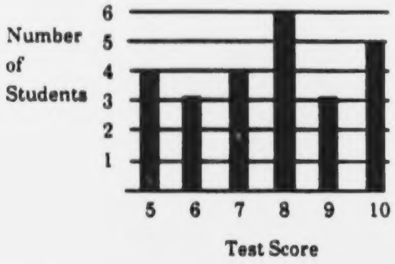
Adapted with permission from "Realistic Examples in Elementary Statistics," Mathematics Teacher, December 1987 copyright by The National Council of Teachers of Mathematics.

Problems: The following problems can serve as a partial review of measures of central tendency and of graph interpretation.

(a) Five test scores were lost, but a summary of the five lost scores indicates that the mode was 90, the median 85 and the mean, 83. If the grades were integers and could range from 0 to 100, what is the lowest possible grade from the missing set of scores?

Answer: 50.

(b) Find the average (mean) test score to the nearest integer.



Answer: $\frac{191}{25} \approx 8$

(c) Problem Solving in Mathematics: Focus for the Future, p. 72, #3, 7.

Note: See Exploring Data for stem-and-leaf and box plot instruction and applications. For activities to help develop students' skills in constructing and interpreting stem-and-leaf plots, see "Stem-and-Leaf Plots," in Activities for Implementing Curricular Themes from the 'Agenda for Action'.

See "Exploring Data with Box Plots," Mathematics Teacher, November 1988 for a set of activities designed to develop students' skills in creating box-and-whisker plots.

For an application of stem-and-leaf, and box-and-whisker plots, see the article, "Exploring Baseball Data," Mathematics Teacher, October 1987.

TECHNOLOGY INTEGRATION

Video:

- Math Wise (ACCESS): "Locating/Interpreting 01-Organizing Information." Provides everyday life examples of situations where organizing available information is important.

206

- "Locating/Interpreting 02 – Graphs"
  Provides everyday life examples of the value of being able to create and interpret graphs.

- "Predicting 03 – Averages"
  Provides everyday life examples of the importance of understanding the meaning of average and its relationship to the individual values in the set for which it is the average. Discusses average as mean and mode.

RESOURCE CORRELATION

MMW2:	pp. 86–95
MM2:	pp. 248–266
MPMM2:	pp. 86–95
ED:	pp. 7–82

CONCEPT 1

Students will be expected to demonstrate an understanding that angles can relate to each other in ways that can be determined by identifying particular characteristics of the segments that form them.

Students will be expected to:

- **1.1 use the following terms: parallel, perpendicular, transversal, corresponding angles, alternate angles, interior angles**
- **1.2 use the relationships between vertically opposite angles to calculate the measures of angles**
  - **1.2.1 verify that vertically opposite angles are equal**
- **1.3 use the relationships amongst angles made by parallel lines and a transversal**
  - **1.3.1 verify the relationships amongst angles made by cutting parallel lines with a transversal.**

COMMENTS

Maintenance of skills should be done as needed within the presentation of new material. It should not be done as a separate section at the beginning of a unit. Introduce and use correct vocabulary within the appropriate context. A good definition of the terms can be found in Mathematics Dictionary, 4th. Edition, James & James.

Use the students' knowledge of the sum of the angles of a triangle along with the newly acquired concepts to develop problems involving vertical angles, parallel lines and triangles.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity 1.2: Have students draw several sets of vertical angles, label them appropriately and measure each of the four angles in each diagram. From the measurements, make a generalization.

Activity 1.2: An intuitive, introductory activity that students can perform is simply to draw two intersecting lines, and then place their pencils along one of the two lines, holding the pencil at the point of intersection. Focussing on one of the vertical angles, turn the pencil until it now lies along the second ray of the angle. What is the relationship of the line of the pencil with the ray of the second vertical angle? Was the angle turned through on both sides of the point of holding the pencil the same?

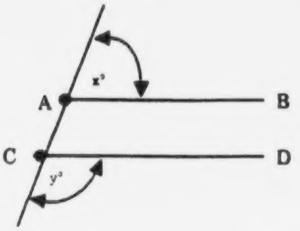
Problem 1.3: Problem Solving in Mathematics: A Focus for the Future, p. 69, #29.

Activity 1.2: Have students draw pairs of parallel lines and a transversal. Measure each of the eight angles and generalize the relationships.

Problem 1.3: Integrating Algebra and Geometry

The following problems extend the geometric aspects of the situations presented by introducing some algebra. They can be solved by guess-and-test. Encourage the students to write the appropriate equations, and to briefly describe the geometric relationships that serve as the basis for the equation.

(a)

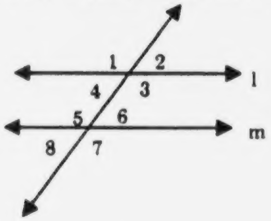


Given AB ∥ CD
$y = 3x$

What is the measure of ∠ACD in degrees?

Answer: 45°.

(b) l ∥ m, m∠4 = 8x + 4, and m∠8 = 5x + 25. Find m∠2.



Answer: 60°.

INTEGRATION TECHNOLOGY

Software:
Geometric preSupposer: Points and Lines (Sunburst) provides exploratory construction and measurement possibilities.

ELECTIVE SUGGESTIONS

Activity 1.3:
Discuss with students why the technique of construction of parallel lines works, i.e., one possible method of construction involves the construction of congruent corresponding angles.

RESOURCE CORRELATION

MMW2:	pp. 263-265, 268-271
MM2:	pp. 290-294, 313-315
MPMM2:	pp. 263-265, 268-271

CONCEPT 2

Students will be expected to demonstrate an understanding that triangles can be uniquely determined by appropriate combinations of sides and angles.

Students will be expected to:

2.1 construct a triangle given two sides and the included angle (SAS), three sides (SSS) and two angles and a side (ASA)

  2.1.1 verify that any of the three conditions SAS, SSS or ASA uniquely determine a triangle.

PROCESS/PROBLEM-SOLVING CONTEXT

Problem: The following problem can serve as a reminder to the students of some of the conditions on the lengths of the sides of triangles.

A wire 8 cm long is to be cut into three pieces that are to be used for the sides of a triangle. If all three pieces have integral lengths, what is the length of the shortest side?

Answer: 2 cm.

What type of triangle is this? Is there only one solution with integral sides? One possible approach is to set up a table of lengths of sides and systematically complete it using a guess-and-test strategy. Have students use a straight-edge and compass and attempt to construct the triangle using the table values.

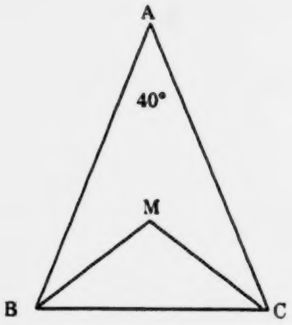
TECHNOLOGY INTEGRATION

Software:
Geometric preSupposer: Points and Lines and Geometric Supposer: Triangles (both from Sunburst) provide extensive exploratory opportunities. "Triangles" best suited.

ELECTIVE SUGGESTIONS

Students may be given a sketch, picture or photograph, or have a "situation" described to them, and be asked to state the ways in which geometry (angles, parallel lines, congruence) plays a role.

Problem: The measure of one angle of a triangle is 40°. What is the measure of the angle formed by the bisectors of the other two angles?



Answer: $m\angle BMC = 110°$.

This problem requires "seeing" some of the relationships that exist among the angles. It may be solved by a guess-and-test strategy, or by algebraic means.

RESOURCE CORRELATION

MMW2:	pp. 286–294
MM2;	pp. 318–319
MPMM2:	pp. 286–294

CONCEPT 3

Students will be expected to demonstrate an understanding that similar triangles have the same shape but may differ in size, that angles are equal and that the ratios between corresponding sides are equal, and that congruent triangles are similar triangles of the same size.

Students will be expected to:

3.1 use the relationship between the lengths of the sides of similar triangles to calculate the lengths of unknown sides
  3.1.1 verify that the ratios between corresponding sides of similar triangles are equal
  3.1.2 solve problems by using the relationships between the sides of similar triangles
3.2 identify the postulate by which a pair of triangles can be shown to be congruent.

COMMENTS

The three conditions for congruence of triangles are the following:

SSS - the corresponding sides are equal
SAS - two of the corresponding sides and the angles they determine are equal
ASA - two of the corresponding angles and the sides between the vertices are equal.

Define similar triangles as triangles that have corresponding angles of equal measure.

Two triangles are said to be congruent if one can be made to coincide with the other by a rigid motion in space. For students, congruence should be shown to occur if one triangle exactly fits on top of another.

Introduce students to the three abbreviations for triangle congruence, SSS, SAS and ASA. At this time students should also be introduced to the conventions used in noting congruences and marking congruences on diagrams.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity 3.1: On a worksheet, give students the "pieces" from which they are to try to construct triangles. The pieces would be:

1) three sides
2) two sides and an angle to be constructed between them
3) two sides and an angle that is not to be between them
4) two angles and the side to be constructed between their vertices
5) two angles and a side that is not between their vertices
6) three angles.

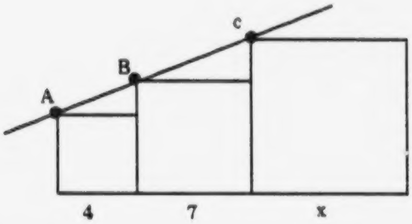
Have students work in groups and compare the construction they do for each of the conditions, both within their own groups and with the class. Which conditions produced congruent triangles? Will these conditions always produce congruences? Which conditions can never guarantee congruence? Why?

Activity: Construct pairs of similar triangles. Measure the lengths of the sides. Find the three ratios between the lengths of the sides of each triangle and compare. Discuss why the ratios will be equal.

Activity: Explore everyday examples of similar triangles. For example, an ironing board uses similar triangles to stay level as it is raised.

Activity: Mention that similarity occurs whenever one triangle is a magnification or reduction of the other. Discuss and discover whether or not the relationships between ratios of corresponding sides will continue for other pairs of similar polygons.

Problem: If the three vertices A, B and C in the three adjacent squares are collinear, find the value of x.



Answer: $x = {}^{49}/_{4}$

To solve the problem we need to know that the triangles are similar. Are the squares also similar? Are all squares similar?

See Problem Solving in Mathematics: Focus for the Future, p. 68, #15; p. 70, #1.

TECHNOLOGY INTEGRATION

Software:
Geometric preSupposer: Points and Lines and Geometric Supposer: Triangles (both from Sunburst) provide extensive exploratory opportunities. "Triangles" is best suited.

ELECTIVE SUGGESTIONS

Activity: Measuring the height of the flagpole (or other tall object).

On a sunny day have a student stand near the pole, and another student measure his or her shadow length. Also measure the length of the shadow of the flagpole, and the height of the first student. Have the students draw appropriate diagrams showing conditions of similiarity, and solve the problem of finding the flagpole height. Compare the results obtained by different groups of students. Compare the calculated height with the known height from school records.

See "How High is a Flagpole?" in Activities for Junior High School and Middle School Mathematics for further discussion.

Rectangular sheets of paper in standard metric form have a special property. If they are folded in half, the two new, smaller rectangles formed are similar in form to the original sheet. Have students investigate the necessary length to width ratio for this to be the case. Have students investigate if this is the case with the notepaper they are using.

RESOURCE CORRELATION

MMW2:	pp. 282–294, 314–317
MM2:	pp. 320–323
MPMM2:	pp. 282–294, 314–317

REVERSE TEXTUAL CORRELATION
GAGE EDUCATIONAL PUBLISHING COMPANY
MATHEMATICS FOR A MODERN WORLD
(Book Two, Third Edition, Revised)

Page	Unit	Learner Expectation Number	TRM Page
2–4	Number Skills	1.1	133–137
6–7	Number Skills	1.1	133–137
10–13	Number Skills	1.1	133–137
14–15	Polynomials and Factoring	1.2	146–149
16–24	Number Skills	1.1	133–137
26–29	Number Skills	1.1	133–137
30–31	Number Skills	1.2	138–142
32–35	Number Skills	2	144–145
36–37	Scientific Notation		
38–40	Number Skills	1	133–137
P2–P5	Polynomials and Factoring	3.1	153–155
P6	Polynomials and Factoring	3.2	156–161
P7	Polynomials and Factoring	4.1	164–167
P8–P9	Polynomials and Factoring	3.2	156–161
P10–P11	Polynomials and Factoring	4.1	164–167
P12–P13	Solving Equations		
44–45	Polynomials and Factoring	3.1	153–155
46–47	Polynomials and Factoring	2.1, 2.2, 2.3	150–152
48–49	Polynomials and Factoring	3.2	156–161
50–52	Solving Equations		
56–59	Number Skills	1.3	143
60–65	Square Root: Pythagorean Theorem, Formulae and Newton's Method		
70–71	Graphing Ordered Pairs		
72–77	Coordinate Geometry and Graphing	1.1, 1.2	168–172
74	Coordinate Geometry and Graphing	2.6	182
78–79	Coordinate Geometry and Graphing	1.3	173–174
82–85	Statistics	1.1, 1.2, 1.3	190–193
86–95	Statistics	2.1, 2.3, 2.4, 2.5, 2.6	194–201
94–95	Statistics	1.3	190–193
100–103	Polynomials and Factoring	3.2, 3.3, 3.4	156–161
104–108	Polynomials and Factoring	4	164–167
112–113	Polynomials and Factoring	3.5	162–163
114–115	Dividing a Polynomial by a Binomial		
116–117	Rational Expressions		
119–126	Review		
128–137	Relations	1	183–187
136–137	Coordinate Geometry and Graphing	2.1	176
138–141	Graphing Non-Linear Relations		
146–149	Coordinate Geometry and Graphing	2.4, 2.5	179–181
148–149	Coordinate Geometry and Graphing	2.2, 2.3	177–178
149	Coordinate Geometry and Graphing	2.6	182
155–166	Solving Pairs of Linear Equations		
167–182	Solving Word Problems		
168–169	Polynomials and Factoring	1.3	146–149
184–198	Number Skills	1.2	138–142
199–218	Personal Finance		
219–234	Earning Money		
235–242	Review		
244–251	Ratio, Proportion, Rates		
252–255	Relations	2	188–189
260–262	Basic Geometric Constructions		
263–265	Geometry	1.1, 1.2	202–203
266–267	Triangles (Review)		
268–271	Geometry	1.1, 1.3	202–203
272–275	Quadrilaterals		
282–294	Geometry	3	206–208
286–294	Geometry	2	204–205
299–312	Circle Geometry		
314–317	Geometry	3.1	206–208
318–332	Trigonometry		
333–342	Matrices		
343–352	Review		

REVERSE TEXTUAL CORRELATION
NELSON CANADA
MATH MATTERS
(Book Two, Alberta Edition)

Page	Unit	Learner Expectation Number	TRM Page
2–5	Diagrams and Patterns in Numbers		
6–22	Number Skills	1.1	133–137
23–27	Number Skills	2	144–145
28–30	Number Skills	1.2	138–142
31–33	Number Skills	1.1	133–137
33–34	Number Skills	1.2	138–142
35	Number Skills	1.1	133–137
41–49	Charts – Reading		
50–53	Number Skills	1.3	143
54–59	Pythagorean Theorem		
60–67	Programming		
64	Number Skills	1.3	143
74–83	Number Skills	1.1	133–137
84–86	Algebra – Vocabulary		
87–88	Polynomials and Factoring	1.2	146–149
89–91	Polynomials and Factoring	3.1	153–155
92–94	Polynomials and Factoring	1.2	146–149
95–100	Polynomials and Factoring	2	150–152
98–100	Number Skills	2	144–145
106–111	Polynomials and Factoring	3.2	156–161
112–113	Polynomials and Factoring	1.2	146–149
114–116	Polynomials and Factoring	3.5	162–163
117	Polynomials and Factoring	1.2	146–149
118–120	Polynomials and Factoring	3.2	156–161
120	Polynomials and Factoring	3.5	162–163
121–123	Polynomials and Factoring	4.1	164–167
124	Number Skills	1.1	133–137
125–128	Polynomials and Factoring	3.3, 3.4	156–161
129–134	Polynomials and Factoring	4	164–167
140–142	Graphing Ordered Pairs		
143–156	Relations	1	183–187
148–150	Coordinate Geometry and Graphing	2.1	176
158–162	Non–Linear Graphing		
163–166	Coordinate Geometry and Graphing	1.3	173–174
167–169	Functions		
175–182	Coordinate Geometry and Graphing	1.1, 1.2	168–172
183–185	Coordinate Geometry and Graphing	2.2, 2.3	177–178
186–189	Coordinate Geometry and Graphing	1.1	168–172
190–195	Coordinate Geometry and Graphing	2.4, 2.5	179–181
196	Coordinate Geometry and Graphing	2.6	182
199–219	Ratio and Proportion		
220–224	Relations	2.1	188–189
225–227	Direct Squared Variation		
228–239	Relations	2.1, 2.2	188–189
245–247	Statistics	1	190–193
248–266	Statistics	2	194–201
267–273	Scatter Plots and Line Fitting		
274–284	Probability		
290–294	Geometry	1.1, 1.2	202–203
295–302	Geometric Constructions: Angles		
303–307	Quadrilaterals		
308–312	Geometric Construction: Perpendiculars		
313–315	Geometry	1.3	202–203
316–317	Geometric Constructions: Congruent Line Segments and Angles		
318–319	Geometry	2	204–205
320–323	Geometry	3	206–208

REVERSE TEXTUAL CORRELATION
ÉDITIONS DU TRÉCARRÉ
MATHÉMATIQUES POUR UN MONDE MODERNE
(livre 2)

Page	Unité	Attentes	TRM Page
2–4	Les nombres	1.1	133-137
6–7	Les nombres	1.1	133-137
10-13	Les nombres	1.1	133-137
14–15	Les polynômes: opération et factorisation	1.2	146-149
16–24	Les nombres	1.1	133-137
26–29	Les nombres	1.1	133-137
30–31	Les nombres	1.2	138-142
32–35	Les nombres	2	144-145
36–37	La notation scientifique		
38–40	Les nombres	1	133-137
44–45	Les polynômes: opération et factorisation	3.1	153-155
46–47	Les polynômes: opération et factorisation	2.1, 2.2, 2.3	150-152
48–49	Les polynômes: opération et factorisation	3.2	156-161
50–52	La résolution d'équations		
56–59	Les nombres	1.3	143
60–65	La racine carrée: le Théorème dePythagore; les formules et la Méthode de Newton		
70–71	La représentation de couples		
72–77	Les coordonnées géométriques	1.1, 1.2	168-172
74	Les coordonnées géométriques	2.6	182
78–79	Les coordonnées géométriques	1.3	173-174
82–85	Notions de statistique	1.1, 1.2, 1.3	190-193
86–95	Notions de statistique	2.1, 2.3, 2.4, 2.5, 2.6	194-201
94–95	Notions de statistique	1.3	190-193
100–103	Les polynômes: opération et factorisation	3.2, 3.3, 3.4	156-161
104–108	Les polynômes: opération et factorisation	4	164-167
112–113	Les polynômes: opération et factorisation	3.5	162-163
114–115	Les division d'un polynôme par un binôme		
116–117	Les expressions et les monbres rationnels		
119–126	La revision		
128–137	Les relations	1	183-187
136–137	Les coordonnées géométriques	2.1	176
138–141	Graphing Non-Linear Relations		
146–149	Les coordonnées géométriques	2.4, 2.5	179-181
148–149	Les coordonnées géométriques	2.2, 2.3	177-178
149	Les coordonnées géométriques	2.6	182
155–166	La résolutions des paires d'équations linéaires		
167–182	La résolution de problèmes écrits		
168–169	Les polynômes: opération et factorisation	1.3	146-149
184–198	Les nombres	1.2	138-142
199–218	Les finances personelles		
219–234	Les salaires et les revenus		
235–242	La revision		
244–251	Les rapports les proportions et les taux		
252–255	Les relations	2	188-189
260–262	Les constructions géometriques de bases		
263–265	La géométrie	1.1, 1.2	202-203
266–267	Les triangles: revision		
268–271	La géométrie	1.1, 1.3	202-203
272–275	Les quadrilatères		
282–294	La géométrie	3	206-208
286–294	La géométrie	2	204-205
299–312	La géométrie circulaire		
314–317	La géométrie	3.1	206-208
318–332	La trigonométrie		
333–342	Les matrices		
343–352	La revision		

MATH 14

Math 14

Program Emphasis

Required Content

The major part of the content of each high school mathematics course consists of topics required of all students who take the course. The required content comprises 80% of the course and contains the concepts, skills and attitudes that all students are expected to acquire. As well, the required portion of all courses includes specific expectations in regard to the problem solving and the use of technology.

Elective Material

Each course will include a compulsory component comprising 20% of the course, made up of elective material which will be consistent with the content and expectations of their required component. The purpose of the elective is to provide for enrichment, remediation, or innovative experimental presentations or activities. It is not intended to provide acceleration or advanced placement. However, horizontal enrichment and extension is appropriate and students should have access to elective material that serves their individual needs and interests.

Suggested Program Emphasis (in %) for Required Portion of Mathematics 14

Unit	Suggested Program Emphasis (in %)
Ratio and Proportion	15
Statistics and Probability	25
Geometry	25
Measurement	15
Algebra and Graphing	20

Ratio and Proportion
Program Emphasis - 15%

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that a ratio is a way of comparing the relative sizes of two quantities.	Students will be expected to compare two quantities using ratios in the form $^a/_b$, a:b and a to b.		pp. 226–227
Students will be expected to demonstrate an understanding that a rate is a ratio between quantities which have different units.	Students will be expected to calculate rates and unit rates.	Students will be expected to solve problems involving the use of rates and unit rates.	pp. 228–229
Students will be expected to demonstrate an understanding that proportions are equations made up of equivalent ratios.	Students will be expected to calculate the unknown value in a proportion.	Students will be expected to solve problems using proportions.	pp. 230–231
Students will be expected to demonstrate an understanding that a percent is a ratio in which a quantity is compared to 100.	Students will be expected to express ratios as percents and decimals and vice-versa. Students will be expected to solve for the unknown element in a percentage calculation.	Students will be expected to solve problems which involve the use of percents.	pp. 232–234

Statistics and Probability
Program Emphasis - 25%

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that data collected from a sample must be organized, presented and analysed in order that valid inferences can be drawn.	Students will be expected to use the terms sample and population.		pp. 235–239
	Students will be expected to determine a suitable sample from a given population.	Students will be expected to access the suitability of a sample chosen from a given population.	
	Students will be expected to organize data using tally sheets, frequency tables and stem-and-leaf plots.	Students will be expected to design and carry out a simple statistical project involving the collection, organization, presentation and analysis of data gathered from an appropriate sample.	
	Students will be expected to interpret and analyse data represented by pictographs, circle graphs, bar graphs, line graphs and histograms.		
	Students will be expected to identify the characteristics of a misleading graphical display.		
	Students will be expected to determine and calculate appropriate measures of central tendency.	Students will be expected to solve problems which involve the use of an appropriate measure of central tendency in the analysis of sets of data.	
	Students will be expected to make inferences and generalizations about a population from which a sample was taken.		
Students will be expected to demonstrate an understanding that a probability is a numerical measure of the likelihood of an event and can take values from 0 through 1.	Students will be expected to list the sample spaces for simple experiments or simulations.		pp. 240–241
	Students will be expected to use counting techniques such as tree diagrams to find the number of possible outcomes of an experiment.		
	Students will be expected to express the probability of an event from a given situation, simple experiment or simulation.	Students will be expected to perform an investigation to find the experimental probability of an event.	

Geometry
Program Emphasis - 25%

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that two- and three-dimensional geometric figures can be classified and have characteristics which uniquely define them.	Students will be expected to classify polygons according to the number of sides.		pp. 242-251
	Students will be expected to classify triangles and quadrilaterals by sides and angles.		
	Students will be expected to determine the sum of the interior angles of a polygon.	Students will be expected to perform an investigation to determine the sum of the interior angles of any polygon.	
	Students will be expected to classify the following common geometric solids: rectangular prisms, pyramids, cylinders, cones and spheres.	Students will be expected to perform an investigation to determine the relationship amongst the edges, faces and vertices of polyhedra.	
		Students will be expected to draw nets which define simple polyhedra.	
		Students will be expected to use nets to construct simple polyhedra.	
	Students will be expected to identify and define the following terms associated with a circle such as the following: radius, diameter, circumference, semicircle.		
Students will be expected to demonstrate an understanding that geometric figures can be defined uniquely and that these unique characteristics can be reproduced using construction techniques and geometric instruments.	Students will be expected to construct a line segment of given length or congruent to a given segment.		pp. 252-253
	Students will be expected to draw an angle of a given size or construct an angle congruent to a given angle.		
	Students will be expected to construct the perpendicular bisectors of segments and angles.		
	Students will be expected to draw a regular polygons.		
	Students will be expected to construct a circle of given radius.		
	Students will be expected to use a combination of construction techniques.	Students will be expected to construct angles of 90°, 45°, 60° and 30°.	
		Students will be expected to use construction techniques in the construction or reproduction of geometric figures or designs.	

222

Geometry
Program Emphasis - 25% (continued)

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that similar geometric figures have the same shape but may differ in size, that corresponding angles are equal and that the lengths of corresponding sides are proportional.	Students will be expected to use the relationship between the lengths of the sides of similar triangles to calculate the lengths of unknown sides. Students will be expected to recognize congruency as similarity in which the size of the figures is the same.	Students will be expected to solve problems using the properties of similar polygons.	pp. 254-255
Students will be expected to demonstrate an understanding of the Pythagorean theorem.	Students will be expected to use the Pythagorean theorem to find the length of a side in a right triangle.	Students will be expected to solve problems involving right triangles by using the Pythagorean theorem.	pp. 256-257
Students will be expected to demonstrate an understanding that angles formed by intersecting lines relate to each other.	Students will be expected to identify parallel, intersecting and perpendicular lines. Students will be expected to identify and recognize the characteristics of the relationship between supplementary, complementary and vertical angles.	Students will be expected to perform an investigation to determine the properties of the angles between intersecting lines. Students will be expected to solve problems involving the relationships amongst intersecting and parallel lines and the angles they form to solve problems involving the measure of unknown angles.	pp. 258-259

Measurement
Program Emphasis - 15%

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that standard units of measure for length, mass and volume provide consistency and accuracy and that the most widely used and accepted measurement system is the SI (metric) system.	Students will be expected to recognize the basic SI units of measurement for length [metre (m)], mass [kilogram (kg)], [tonne (t)] and volume of liquids or gases [litre (L)]. Students will be expected to use the appropriate SI measures for length, mass and liquid/gas volume. (limit: kg, g, L, ,mL, m, cm, mm, km). Students will be expected to express equivalences between units of length, of mass and of volume. Students will be expected to recognize non-metric units of measurement still in common use.		pp. 260-262
Students will be expected to demonstrate an understanding that perimeter is the measure of the distance around a geometric figure.	Students will be expected to calculate the perimeter of polygons. Students will be expected to calculate the circumference of a circle by using a formula.	Students will be expected to calculate the perimeter of irregular shapes and objects. Students will be expected to solve problems which involve finding the circumference of circles and circular regions.	pp. 263-265
Students will be expected to demonstrate an understanding that area is the number of unit squares which are enclosed within the boundaries of geometric figures.	Students will be expected to calculate the area of triangles and quadrilaterals by using formulas (limit: triangles, quadrilaterals). Students will be expected to find the area of a circle by using a formula.	Students will be expected to solve problems which involve finding the area of triangles and rectangles. Students will be expected to solve problems which involve finding the area of circles.	pp. 266-271
Students will be expected to demonstrate an understanding that volume is the number of unit cubes which are within a geometric solid.	Students will be expected to calculate the volume of rectangular prisms and cylinders by using formulas.	Students will be expected to solve problems which involve calculating the volume of rectangular prisms and cylinders.	pp. 272-273
Students will be expected to demonstrate an understanding that time can be measured in seconds, minutes, hours, days, weeks, months and years.	Students will be expected to change units of time from one form to another. Students will be expected to tell time using a 24-hour clock. Students will be expected to write times using proper 24-hour notation. Students will be expected to perform calculations using units of time.		p. 274
Students will be expected to demonstrate an understanding that temperature can be measured in degrees.	Students will be expected to measure temperatures using the Celsius scale. Students will be expected to perform calculations involving the use of temperature measured in Celsius.		p. 275

Algebra and Graphing
Program Emphasis - 20%

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that algebraic expressions can be used to represent situations where one or more values can vary.	Students will be expected to write first degree algebraic expressions to illustrate simple mathematical situations with a maximum of 2 variables.		pp. 276-282
	Students will be expected to evaluate algebraic expressions for replacement values of the variables.	Students will be expected to solve problems which involve the evaluation of expressions for values of the variables.	
	Students will be expected to solve first degree equations in one variable through the use of guess-and-test techniques, concrete manipulatives and formal methods (limit: $x + a = b$, $ax = b$, $ax + b = c$, $x/a = b$).	Students will be expected to solve problems that can be represented by first degree equations in one variable.	
		Students will be expected to use calculators to aid in the solution of equations.	
Students will be expected to demonstrate an understanding that two variables can be related and that those relationships can be expressed by sets of ordered pairs, equations and graphs.	Students will be expected to describe verbally and in writing the relationship between two variables.	Students will be expected to determine the algebraic relationship between two variables by informal means such as guess-and-test.	pp. 283-286
	Students will be expected to plot points on a coordinate plane.		
	Students will be expected to plot points and sketch the graphs which describe the relationship between two variables.		
	Students will be expected to construct tables of values which satisfy a given linear equation in two variables.		
	Students will be expected to read and interpret the graphs which describe the relationship between two variables.	Students will be expected to analyse graphs which describe the relationship between physical variables by the use of interpolation and extrapolation techniques.	

Attitudes

Within the study of the content of this course, it is expected that *students will be encouraged to*

- be confident in their mathematical knowledge and in their ability to acquire new knowledge;
- demonstrate persistence, resolve, flexibility and ingenuity in finding the solution to problems;
- develop an attitude of intellectual curiosity and openness to new ideas, insights and change in the pursuit of mathematical knowledge;
- exhibit an attitude of curiosity and spontaneity and appreciate creativity and innovation in representing situations mathematically;
- value and be aware of the importance of communication skills in mathematics;
- appreciate and develop a positive attitude toward the usefulness of computational competence, mathematical processes and problem solving skills which are used in the decision making and modeling processes in our society;
- value the contributions of mathematics to our culture and civilization.

Problem Solving

Problem solving is to be integrated into all areas of the high school mathematics program because a major purpose of studying mathematics is to learn to solve problems. A student who has successfully completed this course will understand that thinking and problem-solving skills are required in order to explain the world, and that mathematics can provide some of the tools for problem solution.

Students will understand that problems can be solved in many ways, and that problems can be routine, requiring only the application of a known procedure or algoarithm, or non-routine, requiring the development of a process or the conducting of an investigation to solve them.

Students will be expected to use a variety of procedures to help them understand mathematical problems. In particular, they will be expected to

- read the problem thoroughly
- identify and clarify key components
- restate the problem using familiar terms
- evaluate the given information as to whether it is insufficient or extraneous
- interpret pictures, charts and graphs
- determine any hidden assumptions
- ask relevant questions
- identify given, needed and wanted information
- diagram or model the problem situation
- use suitable notation
- determine valid inferences.

Students will be expected to develop a variety of strategies for use in the solution of mathematical problems. In particular, they will be expected to

- conduct an investigation
- use estimation and approximation
- develop equations or use formulae
- use flow charts
- make lists and charts
- look for patterns

- work backward
- break the problem into smaller parts
- look for a simpler or related problem
- make diagrams or models
- use manipulatives
- choose and sequence a series of mathematical operations
- sketch the graph of a problem situation
- establish procedures to gather and organize data.

Students will be expected to develop a variety of skills which can be used to carry out the plan for the solution of a problem. In particular, they will be expected to

- apply selected strategies.
- present ideas clearly
- document the solution process
- utilize appropriate group behaviours
- use calculators and computers
- evaluate problem-solving strategies for effectiveness
- alter or abandon non-productive strategies
- search for additional information
- ask questions
- be open to inspirations, intuitions and "bright ideas".

Students will be expected to employ a variety of skills to help them look back over the solution of a problem. In particular, they will be expected to

- determine the reasonableness of an answer
- explain the solution in oral or written form
- consider the possibility of additional solutions
- search for other strategies and processes of solution
- create and solve similar problems
- note the characteristics that will be identifiable in similar problems
- make a generalization.

Numeration

Numeration concepts and calculation skills are basic in order to successfully address any mathematical situation. Since it is the intent to integrate the development of these concepts and skills into the other expectations of this course, it is not expected that numeration will be taught as a topic on its own.

1. *Students will be expected to* demonstrate an understanding that computation requires skills in addition, subtraction, multiplication and division with whole number, integers and rational numbers and can be done metnally, with paper and pencil, and with electronic calculators, as appropriate to a particular situation.

Mental Computation

Students will be expected to

1.1 add and subtract whole numbers, integers and rational numbers (limit: add three single–digit integers; subtract two single–digit integers; add and subtract two multiples of 5 and 10 up to 100; add and subtract fractions with common denominators; add and subtract tenths and hundredths written in decimal form)
1.2 multiply whole numbers, integers and rational numbers (limit: single digit numbers, multiples of 5 and 10 multiplied by a single–digit multiplier; double any number up to 50; multiply whole numbers, integers and decimals by powers of 10)
1.3 square integers up to 10 and calculate square roots of integral perfect squares up to 100
1.4 divide whole numbers, integers and rational numbers (limit: divide integers to 100 which are perfect multiples of single–digit divisors; halve even integers to 50; divide whole numbers, integers and decimals by powers of 10)
1.5 determine whether a whole number or integer is divisible by 2, 3, 4, 5 or 10
1.6 transform fractions to percents (limit: denominators of 2, 3, 4, 5, 10, 100) and whole number percents
1.7 calculate the percent of given whole number quantities (limit: 1%, 10%, 25%, 50%, 100%, 200%)
1.8 recognize "families" of fractions (tenths, twelfths, sixteenths) and transform fractions within families to common denominators.

Paper and Pencil Computation

Students will be expected to

1.9 add and subtract whole numbers, integers and rational numbers (limit: add three 2–digit integers; subtract 3–digit integers; add and subtract two fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 16; add three 2–digit decimals; subtract 2–digit decimals)
1.10 multiply whole numbers, integers and rational numbers (limit: 2–digit integers, 2–digit decimals, fractions with single–digit denominators and denominators which are multiples of 10)
1.11 divide whole numbers, integers and rational numbers (limit: 3–digit integers which are perfect multiples of single–digit divisors; halve even numbers up to 500)
1.12 calculate the percent of given quantities (limit: 5%, 15%, 25% and multiples of 10%)
1.13 calculate the value of a power (limit: whole number base < 10 and exponent of 2 or 3)
1.14 transform mixed numbers to decimals and percents (limit: denominators of 2, 3, 4, 5, 6, 8, 10).

Calculation with Calculators

Students will be expected to

1.15 calculate the percent of given quantities
1.16 calculate the value of a power (limit: integral base and whole number exponent)
1.17 calculate the square root of any positive rational number
1.18 transform any rational number to decimal and percent form.

2. *Students will be expected to* demonstrate an understanding that rounding, estimation and approximation are fundamental to facility in calculation.

Students will be expected to

2.1 round given values to a specified level of precision
2.2 round solutions to an appropriate level of precision
2.3 use estimation techniques to predict solutions to calculations.

3. *Students will be expected to* demonstrate an understanding that checking an answer for reasonableness and accuracy is a fundamental step in the calculation process and that the level of accuracy required is dependent upon the particular situation.

Students will be expected to

3.1 use estimation techniques to check a calculation for reasonableness
3.2 determine if the accuracy of a calculated answer is adequate for a given situation.

Estimation and Mental Calculation:

The 1986 NCTM Yearbook, Estimation and Mental Computation, focuses on these topics. Many of the articles are appropriate for Grade 10 mathematics. Those cited below are particularly appropriate as teacher resources.

"Reasons for Estimating" and "Mental Calculation: Anachronism or Basic Skill?" provide excellent rationales for actively encouraging the development of estimation and mental computation skills:

The following two references discuss the need for (i) the development of an awareness for, and an appreciation of, estimation; (ii) the development of number sense; (iii) the development of number concepts; and (iv) the development of estimation strategies. They also provide teaching suggestions.

"Teaching Computational Estimation: Establishing an Estimation Mind-set"
"Teaching Computational Estimation: Concepts and Strategies"

"Evaluating Computational Estimation" takes up the question of how to evaluate students' ability to estimate. Testing guidelines are presented.

The NCTM book Activities for Implementing Curricular Themes from the 'Agenda for Action' contains a variety of estimation-related activities. Some of these include percent estimations.

"Calculators and Estimation" contains activities designed to help develop skills in estimating factors and divisors.

See also "Estimating with 'Nice' Fractions," Mathematics Teacher, November 1986 for activities designed to help students become better estimators when working with fractions.

Mathematics 14
Integration of Numeration Concepts and Skills into Ratio and Proportion

The numeration concepts and skills identified in the shaded area represent those skills that students should use, reinforce or learn in this unit of study. A detailed description of each skill can be found on pages 223–225.

	Ratio and Proportion	Statistics and Probability	Geometry	Measurement	Algebra
Mental	**1.1, 1.2, 1.4, 1.5, 1.6, 1.7, 1.8**	1.1, 1.4, 1.5,	1.1, 1.2, 1.3, 1.4, 1.5	1.1, 1.2, 1.3, 1.4, 1.5	1.1, 1.2, 1.3, 1.4, 1.5, 1.8
Estimation	**2.2, 2.3, 3.1, 3.2**	2.1, 2.2, 2.3 3.1, 3.2	2.1, 2.2, 2.3, 3.1, 3.2	2.1, 2.2, 2.3, 3.1, 3.2	2.1, 2.2, 2.3, 3.1, 3.2
Technology	**1.15, 1.18**	1.18	1.16, 1.17	1.16, 1.17, 1.18	1.16, 1.17, 1.18
Paper and Pencil	**1.9, 1.10, 1.11, 1.12, 1.14**	1.9, 1.10, 1.11	1.9, 1.10, 1.11, 1.13, 1.14	1.9, 1.10, 1.11, 1.13, 1.14	1.9, 1.10, 1.11, 1.13, 1.14

CONCEPT 1

Students will be expected to demonstrate an understanding that a ratio is a way of comparing the relative sizes of two quantities.

Students will be expected to:

1.1 compare two quantities using ratios in the form $^{a}/_{b}$, a:b, and a to b.

COMMENTS

To build an understanding of ratio, students should experience the meaning of ratio as a comparison.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students count the number of people in their class with blue eyes and compare the number to the total number of eyes in the room; have students count the number of red "Smarties" and compare it to the total number of "Smarties" in the box. Each of these comparisons should be written in all forms to demonstrate the "equality" between forms.

Problem: About how many times will your heart beat this month?
Answer: About 3 200 000 times.

If the students do not know what the average heart rate is, have them research the problem.

(Note: This type of estimation problem is sometimes referred to as a Fermi-type problem.)

Activity: A full class activity designed to develop mental calculation skills:
Create a set of index cards sufficient to provide at least one card to every student. On the front of each card write a number. On the back, write computational instructions that will connect the number on the front to the number on a second card. For example, suppose the number on the front is 15 and the instruction on the back is "Who has $^{2}/_{3}$ of my number?" The next card will have a 10 on the front. Continue in this way until the answer to the instructions on the last card is provided by the number on the first, thus closing the loop. Distribute all the cards. Choose a student to begin by reading his or her number and the instruction. The student holding the card with the correct answer then reads the new instruction. The activity is complete when the first student closes the loop.

Adapted with permission from "I have a Number...," Mathematics Teacher, October 1988, copyright by the National Council of Teachers of Mathematics.

TECHNOLOGY INTEGRATION

Software:
SRA Computer Drill and Instruction, Mathematics: Level D (Rev. ed.) "Ratio and Percent" section. Selected lessons may be used for drill.

RESOURCE CORRELATION AM: p. 47

CONCEPT 2

Students will be expected to demonstrate an understanding that a rate is a ratio between quantities which have different units.

Students will be expected to:

2.1 calculate rates and unit rates
    2.1.1 solve problems involving the use of rates and unit rates.

COMMENTS

In the initial discussion of rates, students should build on the ratio concept.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have the students take the number of blue eyes and rather than compare it to the total number of eyes, compare it to the number of people. This would constitute a rate. The number of red "Smarties" compared to the number of boxes, once again compares two quantities that have different units, hence is a rate. This should lead into a discussion of the differences and similarities of ratios and rates.

Activity: Extend the introduction of rates into the discussion of how rates are used, especially in comparisons; e.g., which car would be a better buy: Car A where its consumption is 24 km/L or Car B with a consumption of 35 km/L? Note that each of these is an unit rate and hence can be compared.

Activity: Have students examine the newspaper to evaluate the "better buy" for similar products. Students should share their results with other class members to see if they agree with one another.

Activity:

Timing Walks to Get Distance

A person walking at "average" speed can walk a kilometre in about 10 minutes. Use this information to estimate the following distances:

1. The length of the school hall.
2. The length of a city block.
3. The distance around your yard, or the lot on which your home is located.
4. The length of the basketball court.
5. The distance from your home to your school (if you walk).
6. The distance from your house to your friend's house.

Check your estimates if possible.

Reprinted with permission from "Estimation in Measurement," Estimation and Mental Computation, copyright 1986 by the National Council of Teachers of Mathematics.

Problem: A mini-bike can go 38 km on one litre of gasoline. Its tank holds 5.8 L. How many kilometres can the bike go on 0.6 of a tank of gas. Answer: 132.24 km. Have students estimate this answer before calculating.

Problem: At 48 kilometres an hour, a car travels how far in a second? Answer: 13 $^1/_3$ m/s.

From Problem Solving Challenge for Mathematics, See also page 37, #9.13.

Problem: Application of rate in everyday life:
If you lose 6 kg in three months, what is your rate of mass gain? If three months ago you weighed 6 kg more, what has been your rate of mass gain? (Answer to both: -2 kg/mo). These two questions, asked differently, could relate to the same situation.

Reprinted with permission from "Applications in Elementary Algebra and Geometry," Applications in School Mathematics, copyright 1979 by The National Council of Teachers of Mathematics.

Problem: The Manx Shearwater is a seabird. Consider the following real life case: A Manx Shearwater was taken from its nesting burrow on an island off Britain and transported to North America. It was released in Boston, Massachusetts, and 13 days later, returned to its island burrow. Assuming that the distance flown across the Atlantic Ocean was approximately 4800 km, what was the Manx Shearwater's average speed? Answer: Approximately 370 km/d or 15.4 km/h.

As a class project, investigate other such homing or migratory patterns of birds and other animals, and their average rates of speed.

Adapted with permission from "Applications through Direct Quote Word Problems," Applications in School of Mathematics, copyright 1979 by The National Council of Teachers of Mathematics.

TECHNOLOGY INTEGRATION

Students should use the calculator in the calculation of unit rates.

Software:
SRA Complete Drill and Instruction, Mathematics: Level D (Rev. ed.). Selected lessons may be used for drill.
Video:
Math Wise: "Comparing 01 - Numerical Comparisons"
Math Wise: "Comparing 03 - Finding a Common Unit".
Plus Teacher's Guide discussion and student worksheets.

These videotapes provide everyday examples of the use and importance of ratio and rate.

RESOURCE CORRELATION AM: pp. 48–49

CONCEPT 3

Students will be expected to demonstrate an understanding that proportions are equations made up of equivalent ratios.

Students will be expected to:

3.1 calculate the unknown value in a proportion
    3.1.1 solve problems using proportions.

COMMENTS

Students should be involved in calculating unknown values in a proportion as well as solving problems involving proportions. It is further important to note that proportion should be defined within the students' realm of experience. The "cross-product" method should not be introduced until students have an understanding of "proportion."

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Using the same examples as used with rate and ratio, students could see that they were actually calculating a proportion when they were calculating unit rates. The extension is that one may not want to be restricted to unit rates; e.g., if a recipe calls for 600 ml of flour and one wanted to double the recipe than the proportion would look something like:

$$\frac{600\text{ ml}}{1\text{ recipe}} = \frac{x\text{ ml}}{2\text{ recipe}}$$

Canada's Food Guide Handbook is a good source of material for ratio and proportion problems and activities.

Problem: Dave computed the answer to a ratio and proportion problem to be 22.5. However, in the last step of the computation he multiplied by 0.3 instead of dividing by 0.3. What is the correct answer?

Answer: 250.
(What form of problem-solving strategy is employed here?)

For a discussion of proportional reasoning see "Proportionality and the Development of Prealgebra Understandings," The Ideas of Algebra, K-12.

TECHNOLOGY INTEGRATION

Software:
Ratios and Proportions (Media Materials)
"Introductory," "Skill" and "Mastery" student levels of activity. Teacher management based. Teacher documentation and student worksheets provided.

SRA Computer Drill and Instruction, Mathematics: Level D (Rev. ed.) "Ratio and Percent" section. Selected lessons may be used for drill.

Video:
Math Wise: "Comparing 04 – Proportions".
Plus Teacher's Guide discussion and student worksheet. This videotape demonstrates the use and importance of proportionality in an everyday life context.

ELECTIVE SUGGESTIONS

Problem: (with no numbers): If *p* pencils cost *c* cents, how many pencils can be purchased for *d* dollars?

Answer: $100 \times {}^{d}/_{c} \times p$

RESOURCE CORRELATION AM: pp. 50–51

CONCEPT 4

Students will be expected to demonstrate an understanding that a percent is a ratio in which a quantity is compared to 100.

Students will be expected to:

4.1 express ratios as percents and decimals and vice-versa

4.2 solve for the unknown element in a percentage calculation

  4.2.1 solve problems which involve the use of percents.

COMMENTS

It is often assumed that students know the meaning of percent as they have seen it used throughout their school experience. This may not be the case and care must be taken to develop a concrete definition of percent. Expressing ratios as percents and decimals and vice-versa and solving for the unknown element in a percentage calculation should follow an extensive "search" on meaning.

Limit: fractional percents that are derived from the following fractions: thirds, eighths.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: A manipulative transparency model can be used to solve a limited number of percent problems. On a sheet of clear acetate divide a 5 cm by 5 cm square into 100 smaller congruent squares with an indelible marker. This is the "percent" sheet. On a separate worksheet of activities, construct 5 cm by 5 cm squares. Subdivide each of these into the number of parts equal to the "whole" of the respective percent problems. For example, in the problem 12 = ____% of 20, the large square should be subdivided into 20 congruent parts (rectangles); in the problem 80% of 25 = ____, create 25 congruent rectangles.

To solve 12 = ____% of 20, the student shades in 12 of the 20 parts. The percent transparency is then superimposed, and the number of one percent squares covering the shaded portion is counted, showing the solution of 60 percent.

To solve 80% of 25 = ____, the student shades in 80 of the 100 squares on the percent sheet using an erasable marker. The sheet is then superimposed on the 5 cm by 5 cm square that has been subdivided into 25 congruent parts. The number of rectangles covered by the shading is counted, producing an answer of 20.

This model cannot be used to solve problems of the type 30% of____ = 15. A second manipulative model, can be used to solve all three "types" of percent problems.

Adapted with permission from "Manipulating Percentages," in Mathematics Teacher, January 1986, copyright by the National Council of Teachers of Mathematics.

Activity: The second model requires a metre stick and a piece of 2.5 cm wide braided sewing elastic sufficiently long that when fully stretched it will be at least 1 m. Stretch the elastic to its maximum length, and mark it off in units of 1 cm with an indelible black marker. Highlight multiples of 10 percent. Finally, trim off the excess elastic.

The 100% mark on the elastic represents the "whole" in the general form of the problem "% of whole = part." The percentage value on the elastic will be aligned with the "part." The elastic with its one hundred units is used to calibrate a particular "whole" (as indicated on the metre stick) into a like number of one hundred equal parts. Thus, the model corresponds to the concepts involved in percentage problems.

a) Problems of the form 60% of 50 = _______
Align 0 cm on the metre stick with 0% on the elastic. Stretch the tape so that 100% is aligned with 50 on the stick. Read the metre stick marking (30) opposite the 60% mark on the elastic.

b) Problems of the form ___% of 80 = 20?
Align 0 cm with 0% and 80 with 100%. Read on the elastic the percent value (25%) corresponding to 20 on the metre stick.

c) Problems of the form 30% of ___ = 24
Align 0 cm with 0% and stretch the elastic until 30% coincides with 25 on the metre stick. Read on the metre stick the value of the "whole" that corresponds to 100%, that is, 80.

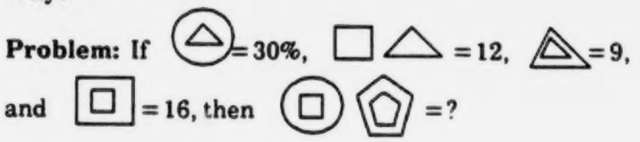
Notice that the elastic and metre stick model corresponds very closely to the more symbolic (and abstract) proportions method of solving percentage problems. As students become familiar with the use of this concrete approach, have them also describe the proportions being represented.

Both models clearly have their limits. It is best to work within the range for which the models were intended. Answers will also often be inexact. This should be acknowledged; encourage students to estimate.

Problem: A class of 20 students averages 66% on an examination; another class of 30 students averaged 56%. Find the average for all students.
Answer: 60%. From Problem Solving Challenge for Mathematics. See also page 50, #15.

Activity: Have students bring in sales flyers and newspaper advertisements, and check the stated discount and sale prices for accuracy. Also bring in other examples from newspapers and magazines that illustrate the use of percent. Discuss the items in class.

Problem: In an economy move at a factory, all the workers had their salaries cut by 10%. They were all very unhappy about this and threatened to go on strike. At this point, the factory president stepped in and agreed to a 10% raise on their new salary. "Everyone should be happy again," he said with a smile. "Things are back the way they were to start with." Were they? Why?



Answer: 10.

Estimation and Mental Calculation:

"Estimation and Mental Arithmetic with Percent," Estimation and Mental Computation. provides a number of teaching strategies.

TECHNOLOGY INTEGRATION

"Developing Estimation Strategies" and "Percent and the Hand Calculator" in Activities for Implementing Curricular Themes from the 'Agenda for Action' provide student activity sheets.

Software:
Algebra Drill and Practice I: (Conduit) – "Percentage" option. Provides practice in percent calculations and percent increase/decrease problems.

Percents (Media Materials)
"Introductory," "Skill," and "Mastery" student levels of activity. Teacher-management based. Teacher documentation and student worksheets provided.

SRA Computer Drill and Instruction, Mathematics: Level D (Rev. ed.) "Ratio and Percent" section. Selected lessons may be used for drill.

Video:
MathWays: "The Percent." Focus is on understanding the concept of percent, and determining what a given percent of a number or quantity represents. Includes some conversion of fraction and decimal notation to percent.

Math Wise: "Comparing 02 – Percent."
Plus Teacher's Guide discussion and student worksheet. This videotape provides examples of the everyday use of percent.

RESOURCE CORRELATION AM: pp. 56–67

Mathematics 14
Integration of Numeration Concepts and Skills into Statistics and Probability

The numeration concepts and skills identified in the shaded area represent those skills that students should use, reinforce or learn within this unit of study. A detailed description of each skill can be found on pages 223–225.

	Ratio and Proportion	Statistics and Probability	Geometry	Measurement	Algebra
Mental	1.1, 1.2, 1.4, 1.5, 1.6, 1.7, 1.8	1.1, 1.4, 1.5,	1.1, 1.2, 1.3, 1.4, 1.5	1.1, 1.2, 1.3, 1.4, 1.5	1.1, 1.2, 1.3, 1.4, 1.5, 1.8
Estimation	2.2, 2.3, 3.1, 3.2	2.1, 2.2, 2.3 3.1, 3.2	2.1, 2.2, 2.3, 3.1, 3.2	2.1, 2.2, 2.3, 3.1, 3.2	2.1, 2.2, 2.3, 3.1, 3.2
Technology	1.15, 1.18	1.18	1.16, 1.17	1.16, 1.17, 1.18	1.16, 1.17, 1.18
Paper and Pencil	1.9, 1.10, 1.11, 1.12, 1.14	1.9, 1.10, 1.11	1.9, 1.10, 1.11, 1.13, 1.14	1.9, 1.10, 1.11, 1.13, 1.14	1.9, 1.10, 1.11, 1.13, 1.14

CONCEPT 1

Students will be expected to demonstrate an understanding that data collected from a sample must be organized, presented and analyzed in order that valid inferences can be drawn.

Students will be expected to:

1.1 use the terms sample and population
1.2 determine a suitable sample from a given population
    1.2.1 assess the suitability of a sample chosen from a given population
1.3 organize data using tally sheets, frequency tables and stem-and-leaf plots
    1.3.1 design and carry out a simple statistical project involving the collection, organization, presentation and analysis of data gathered from an appropriate sample
1.4 interpret and analyze data represented by pictographs, circle graphs, bar graphs, line graphs, and histograms
1.5 identify the characteristics of a misleading graphical display
1.6 determine and calculate appropriate measures of central tendency
    1.6.1 solve problems which involve the use of an appropriate measure of central tendency in the analysis of sets of data
1.7 make inferences and generalizations about a population from which a sample was taken.

COMMENTS

It is important that students experience the five "processes" involved in solving a statistical problem – collection, organization and presentation, analysis, inference, and evaluation for confidence.

Collection – students are expected to use the terms sample and population and determine an appropriate sample from a given population, in a given situation. Further, students should be able to conduct a survey to collect data.

Organization and Presentation – of data should take place in two forms – organizing the raw data using tally sheets, frequency tables and stem–and–leaf plots, and presenting the data in a pictorial form – using one of a pictograph, circle graph, bar graph, line graph or histogram.

Analysis – students are expected to read and interpret the graphs used in the presentation of data, and to identify characteristics of a graphical display that is misleading. Further, students are expected to be able to determine and calculate appropriate measures of central tendency.

Inference – is the process whereby students make generalizations to the population from which the sample was taken, based on the data analysis of the sample.

Evaluation of Confidence – This process involves students questioning their generalizations and, in discussion with their peers, arrives at the conclusion that their study was either valid or invalid – if invalid, identify some reasons; e.g., bias in sampling, insufficient data.

PROCESS/PROBLEM-SOLVING CONTEXT

Expose students to what statistics are and how they see and use them regularly; e.g. how the decision to drop a T.V. show is made; in T.V., commercials where the company advertises that 77% of the population prefer a type of antihistamine; how do they decide whether or not to record a second record if the band "Apple Town's" first record went platinum. Have students examine the newspaper, T.V. and radio to identify where statistics are used.

Students should be given a problem and work through the five processes; e.g., Do you think it's true that three out of four people prefer "ChewChew Gum," as advertised on T.V.?

Given this problem, the collection stage would discuss what group of people the "ChewChew Gum Company" would have surveyed to get this number. If students were to examine their school for this statistic, what would be an appropriate sample and population. For instance, if the population is the school – are one class, two classes, people in your neighbourhood, all the football players in the school, or the "Bubble Gum Champions Club" appropriate groups to call your "sample"?

Organization and Presentation – Once the sample is decided upon, a survey of the sample should take place and have students record the information using tally sheets, frequency tables, and stem and leaf plots (Note: data from all problems may not lend themselves to a stem and leaf plot).

Stem and Leaf Plot:

If the following numbers are included in your data:

77	87	92	66	85	72
68	95	86	94	92	66

The "tens" digits become the stem and the "ones" become the leaves:

$$\begin{array}{c|cccc} 6 & 6 & 6 & 8 & \\ 7 & 2 & 7 & & \\ 8 & 5 & 6 & 7 & \\ 9 & 2 & 2 & 4 & 5 \end{array}$$

For more activities to help develop students' skills in constructing and interpreting stem-and-leaf plots, see "Stem-and-Leaf Plots," Mathematics Teacher, October 1985 and Exploring Data (Recommended Resource, Mathematics 10 and 13.)

Analysis – Have students construct the types of graphs, given their data, and examine the differences and similarities between the graphs. Questions such as: which graph best describes their problem? and which graphs misrepresent the data? assist in the analysis stage. At this point a discussion of misleading graphs is appropriate. Examination of measures of central tendencies is also appropriate at this stage, however not all data lend themselves to this.

Inference – Students have the results of their survey. Do three out of four people prefer "ChewChew Gum" according to their survey? Based on their survey, what proportion of people prefer "ChewChew Gum"? Do you think the T.V. commercial is accurate?

Confidence – (How confident am I in my results?) Have students discuss and ask questions regarding the bias (if any) in the sample, how could their study be more representative of the total population in their town or city, in the province or in the country?

The following is a list of some discrete human characteristics that may be investigated.

i Genetic Characteristics

1. Eye colour (specify)
2. Hair colour (specify)
3. Sex
4. Tongue type (curl)
5. Hair
6. Thumb type ("Hitchhiker's thumb")
7. Ear type (lobe)
8. Dimple (chin/cheek)
9. Freckles (none, few, lots)
10. Handedness
11. Eyedness (left or right dominance)
12. Finger relationship (relative lengths of ring and little fingers)
13. Handclasp (folded hands, interlocked fingers; which thumb is on top of the other?)

ii Acquired Characteristics

1. Preferences
2. Ownership (e.g., of a guitar)
3. Number of certain possessions
4. Hours of sleep
5. Name (the beginning letter, or the number of letter/s
6. Clothing sizes (e.g., shoes)
7. Birthday (use a perpetual calendar and determine day of the week of birth; hours of the day)
8. Age (e.g., year and month – 15.6 years old)

Reprinted with permission from "Applications in School Mathematics: Human Variability" in Applications in School Mathematics copyright 1979 by The National Council of Teachers of Mathematics.

See "Projects" in Teaching Statistics and Probability (1981 NCTM Yearbook) for descriptions of class projects.

TECHNOLOGY INTEGRATION

Students should use the calculator in any calculations.

Video:
The following Math Wise videos provide everyday life examples of using the various mathematics listed.
Math Wise: "Locating/Interpreting 01 – Organizing Information"
Math Wise: "Locating/Interpreting 02 – Graphs"
Math Wise: "Predicting 01 – Choosing a Sample"
Math Wise: "Predicting 03 – Averages".
Plus Teacher's Guide discussion and student activities.

RESOURCE CORRELATION AM: pp. 25–27, 30–33, 36–41, 73–81, 84–89

CONCEPT 2

Students will be expected to demonstrate an understanding that a probability is a numerical measure of the likelihood of an event and can take values from 0 through 1.

Students will be expected to:

- 2.1 list the sample spaces for simple experiments or simulations
- 2.2 use counting techniques such as tree diagrams to find the number of possible outcomes of an experiment
- 2.3 express the probability of an event from a given situation, simple experiment or simulation
  - 2.3.1 perform an investigation to find the experimental probability of an event.

COMMENTS

Students should be actively involved in experiments and simulations when listing sample spaces, using counting techniques and expressing the probability of an event occurring and interpreting the probability of an event as odds (and vice-versa). Have students spinning spinners (these are easy to make and students can make their own), tossing dice, tossing coins, drawing cards from decks to illustrate "probability."

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Thumbtacks: Have students speculate on the probability of a thumbtack landing point-up. Give each student a thumbtack and have them flip it 10 times, keeping track of "point-up" and "side" occurrences. Record the relative frequency of the "point-up" event for each student, and construct a cumulative relative frequency graph. Compare and discuss the speculative and experimental probabilities.

See "Out of Thin Air," in Activities for Implementing Curricular Themes from the 'Agenda for Action' for more probability activities using thumbtacks.

Activity: Cylinders: Have students each bring in three sturdy cylinders of various sizes (paper towel tubes, clean metal cans with the labels and ends removed and so on.) Mark the ends "heads" and "tails." Have students determine the height to diameter ratio, and use this value to classify the cylinders.

Speculate on the probability of each cylinder landing on its "head," "tail," or side when flipped. Flip each of the cylinders 20 times, recording the frequencies of the three landing orientation possiblities.

Were there any unfair heads/tails cylinders? Is there a pattern of landing emerging related to the ratio classification? What is the height to diameter ratio of a coin? What is the likelihood of it landing on its side?

Problem: A coin is flipped 10 times, showing nine heads and one tail. On the next flip, what should you do?
a) Choose heads b) Choose tails c) It doesn't matter

Answer: a) choose heads. The coin may be biased in favour of heads. If not, then the probability of showing heads is still 50%.

See also probability-related activities in Student Math Notes.

TECHNOLOGY INTEGRATION

Software:
Quantitative Literacy "Exploring Probability" option. A utility program for simulating coin and dice probabilities, flipping fair and biased coins, etc.

Video:
Math Wise: "Predicting 01 – Probability."
Plus Teacher's Guide discussion and student worksheet. This videotape provides some examples of the use of probability in everyday life contexts.

RESOURCE CORRELATION AM: pp. 95–101, 117–123

Mathematics 14
Integration of Numeration Concepts and Skills into Geometry

The numeration concepts and skills identified in the shaded area represent those skills that students should use, reinforce or learn within this unit of study. A detailed description of each skill can be found on pages 223–225.

	Ratio and Proportion	Statistics and Probability	Geometry	Measurement	Algebra
Mental	1.1, 1.2, 1.4, 1.5, 1.6, 1.7, 1.8	1.1, 1.4, 1.5,	**1.1, 1.2, 1.3, 1.4, 1.5**	1.1, 1.2, 1.3, 1.4, 1.5	1.1, 1.2, 1.3, 1.4, 1.5, 1.8
Estimation	2.2, 2.3, 3.1, 3.2	2.1, 2.2, 2.3 3.1, 3.2	**2.1, 2.2, 2.3, 3.1, 3.2**	2.1, 2.2, 2.3, 3.1, 3.2	2.1, 2.2, 2.3, 3.1, 3.2
Technology	1.15, 1.18	1.18	**1.16, 1.17**	1.16, 1.17, 1.18	1.16, 1.17, 1.18
Paper and Pencil	1.9, 1.10, 1.11, 1.12, 1.14	1.9, 1.10, 1.11	**1.9, 1.10, 1.11, 1.13, 1.14**	1.9, 1.10, 1.11, 1.13, 1.14	1.9, 1.10, 1.11, 1.13, 1.14

CONCEPT 1

Students will be expected to demonstrate an understanding that two- and three-dimensional geometric figures can be classified and have characteristics which uniquely define them.

Students will be expected to:

1.1 classify polygons according to the number of sides
1.2 classify triangles and quadrilaterals by sides and angles
1.3 determine the sum of the interior angles of a polygon
    1.3.1 perform an investigation to determine the sum of the interior angles of any polygon
1.4 classify the following common geometric solids: rectangular prisms, pyramids, cylinders, cones and spheres
    1.4.1 perform an investigation to determine the relationship amongst the edges, faces and vertices of polyhedra
    1.4.2 draw nets which define simple polyhedra
    1.4.3 use nets to construct simple polyhedra
1.5 identify and define the following terms associated with a circle such as the following: radius, diameter, circumference, semicircle.

COMMENTS

The intent of this concept is to have students build an understanding of two- and three-dimensional geometric figures. In building this understanding, students should be involved in classifying polygons and geometric solids and should participate in activities that expose the properties of the figures. The examination of polygons should be limited to the decagon, while triangles and quadrilaterals should be classified not only by sides, but also by angles. Besides the classification of polygons, students should perform investigations to determine the sum of the interior angles of any polygon. The classification of geometric solids should be limited to rectangular prisms, pyramids, cylinders, cones and spheres. A further examination of polyhedra should include students drawing and using nets to construct polyhedra and performing investigations to determine the relationship amongst the edges, faces and vertices of polyhedra. (Euler's formula: $E + 2 = F + V$ describes this relationship). The final aspect of this concept is an awareness of the characteristics and properties of the circle (radius, diameter, circumference, semicircle).

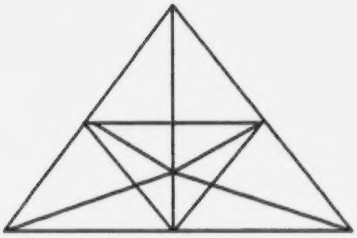
Note: An 11-sided figure is known as a hendecagon (the Loonie is a hendecagon).

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Polygon – Classification and Identification by Sides – have students examine the definition of the terms we use, using a dictionary and identify other words that also use the prefixes used in identifying polygons; e.g., examine octagon in the dictionary and then review the definitions of other words using the prefix "oct" and recognize that all the words relate to eight, then have students draw an eight-sided figure. This will help in the link between the meaning of the prefix and the number of sides in a figure.

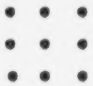
Activity: Triangles and Quadrilaterals - students should be actively involved in measuring sides and angles. In performing these activities, students should be drawn to the similarities and differences in classifying triangles and quadrilaterals by sides and angles. The discussion of sides and angles should be extended to include whether or not particular qualities can describe the same triangle; e.g., can a scalene triangle be a right triangle? Students should not only measure the angles and sides of such triangles, but should also note the differences/similarities between the triangles.

Problem: How many triangles can you count?



Answer:
47

Problem: Using only the nine points of the geoboard as endpoints, how many isosceles triangles can you make on this 3 x 3 geoboard?

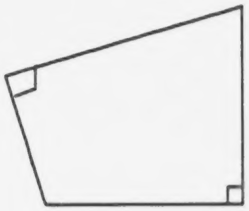


Answer: 36

Problem: A wire 8 cm long is to be cut into three pieces that are to be used for the sides of a triangle. If all three pieces have integral lengths, what is the length of the shortest side?

Answer: 2 cm.

Activity: Make four copies of the following shape, then arrange them to form (*a*) a parallelogram and (*b*) a square.



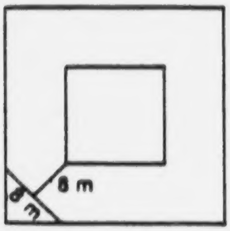
Problem: Cut out an equilaterial triangle. Fold the vertices to meet in the centre. What figure do you have?

Answer: A regular hexagon.

Problem: Given a square, how can you demonstrate with a piece of construction paper that the sum of the angles equals 360° without using a protractor?

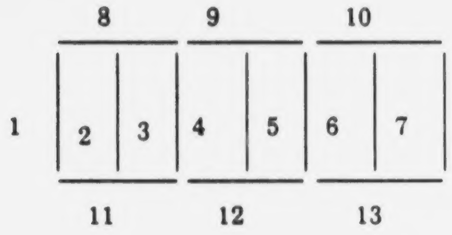
Problem: Mr. Jackson had a garden in the shape of a square. Each year some of his melons from the garden were stolen. Since he depended on the produce for a living, he had a ditch 10m wide and 10m deep dug around the patch. He had the ditch filled with water. However, after only a night or two some of the melons were gone again and the thieves left behind two 8-m planks that they had used to gain access to the patch and to get out again. How did they do this?

Answer:



From Problem Solving Challenge for Mathematics.

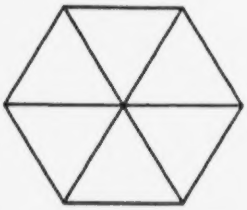
Problem: At a 4H Club Show six pig pens were made with 13 panels, all of equal length as shown:



From Problem Solving Challenge for Mathematics.

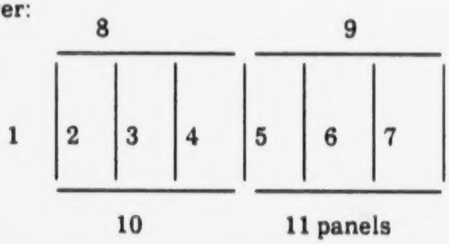
a. One of the panels was broken when a truck backed into it. The club members arranged the 12 panels so that they could still form six pens of equal size and same shape. How did they do it?

Answer:

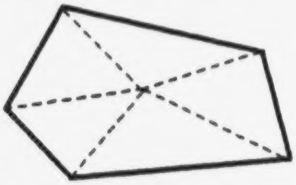


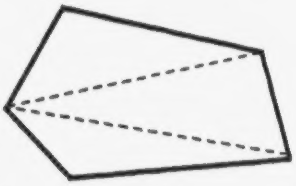
b. Later the manager needed one of the panels in another part of the show. The club members then made six pens of equal size and of the same shape with 11 panels. How did they do it?

Answer:



Activity: Interior Angles of Polygons – Students' initial discussion of this should be with triangles. Students should measure, using the protractor, the number of degrees in several "types" (e.g., scalene, right, obtuse, acute) of triangles. This will give the student practice in recognizing where measurements are taken when measuring the degrees of a polygon. The investigation should then be extended to quadrilaterals – with eventual reference made to the fact that a quadrilateral can be divided into two triangles. Further extension should be made to the remainder of the polygons, initially measuring the angles and then comparing it to the number of triangles that can be constructed within the polygon. Have students consider the two approaches shown. What differences, if any, are there in these procedures? By the end of the investigation, students should have discovered "the rule" and be able to find the number of degrees in a polygon of n sides.





See "Polygons Made to Order," Mathematics Teacher, January 1987 for a series of activities designed to help students understand the relationships among the angles of polygons, and draw regular polygons.

Problem: How many degrees does the minute hand of a clock pass through between 9:30 a.m. and 10:17 a.m.?

Answer: 282 degrees.

From Problem Solving Challenge for Mathematics.

Activity: Polyhedra - An examination of polyhedra that exist in the student's life should be used as an introduction; e.g., a cereal box or cracker box are examples of polyhedra. Separate at the "glue" spots to understand what a net is. Encourage students to examine similarities and differences of the representation of different nets with different polyhedra. Have students reconstruct the boxes to see "how" a net produces a polyhedron. From this point, students should be involved in investigating different polyhedra to examine the relationship between the edges, faces and vertices - this helps and further reinforces patterns in mathematics.

Problem: A wooden cube that measures 10 cm along each edge is painted red. The painted cube is then cut into 2-cm cubes. How many of the 2-cm cubes do not have red paint on any face? Students should make a model (sugar cubes are good for this) using smaller measurements and then examine how many cubes would not have any red paint on any face initially and then expand it to a larger cube. A pattern should occur.

Answer: 27

From Problem Solving Challenge for Mathematics.

See "Discovery with Cubes" in Mathematics Teacher, May 1988 for activities related to this problem.

Activity: Circles - The realization that a circle is a special polygon should be identified by students. Students should have the opportunity to draw circles and then measure the radius and diameter to build an understanding of the relationship between the two. Have students identify whether or not a circle can be drawn only given a radius, a diameter? It is important to note that a discussion of circumference at this point should not include the calculation of circumference, rather an understanding that circumference identifies the outside edge of the circle. The understanding of "semicircle" begins with students' awareness of the word "semi" - once again a discussion of "semi" in their lives will help to realize the relationship with and the "semi" in mathematics. An examination of the number of degrees in a circle is also appropriate - this can be accomplished by students using a protractor to examine the degree measures.

TECHNOLOGY INTEGRATION

Calculators should be used for all addition.

For an example of how spreadsheets can be used in a student exploration of the measures of the angles of regular polygons, see "Integrating Spreadsheets into the Mathematics Classroom," Mathematics Teacher, November 1988. Appleworks and Microsoft Works are Alberta Education Basic Resources.

Software: Geometric Supposer: Quadrilaterals (Sunburst) may be used as an elective exploratory activity.

Problem: A square piece of paper was folded as drawn in Figures I, II and III. Figue IV shows where a small hole was punched through the paper

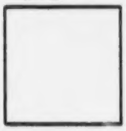


Fig. I

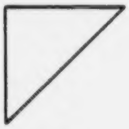


Fig. II

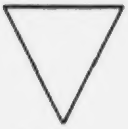


Fig. III

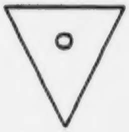
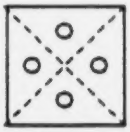


Fig. IV

If the paper was completely unfolded what would it look like ?

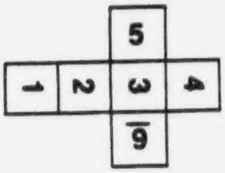
Answer:



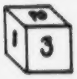
From Problem Solving Challenge for Mathematics.

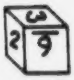
Triangles - Have students draw, using a protractor and ruler, triangles of different names.

Polyhedra - The first figure below shows the network for a certain cube. One of the figures below the network is a drawing of this number cube. Which one is it?

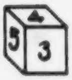


Answer:





(✓)



From Problem Solving Challenge for Mathematics.

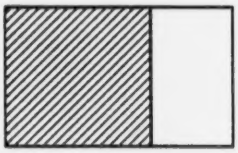
ELECTIVE SUGGESTIONS

Quadrilaterals: (Note: This activity helps develop students' skills with ratio also.)

Have students draw a variety of rectangles, and then choose what each considers to be the most "beautiful." Measure and calculate the ratio of the length to the width. Mention that the Greeks considered that the ratio between 1.4 and 1.8 represented beautiful rectangles.

Are there disagreements with the Greek definition? What objects in everyday life represent examples of beautiful rectangles. What are examples of "ugly" rectangles?

Have students construct a square on their own beautiful rectangle, as shown:



Compare the ratio of the length and width of the unshaded portion to that of the original rectangle. (The width of the original rectangle always becomes the length of the new rectangle.) The students should find that for ratios between 1.4 and 1.8, the form of the two rectangles should be nearly the same.

Have the students construct "golden rectangles," with a length to width ratio of 1.618, and the draw the square as above. What do they find when they compare ratios?

For more information, see "Reinforce Division by Learning Ratios," in Activities for Junior High School and Middle School Mathematics.

Circles – given circles have students identify whether certain chords are radii or diameters or have students draw radii and diameters – writing why a certain chord is not a radius or a diameter.

Triangles – Have students examine the relationship between the size of an angle and the length of the side opposite the angle with the whole triangle.

Problem: The measures of the angles of a triangle are in the ratio 2:3:5. What is the number of degrees in the measure of the smallest angle?

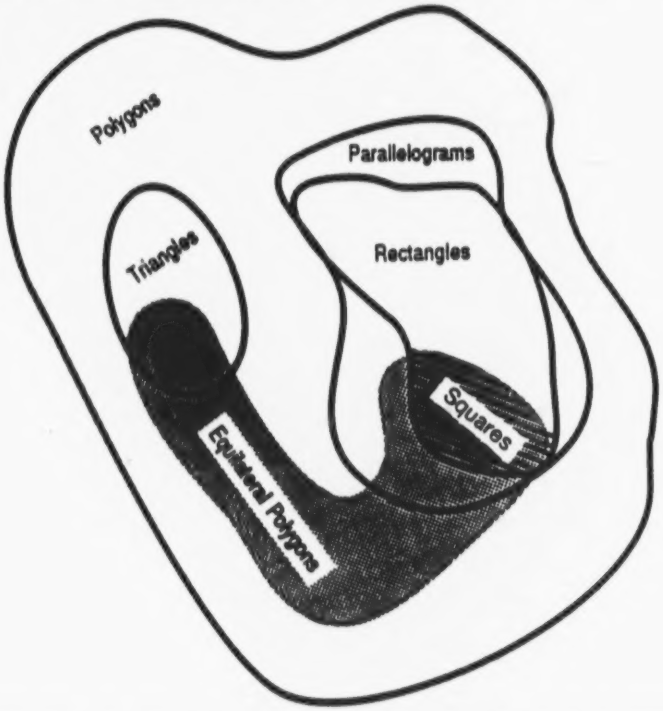
Answer: 36°, 54°, 90°.

Polygons - In discovering the definition "regular polygons" students should be involved in measuring angles and sides of regular and non-regular polygons and drawing comparisons.

Further extension is the discussion of tesselations using polygons.

Activity: Circles and Quadrilaterals
Cut squares and circles out of tagboard. Using a candle or a flashlight, try projecting the shadow of the figure onto the wall of the darkened classroom. What shapes can be created by turning the figures at various angles? Discuss how this is possible.

Activity: Polygons
Allow the students to work in teams to produce a picture illustrating the relationships between sets of polygons. They need to use defining characteristics for whatever figures are chosen. Suppose the choices are polygons, triangles, squares and rectangles. The team would decide which set includes all the others and make that cutout largest. Then team members would pick a set, and together they would plan their picture to depict the relationships between their sets. A finished product may look like the following figure.



Reprinted with permission from "Problem Solving in the General Mathematics Classroom," Activities for Junior High School and Middle School Mathematics, copyright 1981 by The National Council of Teachers of Mathematics.

For related paper-folding activities, see "Discoveries in Geometry by Folding and Cutting," Activities for Junior High School and Middle School Mathematics.

Polyhedra - Show students examples of other figures (e.g., tetrahedron, dodecahedron and so on. Explore the nets for these.

For more **Problems** see: Problem Solving Challenge for Mathematics, p. 27, #8.8 and p. 48, #12.

RESOURCE CORRELATION AM: pp. 212 - 215, 221–222 255–257, 262, 281–282, 284, 286 (identification and classification only 296)

CONCEPT 2

Students will be expected to demonstrate an understanding that geometric figures can be defined uniquely and that these unique characteristics can be reproduced using construction techniques and geometric instruments.

Students will be expected to:

- **2.1** construct a line segment of given length or congruent to a given segment
- **2.2** draw an angle of a given size or construct an angle congruent to a given angle
- **2.3** construct the perpendicular bisectors of segments and angles
- **2.4** draw regular polygons
- **2.5** construct a circle of given radius
- **2.6** use a combination of construction techniques
  - **2.6.1** construct angles of 90°, 45°, 60° and 30°
  - **2.6.2** use construction techniques in the construction or reproduction of geometric figures or designs.

COMMENTS

The student activities involved in this concept may be incorporated in Concept 1. This concept involves students drawing line segments and angles of a given size; constructing congruent line segments, congruent angles, bisectors of angles, perpendicular bisectors of line segments, circles of given radii, angles of 90°, 45°, 60°, 30° and regular polygons. Further it is expected that students will use these construction techniques in constructing or reproducing geometric figures and designs. It is important that meaning be established for several geometric phrases that we assume students understand; e.g., perpendicular, bisector, regular polygons.

The geometric instruments that could be used are the compass, straightedge, ruler, protractor, MIRA or computer. **2.4** should be limited to polygons with 3,4,5,6 or 8 sides only.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: As the drawing and constructing of these figures is occurring, students should be actively involved in the measurement of angles and segments in order to actually experience the "bisection" of an angle or line segment and to "see" that the perpendicular bisector forms a 90° angle with the line.

Once the concept of "bisect" is understood and has been experienced, students should be able to construct a 45° angle without further explanation.

Activity: Given a geometric figure such as a triangle – have students construct a congruent triangle or a figure of half the size with only a compass and a straightedge.

TECHNOLOGY INTEGRATION

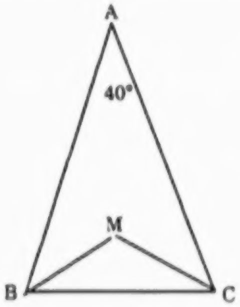
The use of the computer is desirable here to demonstrate that a computer can construct/draw the same figures that a student can.

Software:
The Geometric preSupposer: Points and Lines (Sunburst)
Suitable for teacher demonstration and student exploratory use.

ELECTIVE SUGGESTIONS

- Construct 90°–45°–45° or a 30°–60°–90° triangle.

- The measure of one angle of a triangle is 40°. What is the measure of the angle formed by the bisectors of the other two angles?



Systematic guess and test.

One possible solution:

Answer: 110°

- Construct a perpendicular line to a given line from any point not on the line.

- *M* is the midpoint of *AB*, *C* and *D* are on *AB* so that $AC = CM$ and $MD = DB$. What percent of *AM* is *AD*?

  Answer: 150%

- Find the centre of a circle by constructing the perpendicular bisectors of two chords.

- Divide students into pairs. Give each group a model of a circle, and a piece of square-shaped tagboard. Have them find the centre of the circle using only the materials provided.

One strategy:

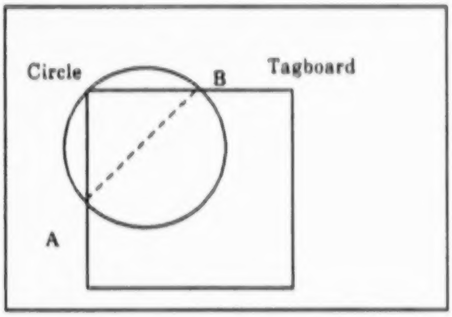


Figure reprinted with permission from "Problem Solving in the General Mathematics Classroom," Activities for Junior High School and Middle School Mathematics, copyright 1981 by The National Council of Teachers of Mathematics.

RESOURCE CORRELATION

AM: pp. 196–197, 234–239, 242 – 243

CONCEPT 3

Students will be expected to demonstrate an understanding that similar geometric figures have the same shape but may differ in size, that corresponding angles are equal and that the lengths of corresponding sides are proportional.

Students will be expected to:

- **3.1 use the relationship between the lengths of the sides of similar triangles to calculate the lengths of unknown sides**
  - **3.1.1 solve problems using the properties of similar polygons**
- **3.2 recognize congruency as similarity in which the size of the figures is the same.**

COMMENTS

This concept is intended to expand students' knowledge and understanding of geometric figures. Students should be able to calculate the lengths of unknown sides and solve problems given similar figures and should recognize congruency as similarity in which the size of the figures is the same.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Many construction and drawing techniques can be used in the development of this topic. To experience the difference between similarity and congruence, give students sets of parameters and have them construct/draw triangles; e.g., give each student three proportional side measurements to form a triangle – have students draw their own triangles with a ruler and then have them measure the angles. What occurs when they measure the angles? (All angle measures should be the same.) Then note how the angle measures are the same but side measures are not. What is the relationship between the side measures for each person's triangle? The same exercise could occur giving the students two side measures and an angle measure, two angle measures and a side measure, or three angle measures (SSS, SAS and ASA, although these relationships do not need to be discussed formally with students). This can be extended to other polygons where students are measuring to discover congruent and similar polygons.

Before students use the algorithm to calculate unknown sides of similar figures, students should be involved in measuring and guessing measures of sides of similar figures.

Activity: Discuss examples of similar triangles in everyday life. For example, an ironing board uses similar triangles to stay level as you raise it. This can serve as an introduction to an outside–of–classroom project.

Activity: Have students perform experiments examining the effect of "halving" one angle in a triangle. Are all of the other angles automatically "halved"? What is the effect of "halving" a side measure? What happens to the measure of the other sides? What happens to the measure of the angles?

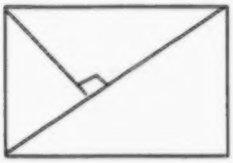
Problem: Can circles be similar? or congruent? What things would have to change to create "similar" circles?

Problem: A man 180 cm tall casts a 45 cm shadow. If a telephone pole casts a 300 cm shadow, then the height of the pole is ____ cm.

Answer: 1200 cm.

Problem: Draw lines to separate a rectangle into three similar triangles.

One solution:



TECHNOLOGY INTEGRATION

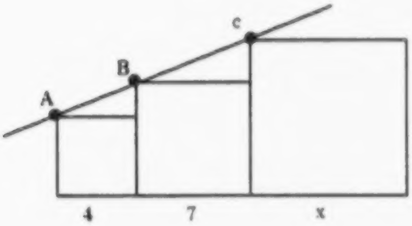
Software:
Geometric preSupposer: Points and Lines (Sunburst) Suitable for teacher demonstration and student exploratory use.

Geometric Supposer: Triangles (Sunburst)
Suitable for elective student exploratory use.

ELECTIVE SUGGESTIONS

- Formally examine the SSS, SAS and ASA cases of similarity and congruency.
- Understand and use the concept of symmetry with respect to geometric figures. This includes examining lines of symmetry.
- **Problem:** If the three vertices A, B and C in the three adjacent squares are collinear, find the value of x.

Answer: $x = {}^{49}/_{4}$



RESOURCE CORRELATION

AM: pp. 258–259, extension 246 – 249

CONCEPT 4

Students will be expected to demonstrate an understanding of the Pythagorean Theorem.

Students will be expected to:

- 4.1 use the Pythagorean Theorem to find the length of a side in a right triangle
  - 4.1.1 solve problems involving right triangles by using the Pythagorean Theorem.

COMMENTS

Students will be expected to use the Pythagorean Theorem to find the length of a side in a right triangle and to solve problems involving right triangles.

Note: It is important that students NOT only be exposed to $a^2 + b^2 = c^2$ as the only formal form of the Pythagorean Theorem. An understanding of what a, b and c represent is more important. Hence students should not only see but use different letters in representation of this theorem.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students measure the sides of perfect "right" triangles (e.g., a triangle with side measurements of 3–4–5) before the discussion of the Pythagorean Theorem. Once the algorithm has been discussed, have students calculate the measure of missing sides, and then measure the sides to confirm the measurement.

Activity: Have students identify situations in which the Pythagorean Theorem can be used.

Problem: A woman paces off 30 steps east, followed by 10 steps north, 10 steps east, and 20 steps north. How far is she from her starting point?

Answer: 50 steps.

Problem: Two vertical poles, 10 m and 15 m high, stand 12 m apart. Find the distance, in metres, between the tops of the two poles.

Answer: 13 m.

For a concrete approach to verifying the Pythagorean Theorem, see "Pythagorean Puzzles," in Activities from 'Mathematics Teacher'.

Activity: Have students describe, in their own words, how the Pythagorean Theorem could be used to find out if the corner of a fence is a right angle.

TECHNOLOGY INTEGRATION

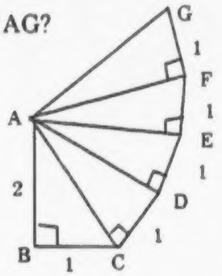
Software:
Geometric preSupposer: Points and Lines (Sunburst)
Suitable for teacher demonstration and student exploratory use.

ELECTIVE SUGGESTIONS

- A study of Pythagoras, by visiting the library is appropriate here.

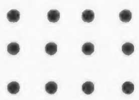
- What is the length of AG?

Answer: 3 units.

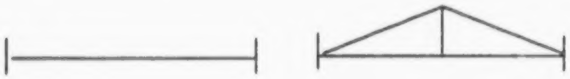


- **Problem:** The dots in a 3-by-4 grid are 1 cm apart. Connect two dots to form a line segment of length $\sqrt{5}$ cm. How many such $\sqrt{5}$ cm line segments can be drawn on the grid?

Answer: 14.



- **Problem:** Suppose the summer heat caused a bridge 2.5 km long to expand 1 m. The bridge is anchored only at its ends. It has never collapsed and only bulges upward during expansion. How high would the bridge bulge? Would you cross the bridge?



Answer: Assuming that the bridge bulges only in the centre and that the sides are straight, the height in the middle is $\sqrt{1250.5^2-1250^2} \approx 35.35$ m.

RESOURCE CORRELATION AM: pp. 218-220, 367-368

CONCEPT 5

Students will be expected to demonstrate an understanding that angles formed by intersecting lines relate to each other.

Students will be expected to:

5.1 identify parallel, intersecting and perpendicular lines

5.2 identify and recognize the characteristics of the relationship between supplementary, complementary and vertical angles

  5.2.1 perform an investigation to determine the properties of the angles between intersecting lines

  5.2.2 solve problems involving the relationships amongst intersecting and parallel lines and the angles they form to solve problems involving the measure of unknown angles.

COMMENTS

Many of the topics covered within this concept will follow the previous discussions. Students should be able to identify parallel, intersecting and perpendicular lines; supplementary, complementary and vertical angles. As opposed to students being told the definitions of each of these, they should perform investigations and activities that provide meaning to each of the terms.

PROCESS/PROBLEM-SOLVING CONTEXT

The use of specific names; e.g., "corresponding angles" of angles involved in parallel lines nor the "patterns"; e.g., Z formed is not encouraged.

Activity: When examining lines, have students identify the difference between parallel lines and intersecting lines. Then have students examine the differences between intersecting and perpendicular lines. This is an appropriate time for students to be involved in indentifying places, in their experience, that they have seen lines. An awareness that the intersection of lines form angles is important in developing further understanding.

Activity: This activity could start off by students measuring pairs of angles before measuring the angles around intersecting lines. Given pairs of angles, formed by intersecting lines or line segments, have students measure each angle and then group the pairs. Once the groups have been established; i.e., those that have the same measure, those whose measures add to 180° and those whose measures add to 90°, attach the names of vertical, complementary and supplementary to each group. Then have the students write, in their own words, the definition of vertical, complementary and supplementary angles.

See "Games, Geometry, and Teaching," in Mathematics Teacher, April 1988, for an in-depth description of games designed to help develop students' understanding of angles, parallel lines and polygons.

TECHNOLOGY INTEGRATION

The calculator should be used in computation.

Software:
The Geometric preSupposer: Points and Lines (Sunburst)
Suitable for teacher demonstration and student exploratory use.

ELECTIVE SUGGESTIONS

- Examine and discuss that the intersection of two lines form a plane.
- Examine the definition of adjacent angles and where they would "fit" into this discussion; i.e., what are the similarities and differences?
- Examine skew lines.

RESOURCE CORRELATION AM: pp. 208–211

Mathematics 14
Integration of Numeration Concepts and Skills into Measurement

The numeration concepts and skills identified in the shaded area represent those skills that students should use, reinforce, or learn within this unit of study. A detailed description of each skill can be found on pages 223–225.

	Ratio and Proportion	Statistics and Probability	Geometry	Measurement	Algebra
Mental	1.1, 1.2, 1.4, 1.5, 1.6, 1.7, 1.8	1.1, 1.4, 1.5,	1.1, 1.2, 1.3, 1.4, 1.5	1.1, 1.2, 1.3, 1.4, 1.5	1.1, 1.2, 1.3, 1.4, 1.5, 1.8
Estimation	2.2, 2.3, 3.1, 3.2	2.1, 2.2, 2.3 3.1, 3.2	2.1, 2.2, 2.3, 3.1, 3.2	2.1, 2.2, 2.3, 3.1, 3.2	2.1, 2.2, 2.3, 3.1, 3.2
Technology	1.15, 1.18	1.18	1.16, 1.17	1.16, 1.17, 1.18	1.16, 1.17, 1.18
Paper and Pencil	1.9, 1.10, 1.11, 1.12, 1.14	1.9, 1.10, 1.11	1.9, 1.10, 1.11, 1.13, 1.14	1.9, 1.10, 1.11, 1.13, 1.14	1.9, 1.10, 1.11, 1.13, 1.14

264

CONCEPT 1

Students will be expected to demonstrate an understanding that standard units of measure for length, mass, and volume provide consistency and accuracy and that the most widely used and accepted measurement system is the SI (metric) system.

Students will be expected to:

1.1 recognize the basic SI units of measurement for length (metre[m]), mass (kilogram[kg]), tonne [t]), and volume of liquids or gases (litre[L])

1.2 use the appropriate SI measures of length, mass and liquid/gas volume. (limit: kg, g, L, mL, m, cm, mm, km)

1.3 express equivalencies between units of length, of mass and of volume

1.4 recognize non-metric units of measurement still in common use.

COMMENTS

Students are expected to use, recognize and express equivalencies within the basic SI units of measurement for length (metre (m)), mass (kilogram (kg)), and volume of liquids or gases (litre (L)). In addition to an exhaustive study of the SI metric system, students should be able to recognize non-metric units of measurement still in common use.

Note: The terms "weight" and "mass" cannot be interchanged. Exercise caution when referring to them.

The Canadian Metric Practice Guide indicates that capacity and volume are the same measures. Specifically, "The cubic metre (m^3) is the SI unit of volume. It may be used for space or for a contained substance, whether solid, liquid, or gas. The litre (L) is the special name for the cubic decimetre ($1L = dm^3$) and is used for volumes of solid, liquid, or gas. The millilitre (mL) is used for medicines, in cooking, laboratories, etc.; the cubic centimetre (cm^3) is also used for laboratory work." (1979, p.19)

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: It is important to note that students should be introduced to the term "measurement" and that it does not only involve the use of m, kg or L. Have the students identify things that are measured in their lives; e.g., their age is measured, temperature is measured, the amount of food consumed is measured, time is measured.

Activity: Further it is important to establish a reason for the popularity of a standard measurement system. Have students measure the thickness of their math book using their thumbs. Each student should have measurements such as $^1/_2$ a thumb, 1 $^1/_2$ thumbs, etc. In actuality, the thickness of the math book is the same, but because we used thumbs that are of differing lengths, the book's thickness is different. If we were to communicate to people outside of the class that the thickness of the math book was $^1/_2$ a thumb, would they know and be able to understand the thickness of the math book? Hence, the need for a standard measurement system so that if we refer to 6 cm, all people will know the thickness.

Activity: Using the appropriate measuring tool in circumstances is an important skill for students. Devise experiments where students are asked to measure, a cm for instance, using a metre stick. This leads into a discussion regarding the accuracy of the tools. Following experience with measuring, give students situations where they must decide, not only the best measuring device, but also the most appropriate units for this circumstance.

Activity: In building an understanding of the different prefixes for the metric system, students should be involved in measuring lines (for instance) using km, mm, cm and then describing the difference, in their own words.

Problem: A snail starts at the bottom of a well 16 m deep and crawls up 4 m each day. Each night, however, the poor thing slips back 3 m. How long will it take the snail to reach the top of the well?

Answer: 13 days.
From Problem Solving Challenge for Mathematics

Problem: A mural 3 m across is painted in the centre of a wall that is 19 m wide. How many metres from the end of the wall is the nearest edge of the picture?

Answer: 8 m.

Problem: A brick has a mass of 600 g plus $^{1}/_{2}$ of its total mass. What is the total mass of the brick?

Answer: 1200 g.
From Problem Solving Challenge for Mathematics

Activity: Find a dripping faucet and measure in millilitres the amount of water that is wasted in 10 minutes. At this rate, how much water would be lost in an hour, a day, a week?

TECHNOLOGY INTEGRATION

Software:
SRA Computer Drill and Instruction, Mathematics: Level D (rev. ed.), "Measurement" section. Selected lessons may be used as drill.

Video:
Math Wise: "Measuring 01 – Measuring Instruments". Plus Teacher's Guide Discussion and student worksheet. (Distributed by ACCESS). Best used as an introduction to the unit.

ELECTIVE SUGGESTIONS

- Discuss the cL, dL, dg, etc.

RESOURCE CORRELATION

AM: pp. 139–143, 152–153, 161–166, 176–177

CONCEPT 2

Students will be expected to demonstrate an understanding that perimeter is the measure of the distance around a geometric figure.

Students will be expected to:

2.1 calculate the perimeter of irregular shapes and objects
2.2 calculate the circumference of a circle by using a formula
  2.2.1 solve problems which involve finding the circumference of circles and circular regions.

COMMENTS

Students are expected to solve problems involving both the perimeter and circumference and use formulas to calculate these measures. Students are often expected to know "perimeter," however, they may only know perimeter as $2l + 2w = P$ and not be able to relate perimeter to circumference, for instance. Hence, by no means should the formula be the introduction to this concept. Students should build an understanding of and be able to describe perimeter, before using the formula.

Polygons – Have students measure the distance around several physical objects such as a door, desktop, window, before measuring polygons. This provides the student with an experience that demonstrates that "perimeter" not only exists in mathematics. It is important to also discuss at this point the appropriate measuring devices to be used, depending on the accuracy required.

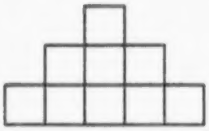
PROCESS/PROBLEM-SOLVING CONTEXT

Activity: To have students realize that figures that have the same perimeter are not necessarily the same figure, have students create a figure with a length of string (the same length to several students) – the perimeter is the same. Questions that could be explored at this point: Does the size of the perimeter determine the number of sides in a polygon? Does the perimeter affect the degree measure of the polygon? If two figures are joined, will the perimter of the new figure be the perimeter of figure 1 plus the perimeter of figure 2 (have students join their figure with another students' and measure the perimeters to discover if this is so).

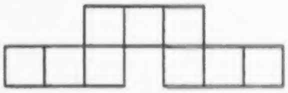
Problem: Figures A and B below are made up of congruent squares. If the perimeter of Figure A is 48 cm, what is the perimeter of Figure B?

Answer: 60 cm.

A



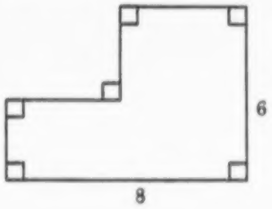
B



From Problem Solving Challenge for Mathematics

Problem: The perimeter of the polygon shown is:
A) 14 B) 20 C) 28 D) 48
E) cannot be determined from the information given.

Answer: 28.



Activity: Circles - Have students discuss the possible ways of measuring the distance around circles. Have students use strings, and possibly measuring tapes, to measure the distance around circular objects - this will help to reinforce how important it is to have a formula for the circumference of a circle.

Activity: To assist in the understanding of π, have students measure the diameters and circumferences of a set of circles and have them notice that the ratio of these two attributes is approximately 3.14 in each case. Students could fill out a chart as follows:

Circle	C	d	$C/_d$
1			
2			
3			
4			
5			

Problem: When a bicycle travels a kilometre its 50 cm-diameter tires make how many revolutions?

Answer: $\approx$ 637 rev.

Problem: The minute hand of a clock is 6 cm long. To the nearest centimetre, how far does the tip of the minute hand move in 35 minutes?

Answer: $7\pi \approx 22$ cm.

TECHNOLOGY
INTEGRATION

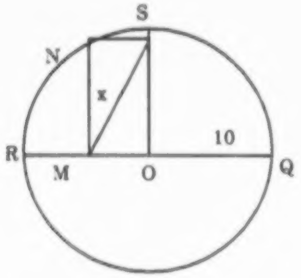
Software:
The Geometric preSupposer: Points and Lines (Sunburst)
Suitable for teacher demonstration and student exploratory use.

ELECTIVE SUGGESTIONS

- **Polygons** – A discussion of "isoperimetric" – polygons that have the same perimeter.
- **Circles and Polygons**

MNPO is a rectangle. O is the centre of the circle. Find x.

This is not a circumference problem, but the focus is the circle.



RESOURCE CORRELATION AM: pp. 260–263

CONCEPT 3

Students will be expected to demonstrate an understanding that area is the number of unit squares which are enclosed within the boundaries of a geometric figure.

Students will be expected to:

- 3.1 calculate the area of triangles and quadrilaterals by using formulas (limit: triangles, quadrilaterals)
  - 3.1.1 solve problems which involve finding the area of triangles and rectangles
- 3.2 find the area of a circle by using a formula
  - 3.2.1 solve problems which involve finding the area of circles.

COMMENTS

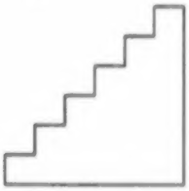
This concept covers the use of the area formulas for triangles, quadrilaterals and circles in calculations and in solving problems. It is important to note once again that students should not be introduced to this concept with the formula, but rather should experience finding the area of these figures without the means of the formula.

Note: composite area problems could be placed on a grid when calculating area.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Polygons – Graph paper and geoboards are useful in building an understanding of area. The need for an indirect measure of area (hence the need for formulas) will become evident when area becomes difficult to "count."

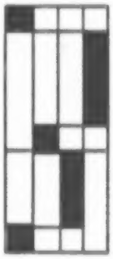
Problem: Find the area of the figure below. The line segments for the "step" formation meet at right angles and are of equal length, 1cm.



Answer: 21 cm^2.

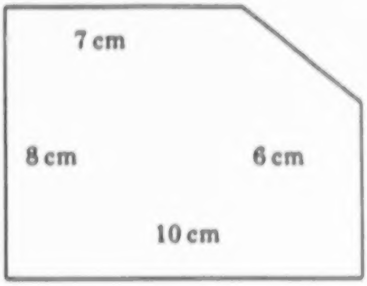
Problem: What fractional part of the figure is shaded (assuming all horizontal lines are parallel, all angles are right, and all vertical lines are equally distant and parallel)?

Answer: $^1/_4$.



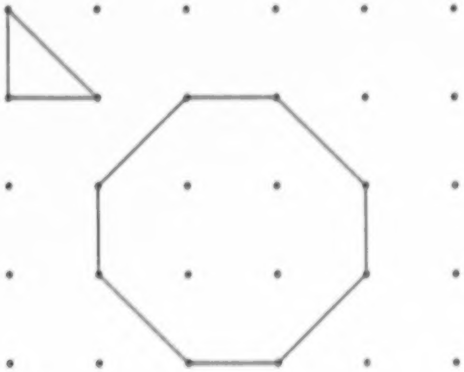
270

Problem: Find the area:



Answer: 77cm^2.

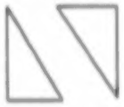
Problem: The area of the triangle is 2 cm^2. What is the area of the octagon?



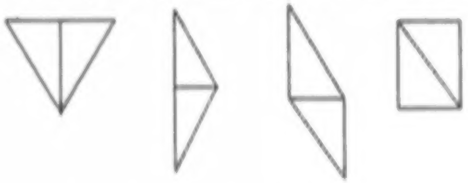
Answer: 28 cm^2.

Problem:

Here are two triangles.



These triangles can be rearranged in several ways to form different figures.



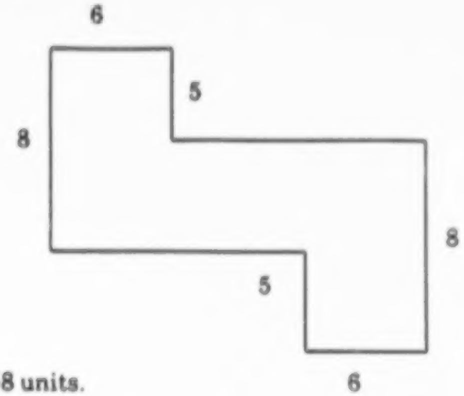
Do these figures have the same shape?

Do these figures have the same area?

Be sure to make the geometry connection as well as those that are measurement related.

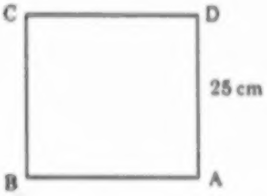
Reprinted with permission from "Problem Solving in the General Mathematics Classroom," Activities for Junior High School and Middle School Mathematics, copyright 1981 by The National Council of Teachers of Mathematics.

Problem: If the area of this figure is 108 square units, then what is its perimeter?



Answer: 58 units.

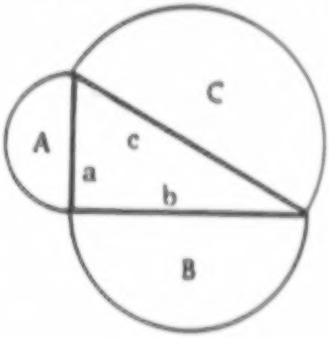
Problem: What are the dimensions of the square EFGH you must cut from the square ABCD so that the area of square ABCD is reduced by 25%.



Answer: 12.5 cm x 12.5 cm.

Activity: Circle - Have students cut circles into portions and form a parallelogram and then attempt to measure the area - once again because of the inaccuracy, it is important to have a formula to find the area of a circle.

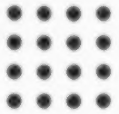
Problem: Pythagoras showed that for a right triangle $a^2 + b^2 = c^2$.



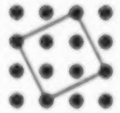
Is it true that for a right triangle the arc of the semicircles are related by Area A + Area B = Area C?

Answer: Yes.

Problem: Make a square with an area of five square units on the grid.



Answer: $S = \sqrt{5}$.



TECHNOLOGY INTEGRATION

For a set of appropriate grade – level area – maximization problems designed to promote the use of tables and calculators, see "Using Calculators to Fill Your Table," Activities for Implementing Curricular Themes from the 'Agenda for Action'.

Software:
Geometric preSupposer: Points and Lines (Sunburst)
Suitable for teacher demonstration and student exploratory use.

SRA Computer Drill and Instruction, Mathematics: Level D, "Measurement" section. Selected lessons may be used for drill.

Video:
Math Ways (Kinetic) – "Areas".
Area of circle and surface area of cylinder are developed. Includes derivation of π as relationship between circumference and diameter. Derives formula $A = \pi r^2$. Surface area is not a component of the Mathematics 14 program of studies.

ELECTIVE SUGGESTIONS

Problem: You know that the perimeter of a certain rectangle measures 22 cm. If its length and width each measure a whole number in centimetres, how many different areas (in square centimetres) are possible for this rectangle?

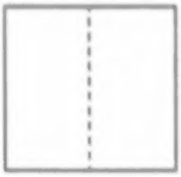
Answer: 5.

Polyhedron – discuss the surface area of pyramids and cones.

Elective Suggestions Involving the Square:

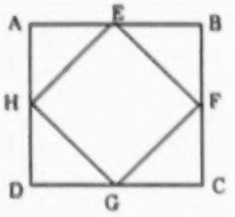
Problem: A square piece of paper is folded in half vertically. If the resulting figure has a perimeter of 12 cm, what was the area of the original square?

Answer: 16 cm^2.



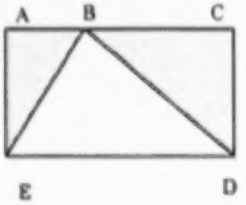
Problem: The area of square ABCD is 625 cm^2. E, F, G and H are midpoints of AB, BC, CD and DA, respectively. Find the area of square EFGH.

Answer: 312.5 cm^2.



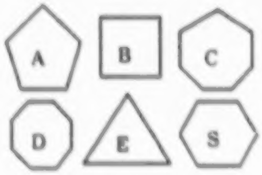
Problem: If ACDE is a square whose edge measures 12 cm, what is the area of the shaded region?

Answer: 72 cm^2.



Explore various strategies. One key strategy: What is the effect of drawing line segment BF parallel to AE, where F lies on line segment ED?

Activity:



Shapely Relations

Equipment: Six cutout polygonal regions labelled A, B, C, D, E and S.
Compare the areas of shapes, A, B, C, D and E with the area of shape S. Decide whether each one is greater than, equal to or smaller than S.

Comments: One might attempt to make a numerical appraisal of the area of each polygon, or superimpose one on another for comparison, or mentally subdivide each into regions of approximately equal size. This type of problem lends itself well to presentation on an overhead projector.

Reprinted with permission from "A Do-It-Yourself Estimation Workshop," Estimation and Mental Computation, copyright 1986 by The National Council of Teachers of Mathematics.

Problem: A rectangular garage is to be built with a fixed perimeter of 30 m. What dimensions produce the smallest floor area? The largest floor area.
Encourage the students to try various strategies. For example:

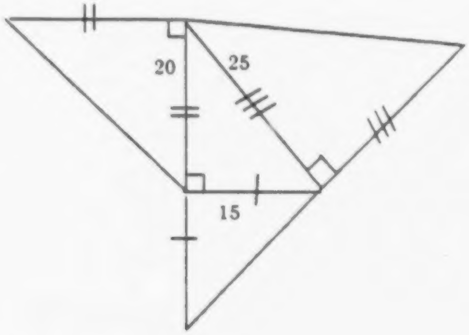
- Guess and test
- Construct a table (of L, W, P, A)
- Graph the relation between L and A. (How will it compare to the graph of W and A?)
- Are decimal values needed to find the largest area?

Answers: 1 m x 14 m (best whole number solution),
7.5 m x 7.5 m.

After the mathematical problem has been solved, discuss the feasibility of a 7.5 m x 7.5 m building as a garage. Might other dimensions (p = 30 m) be more appropriate?

Extension to the Pythagorean problem on page 268. Does the Pythagorean Relationship hold in the situation below?

Answer: Yes.



For more **Problems** see: Problem Solving for Mathematics, p. 13, #4; p. 21, #7.12; p. 24, #8.1; and p. 45, #11. and Problem Solving in Mathematics: Focus for the Future, pp. 67, 68, #7, 20 (extension).

RESOURCE CORRELATION AM: pp. 144–145, 266–273

CONCEPT 4

Students will be expected to demonstrate an understanding that volume is the number of unit cubes which are within a geometric solid.

Students will be expected to:

4.1 calculate the volume of rectangular prisms and cylinders by using formulas
  4.1.1 solve problems which involve calculating the volume of rectangular prisms and cylinders.

COMMENTS

Students are expected to use formulas for the volume of rectangular prisms and cylinders in calculations and in problem solving.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Use the science lab to measure the dimensions of prisms and cylinders and then measure the amount of liquid that the polyhedron will hold. Does this amount equal the calculation using a formula? Have students identify different prisms and cylinders that have equal volumes and examine the dimensions.

Problem: Mr. Brook wants to build a container with a volume of 64 cm^3. He can build it either in the shape of a cube or of a rectangular prism. The rectangular prism would have to be 8 cm high and 4 cm wide. Which would have the smallest surface area, and by how much?

Answer: Cube, 16 m^2.

From Problem Solving Challenge for Mathematics

Problem: A rectangular box has volume 15 cm^3. If the length, width and height of the box are doubled, then what is the resulting volume?

Answer: 120 cm^3.

From Problem Solving Challenge for Mathematics

TECHNOLOGY INTEGRATION

Software:
SRA Computer Drill and Instruction, Mathematics: Level D (rev. ed.), "Measurement" section. Selected lessons may be used for drill.

Video:
Math Ways: "Volume" Develops concept of volume and formulas for determining volume of rectangular prisms and cylinders. Also derives formulas for calculating volume of triangular prisms and cones (elective).

Math Wise: "Measuring 02 - Formulas." Plus Teacher's Guide discussion and student worksheet. Provides everyday application of area and volume. (Distributed by ACCESS)

ELECTIVE SUGGESTIONS

Extend the study of volume to cones, pyramids and spheres.

Activity: Have students arrange five or six commonly used containers of various volumes in order of increasing capacity. Check the estimates on a pairwise basis, or by measuring.

Activity: Have students contact a local grocer to find out the number of cans of pop the store sells in a week. Estimate the number of cans of pop the store would sell in a year. How long will it take to sell one million cans? Determine the volume occupied by one million empty cans of pop. What size of room would they fill? If the cans were placed side by side in one layer, how much floor space would they cover? Suppose the empty cans were flattened. What space would the one million cans now occupy? Estimate using a small sample.

RESOURCE CORRELATION AM: pp. 288–289, 292–293

CONCEPT 5

Students will be expected to demonstrate an understanding that time can be measured in seconds, minutes, hours, days, weeks, months and years.

Students will be expected to:

5.1 change units of time from one form to another
5.2 tell time using a 24-hour clock
5.3 write times using proper 24-hour notation
5.4 perform calculations using units of time.

COMMENTS

In many respects students recognize that "time" not only includes a watch, but that there are other measurement units. It is important to reinforce the interrelationship between seconds and minutes and hours etc.

PROCESS/PROBLEM-SOLVING CONTEXT

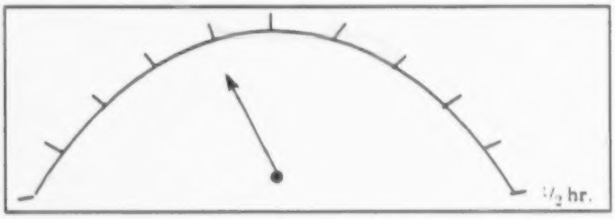
Activity: Have students identify (or suggest) why the 24–hour clock is used. This would provide an opportunity for students to investigate this in the library.

TECHNOLOGY INTEGRATION

Software:
SRA Computer Drill and Instruction, Mathematics: Level D, "Measurement" section. Selected lessons may be used for drill.

ELECTIVE SUGGESTIONS

How much time is left on this parking meter?



Answer: 12 minutes.

RESOURCE CORRELATION AM: pp. 174 – 175, 186–187

CONCEPT 6

Students will be expected to demonstrate an understanding that temperature can be measured in degrees.

Students will be expected to:

6.1 measure temperatures using the Celsius scale
6.2 perform calculations involving the use of temperature measured in Celsius.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students estimate and then actually measure the temperature of rooms and locations around the school. This could also be done in a lab environment where students are asked to estimate and then measure the temperatures of different liquids.

RESOURCE CORRELATION AM: pp. 172 – 173.

Mathematics 14
Integration of Numeration Concepts and Skills into Algebra and Graphing

The numeration concepts and skills identified in the shaded area represent those skills that students should use, reinforce, or learn within this unit of study. A detailed description of each skill can be found on pages 223–225.

	Ratio and Proportion	Statistics and Probability	Geometry	Measurement	Algebra and Graphing
Mental	1.1, 1.2, 1.4, 1.5, 1.6, 1.7, 1.8	1.1, 1.4, 1.5,	1.1, 1.2, 1.3, 1.4, 1.5	1.1, 1.2, 1.3, 1.4, 1.5	**1.1, 1.2, 1.3, 1.4, 1.5, 1.8**
Estimation	2.2, 2.3, 3.1, 3.2	2.1, 2.2, 2.3 3.1, 3.2	2.1, 2.2, 2.3, 3.1, 3.2	2.1, 2.2, 2.3, 3.1, 3.2	**2.1, 2.2, 2.3, 3.1, 3.2**
Technology	1.15, 1.18	1.18	1.16, 1.17	1.16, 1.17, 1.18	**1.16, 1.17, 1.18**
Paper and Pencil	1.9, 1.10, 1.11, 1.12, 1.14	1.9, 1.10, 1.11	1.9, 1.10, 1.11, 1.13, 1.14	1.9, 1.10, 1.11, 1.13, 1.14	**1.9, 1.10, 1.11, 1.13, 1.14**

CONCEPT 1

Students will be expected to demonstrate an understanding that algebraic expressions can be used to represent situations where one or more values can vary.

Students will be expected to:

1.1 write first degree algebraic expressions to illustrate simple mathematical situations with a maximum of 2 variables

1.2 evaluate algebraic expressions for replacement values of the variables

  1.2.1 solve problems which involve the evaluation of expressions for values of the variables

1.3 solve first degree equations in one variable through the use of guess-and-test techniques, concrete manipulatives and formal methods (limit: $x + a = b$, $ax = b$, $ax + b = c$, $^x/_a = b$)

  1.3.1 solve problems which can be represented by first degree equations in one variable

  1.3.2 use calculators to aid in the solution of equations.

COMMENTS

It is expected that students will be able to write and evaluate first degree algebraic expressions to illustrate simple mathematical situations (limit: two variables). This will lead to solving first degree equations in one variable through "guess and test," concrete manipulatives and formal methods. It is not the intention that this concept be strictly dealt with at the abstract level, students should have concrete experiences with which to relate to the manipulation of algebraic symbols.

A Discussion of Variable

"*The meaning of variable is variable*; using the term differently in different contexts can make it hard for students to understand. As teachers, we should be sensitive to this multiple usage." (p. 425).

Complete the following activity yourself, and then have the students do it. Discuss the list generated with the class.

"Complete the following sentence: A variable is a __________. Use *the one* English word that, in your opinion, best captures the meaning of the term *variable*. (If you generate a number of possible candidates, please list them in order of preference.)" (p. 420).

Mere synonyms are not acceptable; try to get to the heart of the concept. Some of the words in the list may be better than others, but there is no *one right* word for "variable."

"Here are three suggestions related to teaching the concept of variable...

1. Variables are tools for expressing mathematical generalizations. It should help, therefore, if students had the habit of verbalizing such generalizations before they were asked to formalize those generalizations using the

language of mathematics. It might be useful to make a habit of asking students to summarize some of their observations about arithmetic in their own words, in short exercises. ("What happens when you add three times a number to two times that number?" or "Can you say whether the sum of two odd numbers will necessarily be even or odd?") By asking students to observe patterns and to summarize them verbally, we may help make the transition from arithmetic to algebra."

2. The dynamic aspects of the variable concept should be stressed whenever it is appropriate and feasible. At the simplest level, students can be led to observe that the cost (variable) that appears on a gasoline pump is a (dynamic, linear) function of the amount of gasoline that emerges from the pump. Other dynamic relationships include time-dependent phenomena, for example, the temperature in a room at time *t*, the distance an object falls in *t* seconds, and so on."

3. Much of the algebra curriculum consists of solutions of equations independent of the context from which they were drawn. Many applications of algebra, for example, the standard word problems, are so neatly packaged, and taught in separate units, that students miss the point—that the language of algebra is a powerful means of capturing the mathematical essence of a wide variety of situations and not just the ones they've been taught to solve. We recommend that students be exposed to a wide range of problems for which algebra, especially in its role of a tool for mathematical generalization, can be useful." (pp. 425–426)

Reprinted with permission from "On the Meaning of Variable," Mathematics Teacher, September 1988 copyright by the National Council of Teachers of Mathematics.

For more examinations of the concept of variable, see "What Are These Things Called Variables?", Mathematics Teacher, October 1983 and "Children's Difficulties Beginning Algebra," The Ideas of Algebra, K–12.

For discussions on teaching algebraic expressions and the concept of equation in meaningful ways, see "Constructing Meaning for the Concept of Equation," Mathematics Teacher, November 1980, and "Teaching Algebraic Expressions in a Meaningful Way," The Ideas of Algebra, K–12.

For a discussion (with illustrative examples) of fostering greater understanding of algebraic concepts through the use of diagrams, see, "Developing Algebraic Representation Using Diagrams," in The Ideas of Algebra, K–12.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: To have students "experience" variable, examples such as: given a group of rulers, their lengths can vary; given a bag of marbles, the number of marbles in the bag can vary; Sally deposited money in her bank, the amount of money can vary; will help to develop an understanding of variable. This experience should lead into a mathematical representation of a situation.

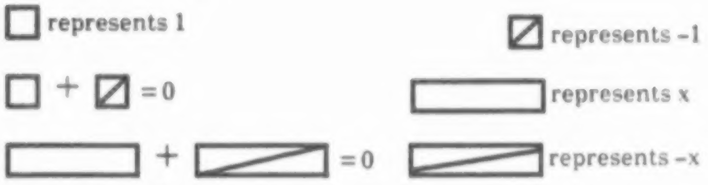
Once an algebraic representation has occurred, have students create their own situations when given an algebraic expression; e.g., given 5x + 3, verbally create a situation or write a situation; 5x + 3 could represent that I bought five hot dogs at a certain amount per hot dog and spent $3 on a T-shirt.

Activity: Evaluation of algebraic expressions can be expanded from here. Have students discuss what happens to the total amount of money spent when the price of hot dogs varies; e.g., if hot dogs were $4 each, how much did I spend; if hot dogs were $1.50 each, how much did I spend.

When solving equations, students must be first exposed to the concept of equality. This can be demonstrated by showing "equal" number of marbles per person, equal amount of weight on each side of a balance.

Activity: Concrete
In an attempt to create a concrete experience in solving equations a form of concrete manipulation is encouraged. One such form is as follows: (It is important to note that this form of concrete manipulation is difficult to realize in some circumstances, hence it is hoped that by the time those cases occur, students will have reached the abstract stage and the algorithm can be used to solve the equations.)

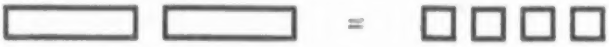
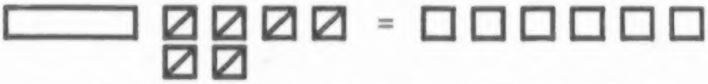
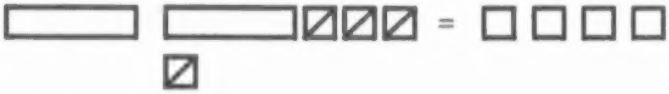
Assumed knowledge:



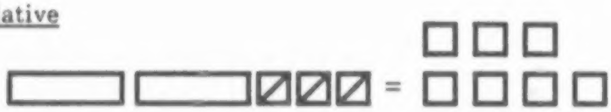
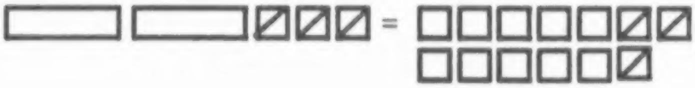
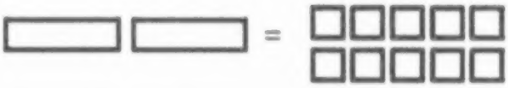
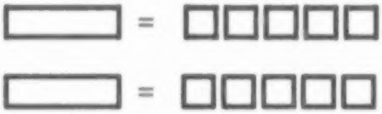
"Game Tactics":

1. What you do to one side of the equation, you must do to the other side.
2. Must use an opposite symbol (e.g., □ is opposite to ⧄) to "remove" the representation.
3. Ultimately, as this is to solve for the variable, the steps that are taken must be to ensure that all of the variables and numbers are on opposite sides of the equation.
4. As each step is performed, you must write down the mathematical statement that illustrates the action taken.

In using concrete manipulatives, students should have experience in "building" equations before solving equations.

Build	Concrete Manipulatives
2x = 4	
x–6 = 7	
2x–3 = 4	

When solving equations; e.g.,
1) solve for x in 2x–3 = 7

Concrete Manipulative		Algebraic Representation
Step 1:		2x–3 = 7
Step 2 "take away –3 from each side"		2x–3–(–3) = 7–(–3)
Step 3 "left with"		2x = 10
Step 4 "divide each side into two parts"		x = 5

e.g. 2) solve for x in $x/_4 = 7$. This is difficult to illustrate, hence, no attempt will be made to do so.

Following the concrete experience in solving equations, students should work from the 5x + 3 example and add the equality; e.g. If I spent a total of $13 and bought five hot dogs and spent $3 on a T–shirt, then the total amount of money I spent can be represented by 5x + 3 = 13.

For more information see "Solving Linear Equations Physically," in Activities for Implementing Curricular Themes from 'The Agenda for Action' for physical and pictorial approaches to solving equations.

TECHNOLOGY INTEGRATION

Software:

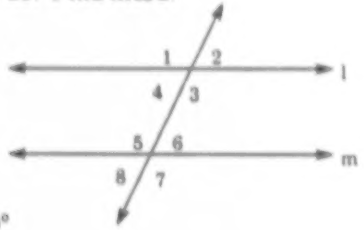
Equations (Mindscape)
Suitable for structured practice and remedial activity in solving equations of the form $Ax + B = C$.

SRA Computer Drill and Instruction, Mathematics: Level D (rev. ed.), " Pre-Algebra" section. Selected lessons may be used for drill in solving equations of the form $Ax + B = C$.

ELECTIVE SUGGESTIONS

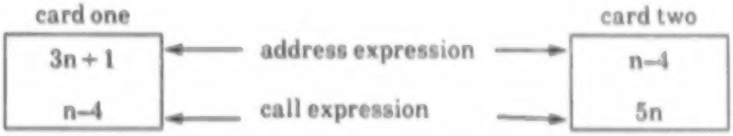
Introduce equations of the form $ax + b = cx + d$

Problem: $l \parallel m$, $m\measuredangle 4 = 8x + 4$, and $m\measuredangle 8 = 5x + 25$. Find $m\measuredangle 2$.



Answer: 60°

Activity: Class activity designed to help develop elementary algebra skills, and listening skills: The following activity is similar to the "I have a Number. . ." activity described on page 137. Divide the class into two groups of students, and with a deck of index cards create a set of "algebraic expression" cards for each group. On the front of each card write an address expression and a call expression. For example:



Each set of cards forms a closed loop. That is, the card that follows card one above will have n–4 as its address expression. If its call expression is 5n, then the card following will have 5n as its address expression. The call expression of the last card in the set is also the address expression of the first card.

A student starts by announcing, in English, his or her address and call expression. Using the above example, the first student would say, "I have one more than three times a number. Who has four less than a number?" The student with the second card shown would follow in a similar manner. Thus, each student has to (a) translate a sentence that is heard into an algebraic expression, and (b) translate an algebraic expression into a spoken sentence.

Variations include:

1) A mixture of algebraic expressions and English sentences for the call and address expressions.

2) Multiple responses on some cards from which a student may choose. Thus some students may be called on more than once. Set rules to avoid "inner loops."

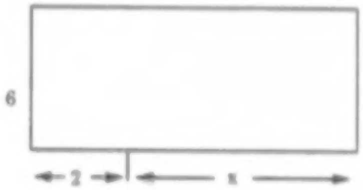
3) Encourage students to create equivalent English sentences for the same algebraic expression. For example the call expression on card two could also be described as "Who has 10 times a number divided by two?"

4) Create more complex expressions, such as $5(x-1)$.

Adapted with permission from, "I have . . . Who has . . ?" Mathematics Teacher, October 1980, copyright by The National Council of Teachers of Mathematics.

See Problem Solving Challenge for Mathematics, p. 28, #8.10 and p. 31, #9.2.

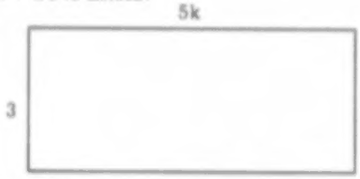
Activity: Provide students with activities of the forms (a) What is the perimeter and area of the following figure?



Answer: $P = 16 + 2x$ units; $A = 12 + 6x$ square units

(b) Draw a rectangle that has an area of 15 k square units and a perimeter of 6 + 10 k units.

Answer:



These activities help strengthen understanding in both measurement and algebra, and help develop problem-solving skills.

Problem: To swim one kilometre in a certain rectangular swimming pool, one must either swim the long length 80 times or negotiate the perimeter of the pool 12.5 times. What is the number of square metres in the area of the swimming pool?

Answer: 375 m^2.

RESOURCE CORRELATION AM: pp. 324–325, 331–339, 342 – 345, 348–349, 355–360, 369–371, extension 361–362

CONCEPT 2

Students will be expected to demonstrate an understanding that two variables can be related and that those relationships can be expressed by sets of ordered pairs, equations and graphs.

Students will be expected to:

- 2.1 describe verbally and in writing the relationships between two variables
  - 2.1.1 determine the algebraic relationship between two variables by informal means such as guess-and-test
- 2.2 plot points on a coordinate plane
- 2.3 plot the points and sketch the graphs which describe the relationship between two variables
- 2.4 construct tables of values which satisfy given linear equations in two variables
- 2.5 sketch graphs from tables of values which satisfy a given linear equation in two variables
- 2.6 read and interpret the graphs which describe the relationship between two variables
  - 2.6.1 analyze graphs which describe the relationship between physical variables by the use of interpolation and extrapolation techniques.

COMMENTS

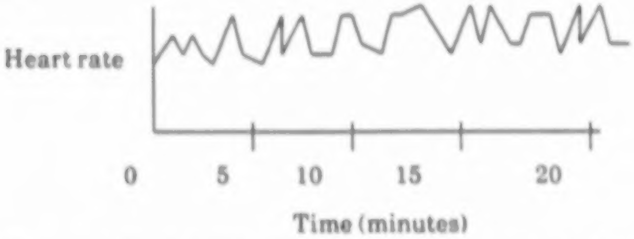
This concept is intended to develop an awareness, among students, of the mathematical representations of relationships between variables. Students should build these relationships based on their experiences and extend it to the abstract. Students are expected to plot points and sketch graphs that describe the relationship between two variables. Given a linear equation, students are expected to construct a table of values and sketch the graph from the table of values. It follows from this, that students are expected to read and interpret the graphs that describe relationships and that they should be able to verbalize the relationships.

Students should draw from their previous experience things that are related, for instance the number of hours worked and the amount of the paycheque to grasp an idea of the concept relationship.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Before having students plot points and sketch graphs of relationships, the student should have plenty of opportunity to read graphs of actual events in such a manner that it describes something that they may have experienced or could imagine the experience; e.g., they read a graph that describes the speed of a car as it is traveling up a hill, and then write the description of the graph.

The following graph describes the heartbeat of a hockey player during the third period of the final game of a championship series.



Can you tell what this player did during the period? When did he play? How much did he play? Was this player in good shape? What could have occurred around the 3 minute mark? the 12 minute mark? Do you think that this player was on the winning team? Why?

Activity: From reading graphs, students should practise sketching graphs that describe relationships; e.g., graph the feeling of hunger over a period (a day); a graph that describes the noise level of the seventh game of the Stanley Cup final, from the singing of the national anthem to the end of the game; given a glass of water (at room temperature) that an ice cube has been dropped into, draw a graph that describes the temperature of the water of time.

Activity: This leads to further discussions regarding a set of points helping to sketch the graph. Have students identify "points" that helped them sketch their graphs.

Activity: This experience of identifying points will help students to relate the graphing to the coordinate plane. In an introduction to the coordinate plane, have students rate their hunger, for instance, give it a number value – this will assist students in making the link. Practice in finding and naming points on the coordinate plane is available by playing "Battleships," creating pictures or puzzles and having classmates discover the picture.

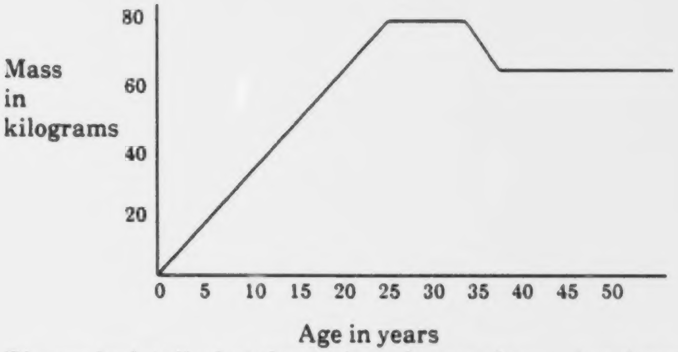
Activity: Graphs presented to the student can eventually reach a straight line stage and, after students have had the opportunity to read information from these graphs, discuss how, in mathematics, we give these lines names. Initially, the "equation" of the line could be the D = st, for instance, as they may have seen this formula before... the equation describing the line. It will be important then for students to be able to read points from these graphs and set up a table of values...from this point a discussion regarding the "rule" to describe the relationship between the numbers can be used. From this point we can then take an equation of a line and ask students to plot the line. The students have some previous experience and understanding of both what the line represents, and the relationship between two variables. Combined with their experience with graphs, this background will assist their understanding of constructing tables of values and graphs.

PROCESS/PROBLEM-SOLVING CONTEXT

With an understanding of relationships, students should now be able to work at guessing the relationship between two variables: e.g., given a table of values, guess how the second column relates to the first.

Activities: The graph illustrates a person's mass from birth to 50 years of age. Make up a story to explain the graph. Include in your story—

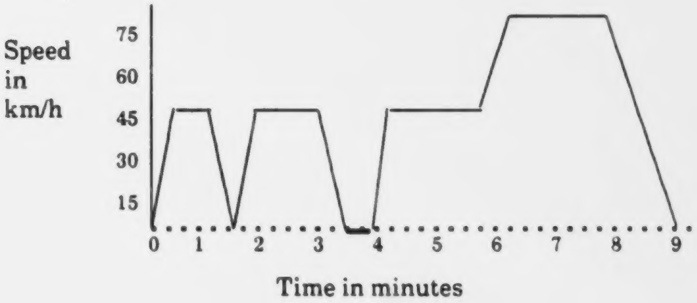
1. whether the person is a man or a woman.
2. about how much that person's mass was at birth,
3. what the person's maximum mass was, and
4. whether the person dieted sometime and, if yes, when.



Discuss in detail what the students have written. Are there misinterpretations that need to be clarified?

A man is driving from home to work. The graph shows his speed at any time during his nine-minute trip. Make up a story about his trip to explain the graph. In your story tell—

1. approximately how far he lives from where he works,
2. what happened one and one-half minutes after he started.
3. what happened between three and one-half minutes after he started and four minutes after he started, and
4. something observed about speed limits.



Discuss in detail what the students have written.

Reprinted with permission from, "Makeup a Story to Explain the Graph," Mathematics Teacher, January 1984, copyright by The National Council of Teachers of Mathematics.

See "Getting the Point!" In Activities from the 'Mathematics Teacher' for a motivational activity designed to give students experience in plotting point, and "Using Linear Functions" in Mathematics Teacher, October 1988 for a series of activities designed to provide students with practice in plotting points and sketching and interpreting graphs in the context of electrician fees.

TECHNOLOGY INTEGRATION

There are several computer programs available for sketching and drawing graphs. If you have some available, it is strongly advised that students have an opportunity to use them, following the building of understanding.

Software:
Graphing Linear Functions (Mindscape), "Easier forms." Students determine and graph three sets of ordered pairs that satisfy the given linear equation.

SRA Computer Drill and Instruction, Mathematics. Level D (rev. ed.) "Pre-Algebra" section. Selected lessons may be used for drill in plotting ordered pairs and creating a table of values for a given equation.

Other graphing programs, such as Master Grapher, (Addison-Wesley, computer Graphing Experiments I (Addison-Wesley), Equation Math (MECC) and Green Globs and Graphing Equations (Sunburst) may be used to graph given linear and non-linear functions and relations.

For an example of how spreadsheets can be used to help develop students' ability to interpret relationships between two variables, see "Integrating Spreadsheets into the Mathematics Classroom," Mathematics Teacher, November 1988. Apple Works and Microsoft Works are Alberta Education Basic Resources.

ELECTIVE SUGGESTIONS

Discussion of "equations of lines" in the formal sense; e.g., slopes of lines, the slope - y - intercept form of graphing.

RESOURCE CORRELATION

AM: pp. 377-381, 397-407, 412-413

REVERSE TEXTUAL CORRELATION
MERRILL PUBLISHING COMPANY
APPLICATIONS OF MATHEMATICS
(Canadian Edition)

Page	Unit	Learner Expectation Number	TRM Page
2–24	Charts and Tables		
25–27	Statistics and Probability	1.1., 1.2, 1.3	235–239
30–33	Statistics and Probability	1.3, 1.6	235–239
36–41	Statistics and Probability	1.6	235–239
47	Ratio and Proportion	1	226–227
48–49	Ratio and Proportion	2	228–229
50–51	Ratio and Proportion	3	230–231
52–53	Scale Drawings		
56–67	Ratio and Proportion	4	232–234
73–81	Statistics and Probability	1.4	235–239
83–89	Statistics and Probability	1.4, 1.5	235–239
94–101	Statistics and Probability	2	240–241
104–111	Independent and Dependent Events, Adding and Estimating Probability		
117–123	Statistics and Probability	2	240–241
126–127	Statistics: Estimation		
128–129	Probability: Expected Winnings		
130–131	Using Venn Diagrams		
139–143	Measurement	1.1, 1.2, 1.3	260–262
144–145	Measurement	3.1	266–271
148–149	Changing Area Units		
152–153	Measurement	1.2	260–262
161–166	Measurement	1.1, 1.2, 1.3	260–262
168–169	Old Units of Measurements		
172–173	Measurement	6	275
174–175	Measurement	5.1, 5.4	274
183–185	Reading Measures: Odometers and Maps		
186–187	Measurement	5	274
188–195	Reading Meters		
196–197	Geometry	2.2	252–253
205–206	Geometry: Basic Terms		
207	Geometry: Right, Acute and Obtuse Angles		
208–211	Geometry	5.1, 5.2	258–259
212–215	Geometry	1.2, 1.5	242–251
218–220	Geometry	4	256–257
221–222	Geometry	1.3.1	242–251
224–225	Conditional Statements		
226–227	Deductive and Inductive Reasoning		
233	Congruent Figures: Definitions		
234–239	Geometry	2.1, 2.2, 2.3	252–253
242–243	Geometry	2.3	252–253
244–245	Constructing Parallel Lines		
246–249	Geometry	3.2 extension	254–255
255–257	Geometry	1.1, 1.2	242–251
258–259	Geometry	3.1, 3.1.1	254–255
260–263	Measurement	2	263–265
262	Geometry	1.5	242–251
266–273	Measurement	3	266–271
281–282, 284, 286	Geometry	1.4	242–251
288–289	Measurement	4	272–273
292–293	Measurement	4	272–273
294–295	Volume of Pyramids and Cones		
296	Geometry	1.4	242–251
296–297	Surface Area and Volume of Spheres		
304–323	Rational Numbers		
324–325	Algebra and Graphing	1.1, 1.3.1, 1.3.2	276–282
331–339	Algebra and Graphing	1.1, 1.3	276–282
342–345	Algebra and Graphing	1.1, 1.3	276–282
346–347	Solving Inequalities		
348–349	Algebra and Graphing	1.1, 1.3.1, 1.3.2	276–282
355–362	Algebra and Graphing	1.2	276–282
365–366	Squares and Square Roots		
367–368	Geometry	4	256–275

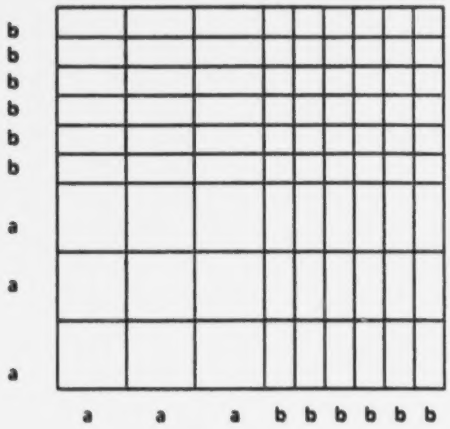
291

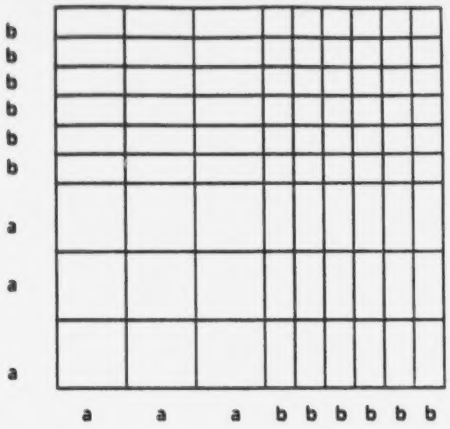
REVERSE TEXTUAL CORRELATION
MERRILL PUBLISHING COMPANY
APPLICATIONS OF MATHEMATICS
(Canadian Edition) (Cont'd)

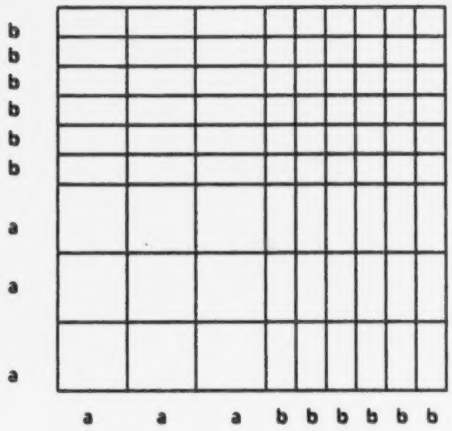
Page	Unit	Learner Expectation Number	TRM Page
369–371	Algebra and Graphing	1.2	276–282
377–381	Algebra and Graphing	2.1	283–286
382–383	Arithmetic Sequences		
386–387	Geometric Sequences		
388–389	Scientific Notation		
397–407	Algebra and Graphing	2	283–286
408–411	Transformation Geometry		
412–413	Algebra and Graphing	2	283–286
419–461	Appendix: Basic Skills Review		
462–464	Tables		
465–472	Glossary		

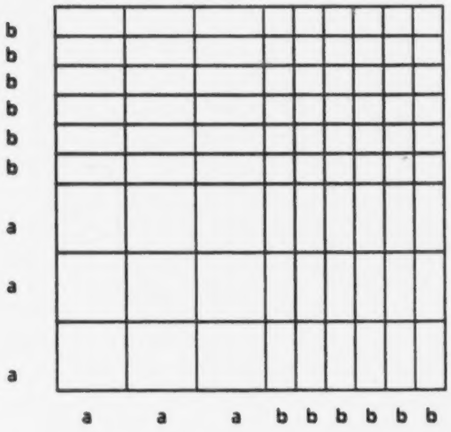
APPENDIX A

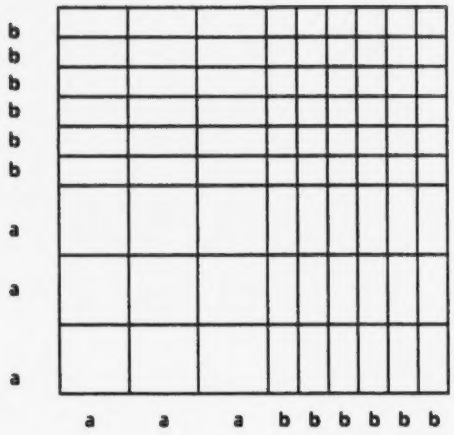
TEMPLATE: BINOMIAL GRID

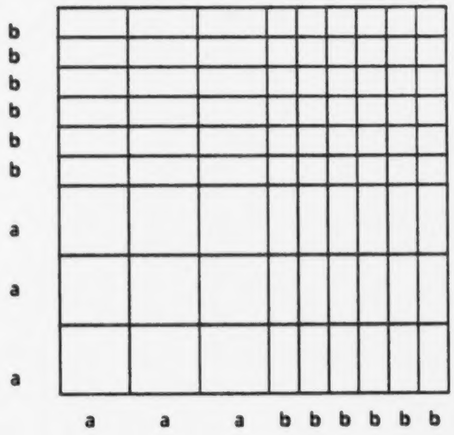

b
b
b
b
b
b
a
a
a
a a a b b b b b b


b
b
b
b
b
b
a
a
a
a a a b b b b b b

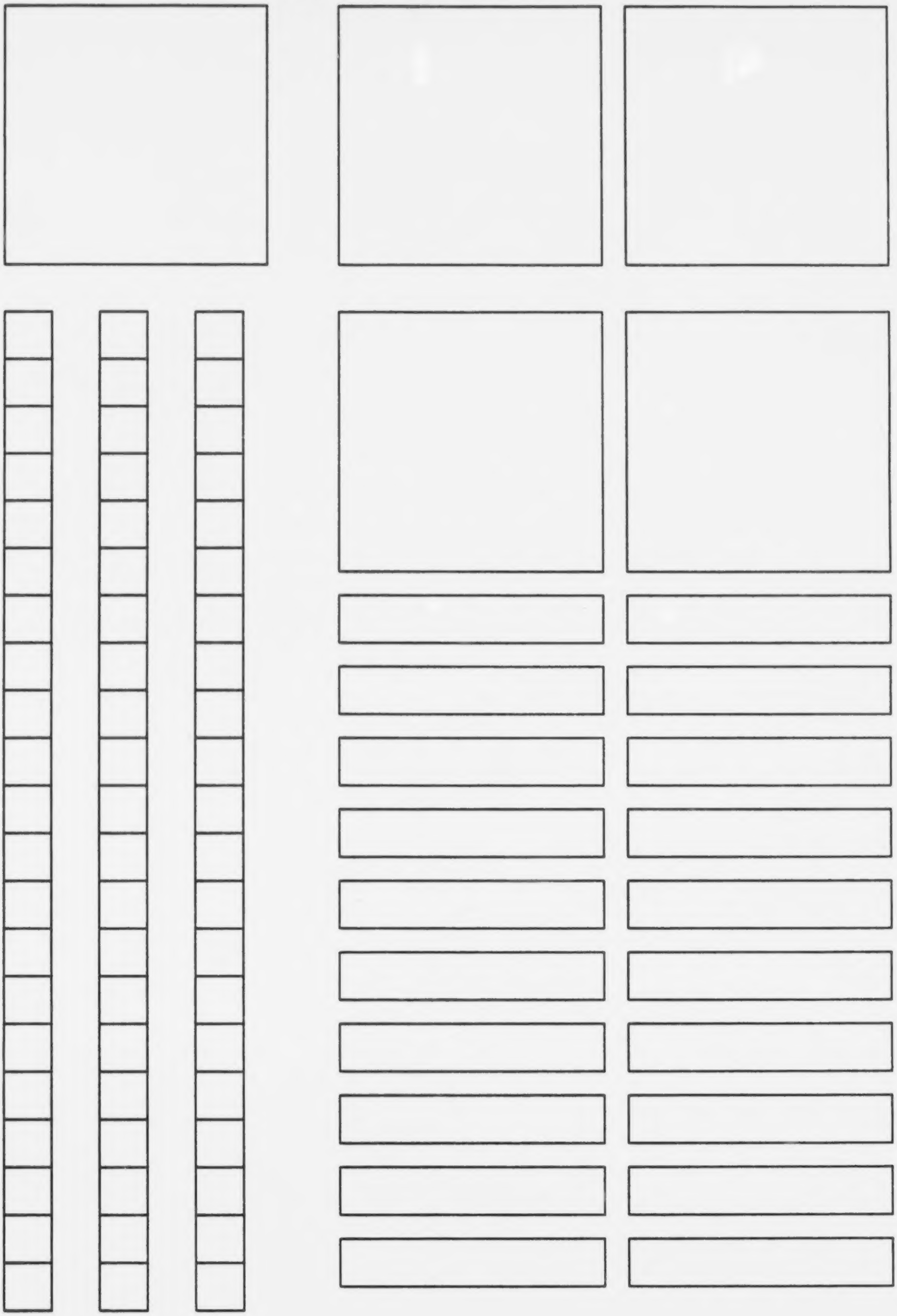

b
b
b
b
b
b
a
a
a
a a a b b b b b b


b
b
b
b
b
b
a
a
a
a a a b b b b b b


b
b
b
b
b
b
a
a
a
a a a b b b b b b


b
b
b
b
b
b
a
a
a
a a a b b b b b b

TEMPLATE: ALGEBRA TILES



APPENDIX B

RECOMMENDED RESOURCES

(A) Print

Activities for Implementing Curricular Themes from the 'Agenda for Action'. (Edited by C.R. Hirsch). Reston, VA: National Council of Teachers of Mathematics. ISBN: 0-87353-229-5. (Student) Provides activities in manipulatives, problem solving, "expanded" basic skills (e.g., exploratory data analysis, estimation), and calculator and computer use. See for example:

"Calculators and Estimation" (Goodman, T.), pp. 116 - 119. Activities promote use of calculator in aiding development of estimation skills.

"Developing Estimation Strategies" (Rubenstein, R.N.), pp. 76-82. Provides activities in estimating fraction, decimal and percent operations.

"Extending Problem-Solving Skills" (Laing, R.A.), pp. 20-24. Describes activities that require various problem-solving strategies.

"Finding Factors Physically" (Hirsch, C.R.), pp. 183-189. Describes manipulative approach using tiles to factor trinomials.

"Microcomputer Unit: Graphing Straight Lines" (Hastings, E.H. et al.), pp. 144-148. Provides computer program listing and activities to investigate effects of m and b in equation $y = mx + b$.

"Out of Thin Air" (Schell, V.), pp. 200 - 204. Describes experimental probability activities using thumbtack simulations.

"Percent and the Hand Calculator" (Coburn, T.G.), pp. 120-126. Activities promote use of calculator in aiding development of percent calculation estimation skills.

"Solving Linear Equations Physically" (Kinach, B.), pp. 176 - 182. Describes manipulative and pictorial approaches to solving simple equations.

"Stem-and-Leaf Plots" (Landwehr, J.M. et al.), pp. 45-51. Provides activities constructing stem-and-leaf plots.

"Teaching Problem-Solving Skills" (Schaaf, O.F.), pp. 4-9. Describes activities that require various problem-solving strategies.

"Using Calculators to Fill Your Table" (Channell, D.E.), pp. 127 - 131. Activities promote use of calculator in determining successive approximations in maximum area problem.

Activities for Junior High School and Middle School Mathematics: Readings from the 'Arithmetic Teacher' and the 'Mathematics Teacher'. (Edited by K.E. Easterday, et al.) Simpson. Reston, VA: National Council of Teachers of mathematics, 1981. ISBN: 0-87353-188-4 (Student) Provides activities in a wide range of mathematics topics. See for example:

"A Concrete Approach to Integer Addition and Subtraction" (Bennett, A. B. Jr. et al.), pp. 24 - 27, 172. Describes a minipulative approach to integer (+ and –) arithmetic.

"Discoveries in Geometry by Folding and Cutting" (Edwards, R. R.), pp. 120 - 122. Describes activities with paper folding to explore triangular, quadrilateral and other polygon relationships.

"Flow Charts for Equation Solving and Maintenance of Skills" (Morrow, L. J.), pp. 31 – 38. Title self-explanatory. At grade 10 level, provides remedial alternative approach.

"Four Television Games for Your Classroom" (Vrbka, M. M.,) pp. 62 – 66. Presents games that provide practice in mental arithmetic, fractions, percent, decimals, proportions, measurement and geometry.

"How High is a Flagpole?" (Hunt, J.D.), pp. 151-152. Describes a number of outdoor activities using similar triangle principles to find height of flagpole.

"Metric Paper" (Kissane, B. V.,), pp. 84 – 86. Folding standard international metric-sized paper produces similar rectangles.

"Problem Solving in the General Mathematics Classroom" (Troutman, A. P. et al.), pp. 211 – 218. Describes geometry-related problem solving activities.

"Quantifying Chance" (Souviney, R. J.), pp. 90 – 92. Describes probability activities using non-standard dice, marbles and alphabet (letter frequency).

"Reinforce Division by Learning Ratios" (Arnold, W. R.), pp. 74 – 76., Applies concept of ratio to measurements of circular objects, squares, rectangles. Interesting discussion and activity related to Greek concept of "beautiful" (similar) rectangles.

"Simulating Problem Solving and Classroom Settings" (Liedtke, W. et al.), pp. 194–197. Describes problem of making box of largest volume out of given sheet of cardboard. Interest is in possible strategies for solving: model, diagram, table, graph, successive approximations, equation.

"The Area of a Parallelogram IS the Product of its Sides" (Lepowsky, W. L.), pp. 123 – 125. Describes area discovery activities using square units (rectangles) and rhombus units (parallelograms;) compare and contrast.

"The Four Operations: Wrapping It Up" (Temple, R.J.), pp. 59–60. Presents alternative drill approach to fractions and decimals.

"The Percent Game" (Pascale, M.), pp. 67–68. Presents a card game to practise fraction–decimal–percent equivalencies.

"Using a Game as a Teaching Device" (Brumfiel, C.), pp. 166–171. Describes "Do you know my number?" game involving algebra and logical reasoning.

Algebra Tiles for the Overhead Projector (Howden, H.). New Rochelle, NY: Cuisenaire, 1985. ISBN: 0–914040–42–1 (Teacher) Describes using tiles on the overhead projector to demonstrate polynomials; the zero principle; adding, subtracting and multiplying polynomials; and factoring polynomials.

Exploring Data. (Landwehr, J.M. et al.) Palo Alto, CAB: Dale Seymour, 1986. ISBN: 0–86651–321–3. (Student) Provides instruction and application problems in stem-and-leaf plots, box-and-whisker plots, scatter plots and line fitting and smoothing.

298

How to Evaluate Progress in Problem Solving. (Charles, R. et al.) Reston, VA: National Council of Teachers of Mathematics, 1987. ISBN: 0-87353-241-4 (Teacher) Discusses goals of problem solving evaluation, evaluation techniques, organizing and managing an evaluation program, and using evaluation results.

Mathematics Dictionary (4th ed.) (James and James.) New York: Van Nostrand Reinhold, 1976. ISBN: 0-442-24091-0 (Teacher) Provides authoritative definitions of mathematics terms.

Problem Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985. (Teacher) Produced as junior high school resource. Provides discussion of the nature of problem solving, strategies for teaching problem solving, a bank of problems, methods of evaluating problem solving performance and a 36-item problem solving bibliography.

Problem Solving in Mathematics: Focus for the Future. Edmonton: Alberta Education, 1987 (Teacher) Produced as senior high school resource. Provides discussion of the nature of problem solving, strategies for teaching problem solving, project and computer related problems, a bank of mathematics problems, methods of evaluating problem solving performance and a 123-item problem solving bibliography.

The Language of Graphs: A Collection of Teaching Materials. (draft). (Swan, M.) Nottingham England: Shell Centre for Mathematical Education, University of Nottingham, n.d. ISBN: 0-906126-11-8 (student) Provides a wide range of graphing activities with emphasis on interpreting for meaning. Topics include modeling and interpreting with points, modeling situations, understanding distance-time graphs and interpreting complex graphs.

(B) Video

Math Ways #1: Available through (a) ACCESS NETWORK, Media Resource Centre, 295 Midpark Way S.E., Calgary, T2X 2A8, 256-1100 in Calgary or 1-800-352-8293, toll free, outside of Calgary and (b) the Regional Resource Centre or Urban Media Centre in your area. Fifteen-minute videos on various topics. Primarily animation, with voice-over.

The Percent: Focus is on understanding the concept of percent and determining what a given percent of a number or quantity represents. Some conversion of fractions and decimals to percent described. A brief closing summary is provided.

Areas: Figures discussed are the circle and cylinder. First derives π as the relationship between circumference and diameter, using examples. Area then introduced by deriving the formula $A = \pi r^2$, followed by area calculation examples. Discussion of surface area of a cylinder follows.

Volumes: Discusses volume of prisms, cylinders and cones. Introduces cubic unit of measure concept with concrete objects. Examines volume of rectangular and triangular prisms, then cyclinders, emphasizing "V = area of X height." Provides detailed examples of calculation. Derives volume of cone from that of cylinder, using concrete demonstration.

Math Wise. Bloomington, IN: Agency for Instructional Television, 1981 (Available through ACCESS NETWORK, Media Resources Centre, 295 Midpark Way S.E., Calgary, Alberta, T2X 2A8. 256-1100 in Calgary and 1-800-352-8293, toll free, outside of Calgary. (Student) A series of 15-minute video vignettes dealing with a number of mathematics topics in the context of contemporary everyday life situations involving teenagers. In the course of these daily activities the actors use mathematics; the videos are not mathematics "lessons." A hostess introduces the mathematics topic of each video and closes by reviewing the mathematically significant points in each of the "stories." A teacher's guide that provides ideas for discussion and a student worksheet for each of the videos are included. Entertaining as well as informative; well done.

The following Math Wise videotapes have been referenced in the Mathematics 10/13/14 Teacher Resource Manual.

"Measuring 01: Measuring Instruments."
"Measuring 02: Formulas."
"Comparing 01: Numerical Comparisons."
"Comparing 02: Percent."
"Comparing 03: Finding a Common Unit."
"Comparing 04: Proportions."
"Locating/Interpreting 01: Organizing Information."
"Locating/Interpreting 02: Graphs."
"Predicting 01: Probability."
"Predicting 02: Choosing a Sample."
"Predicting 03: Averages."

(C) Computer Software

Computer Graphing Experiments I: Addison-Wesley, (Apple II) Capable of graphing linear equations, parabola exponential-logarithm functions, inverse functions special functions such as absolute value and polynomials. "Linear Equations" option provides following sub-options: graphs of the form $y = mx$ and $y = mx + b$, and simultaneous equations. Students specify equation by inputting m and b. Extensive student worksheets provide activities in investigating effects of ma and b, parallel and perpendicular lines, writing equation in $y = mx + b$ form given $Ax + By + C = 0$ form, two points or point-slope. Point of intersection of system of equations identified as ordered pair. Graphs of straight lines slightly ragged, appearance of perpendicularity problematic, no grid. Suitable for teacher demonstration, small group and individual student use. Teacher's guide with extensive set of student activity worksheets.

Master Grapher (Waits, B. et al.) Reading, Mars: Addison-Wesley, 1988. Apple II, IBM PC or PS/2, Macintosh . ISBN: 0-201-18246-7. Versatile utility graphing program capable of graphing a variety of functions, conics, parametric equations and polar form relations. Linear equations can be graphed in "Function Grapher" option. Suitable for exploring effects of m and b in $y = mx + b$, graphing systems of linear equations. Features include sliding vertical and horizontal lines, or cross-hairs that permit identification of x- and y-intercepts and point of intersection, change of viewing window, zoom in/out, grid display. Suitable for teacher demonstration, small group and individual student use.

SRA Computer Drill and Instruction, Mathematics, Level D: Willowdale, On: Science Research Associates, 1981-1983. (Apple II. Recommended resource at the Junior High Level. Also available for IBM PC.) A multi-disk package of drill exercises in computation, numbers and numeration, fractions, decimals, ratio and percent, measurement, pre-algebra and applications. Level appropriate for various copies in Mathematics 14, some in Mathematics 13. Does not provide answers or method of solving. May be used by students in unmanaged or teacher-managed form.

300

SUPPLEMENTARY RESOURCES

(A) Computer Software

Algebra Drill and Practice I. (Detmer, R. et al.) Iowa, City, IA: Conduit, 1981. (Apple II, IBM PC.) Available from Conduit, University of Iowa, Oakdale Campus, Iowa City, IA 52242. $125 US (1988) Two–disk package provides drill and practice in a variety of topics including algebraic fractions, slopes and equations of line, percentages, simplifying linear algebraic expressions and word problems. "Slopes and equations of lines" option offers four types of problems: find slope given two points or given the equation in $Ax + By + C = 0$ form and find equation of line given point and slope or two points. Must be expressed in $Ax + By + C = 0$ form. "Percentage" option give percent problems of the form "20 is 30% of what number?" Provides example solutions, option of seeing solution to current problem. Typically, student inputs answer only, must solve mentally or on paper first. Teacher's guide included.

Algebra Drill and Practice II: (Detmer, R. et al.) Iowa, City, IA: Conduit, 1982. (Apple II, IBM PC.) Two–disk package provides drill and practice in simplifying integral exponent expressions, solving absolute value problems, linear equations and linear systems in two variables, factoring binomials and trinomials, finding roots of quadratic equations and solving linear inequalities. Step–by–step presentation of solution of examples and problems with which student has difficulty provided. Student inputs answer only, must solve mentally or on paper. Optional graphic solution of linear systems and inequalities provided by program following successful algebraic solution by student. Teacher's guide included.

Coordinate Math. St. Paul, MN: Minnesota Educational Computing Corporation, 1987. (Item No. A–192.ISBN: 0–87490–399–8. Apple II. Available from MECC, 3490 Lexington Avenue North, St. Paul, MN 55126) Consists of three educational games that require students to use their skills at problem–solving and locating and using coordinate points on the Cartesian plane to accomplish a stated goal. Teacher's guide, with student worksheets, included.

Equation Math. St. Paul, MN: MECC, 1987. (Item No. 1–193. ISBN: 0–87490–405–6. Apple II.) A versatile graphing program that offers three main options: "Exploring Equations," "Polygraph" and "Polar." The "Exploring Equations–Linear" option provides opportunity to explore effects of ma and b in $y = mx + b$. Equations of form $x = b$ and $y = b$ acceptable. Hard copy of screen contents optional Automatic increment/decrement of parameters through cursor key action provided. "Quadratic equation" option also available.. "Polygraph" option capable of graphing numerous types of relations, including linear. Zoom in/out option provided. Display of ordered pairs that satisfy relation optional. Suitable for teacher demonstration and small group or individual student use. Teacher's guide, with extensive set of student worksheets, included.

Equations. (Ross, D.) Northbrook, IL: Mindscape, 1982. (Apple II, IBM PC, Commodore 64/128, Tandy 1000. Available form Mindscape, Inc., 3444 Dundee Road, Northbrook, IL 60062. $29.95 US (1988)). Provides very structured practice in solving equations of the form $AX + B = C$. Format is interactive step–by–step solution of the problem presented. Presentation of example solution and solutions of problems with which student has difficulty provided.

Equations II. (Appel, M. et al.) Northbrook, IL: Mindscape 1984. (Apple II, IBM PC, Commodore 64/128,Tandy 1000.Available form Mindscape, Inc., 3444 Dundee Road, Northbrook, IL 60062. $29.95 US (1988)).Provides very structured practice and remedial opportunities in solving equations of the form $Ax + B = Cx + D$. See Equation.

Equations III. (Ross, D.) Northbrook, IL: Mindscape, 1985. (Apple II) Provides very structured practice in solving equations of the form $Ax + B(Cx + D)\text{-}Ex + F$. See Equations.

Factoring Algebraic Expressions. Northbrook, IL: Mindscape. (Apple II, $29.95 US (1988)). Provides practice and remedial opportunities in factoring common factor problem, trinomials ("easier" :$a = 1$ and harder :$a \neq 1$) and difference of squares Students must determine factors mentally or on paper, input as directed on screen. Program provides check by multiplication. Optional presentation of solution available.

Geometric preSupposer: Points and Lines. (Schwarta, J. et al.) Scarborough, ON: Sunburst, 1986. (Apple II. Available from Sunburst Communication, P.O. Box 3240, Station F, Scarborough, ON, M1W 9Z9. $132 (1989) Provides extensive possibilities for drawing points, lines, segments, triangles, quadrilaterals, polygons, circles, bisectors of angles and parallel and perpendicular lines. Measurement and/or calculation of length, perimeter, circumference, area or angles possible. Lines tend to be slightly ragged. Will take time for students to develop facility with program. Suitable for teacher demonstration, or geometry exploration by small group or individual students. Extensive teacher's guide, with student worksheets, included.

Geometric Supposer: Triangles. (Schwartz, J. et al.) Scarborough, ON: Sunburst, 1985. (Apple II, IBM PC, Tandy 1000.) Provides extensive possibilities for exploring properties of triangles, which may be created by angle or side classification, or by construction according to SSS, SAS or ASA. Other figures such as line segments, circles, medians, altitudes, parallel and perpendicular lines, bisectors of angles and perpendicular bisectors of lines may be constructed. Measurement of lengths, perimeters, areas, angles and distance possible. Sum, difference, product, square and ratio of various measures also possible. See other comments, "Points and Lines." Extensive Teacher's guide, with student worksheets, included.

Graphing Linear Functions. (Berman, M. et al.) Northbrook, IL: Mindscape, 1984. (Apple II, IBM PC, Commodore 64/128, Tandy 1000. $29.95 US (1988)). Provides structured practice in graphing equations of the form $y = mx + b$ ("Easier" option) and "Harder" forms, for example, $Ax + By + C = 0$, $By = Ax + C$, $(B \neq 1)$, etc. Students then determine three sets of ordered pairs that satisfy equation and plot by means of cursor key movements. Program then draws line In "Harder" forms option, students must first solve for y in step-by-step fashion.

Green Globs and Graphing Equations. (Dugdale, S. et al.) Scarborough, On: Sunburst. 1986. (Apple II, IBM PC, Tandy 1000. $87 (1989)). A versatile graphing program. "Equation Plotter" option is a utility program in which a wide variety of forms of equations can be input and graphed. "Linear" sub-option provides opportunity to explore effects of m and b in $y = mx + b$; input in $Ax + By + C = 0$ form; investigate parallel and perpendicular lines and $y = b$, $x = b$ forms; etc. Systems of equations may also be graphed, although coordinates of intersection point not given. Other modes of "Equation Plotter" permit such things as rational, polynomial, absolute value quadrated and exponential functions and identifying zeros. Suitable for teacher demonstration and small group or individual student use. "Linear and Quadratic Graphs" option provides practice in writing equations. Program draws graph, student must write equation that satisfies graph. Extensive teacher's guide, with student worksheets, included.

Percents. Baltimore: Media Materials, 1983. (Apple II, except II c). Available from Media Materials, 2936 Remington Avenue, Baltimore, MD 21211. $49.95 US (1988)). Provides practice and remedial opportunities in solving percent problems. Teacher's guide, with student worksheets, included.

Ratios and Proportions. Baltimore: Media Materials, 1983. (Apple II, except II c). Provides practice in solving ratio and proportion problems. Teacher's guide, with student worksheets, included.

Solving Quadratic Equations. (Ross, D. et al.) Northbrook, IL: Mindscape, 1982. (Apple II, IBM PC, Commodore 64/128, Tandy 1000. $29.95 US (1988)). Provides structured practice in solving quadratic equations. Two options: "Easier $(A = 1)$" and "Harder $(A = 2 \text{ or } 3)$." Students presented

with quadratic equation, which must be factored mentally or with pencil-and-paper. Program checks by multiplying. Students then must solve for each of the factors in step-by-step manner. Program demonstrates solution for questions student has difficulty with.

Spreadsheets for Mathematics and Science: Appleworks Sampler. St. Paul, MN: MECC, 1987. (Item No. T-260. ISBN: 0-87490-415-3, Apple II.) Provides practice in learning and using spreadsheets (specifically, Appleworks) through its application to mathematics and science problems. Requires time. Teacher must know how to use spreadsheet before introducing to students. Extensive teacher's guide, with student activity sheets, included.

OTHER RESOURCES

(A) Print

A Sourcebook of Applications of School Mathematics. Joint Committee of the Mathematical Association of America and the National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics, 1980. ISBN: 0-87353-164-7. (Teacher) Contains a number of articles that discuss use of applications in mathematics, and provide specific examples of application in advanced arithmetic, algebra, geometry, trigonometry and probability.

Activities from the 'Mathematics Teacher'. (Edited by E.M. Maletsky et al.). Reston, VA: National Council of Teachers of Mathematics, 1981. ISBN: 0-87353-173-6. (Student) Provides activities in a number of mathematics topics. See for example:

"Discovery with Cubes" (Reys, R. E.), pp. 107 – 110. Describes pictorial activities for exploring patterns with cubes.

"Getting the Point!" (Snyder, M. F.), pp. 55 – 58. Describes motivational activities that provide point plotting practice.

"Pattern Gazing" (Aviv, C.A.), pp. 131-134. Describes activities designed to enable students to discover number patterns and make generalizations.

"Probability and Pi" (Hirsch, C.R.), pp. 111-114. Describes manipulative activities using Monte Carlo techniques to calculate estimation of pi.

"Pythagorean Puzzles" (Spaulding, R. E.), pp. 67 – 70. Describes manipulative geometric activities for discovering/verifying Pythagorean Theorem.

Applications in School Mathematics: 1979 Yearbook. (Edited by S. Sharron et al.) Reston, VA: National Council of Teachers of Mathematics, 1979. ISBN: 0-87353-139-6 (Teacher) Numerous examples provided of applications in mathematics in this collection of articles. Extensive bibliography also provided. Partial listing of yearbook articles:

"Applications in Elementary Algebra and Geometry" (Usiskin, Z.), pp. 20-30. Provides ideas and some examples of applications.

"Applications in School Mathematics: Human Variability" (Pagni, D.L.), pp. 43-58. Good source of human characteristics as variables suitable for statistics and probability problems and projects.

"Applications through Direct Quote Word Problems" (Pinker, A.), pp. 31–42. Makes connections between mathematics and other subject areas by using direct quotes from newspapers, journals and books as the basis for mathematics problems.

"Applications: Why, Which, and How" (Lesh, R.), pp. 8–19. Discusses the value of integrating applications in the mathematics lesson.

"Applying Mathematics to Environmental Problems" (Thiessen, D. et al.), pp. 149–158. Provides a number of examples of class activities designed to make students more environmentally conscious while applying mathematics skills and knowledge. Up–dating and "Canadian–izing" occasionally necessary.

"Wildlife, Unemployment and Insects: Mathematical Modeling in Elementary Algebra" (Johnson, D.C.), pp. 137–148. Provides four examples (with sample student worksheets) of applying mathematics to "real–life" situations.

Canada's Food Guide Handbook. Ottawa: Department of National Health and Welfare, 1988. ISBN: 0–662–11947–9 (Teacher) Provides resource material suitable for use in ratio and proportion problems and projects.

Computers in Mathematics Education, 1984 Yearbook. (Edited by V.P. Hansen et al.) Reston, VA: National Council of Teachers of Mathematics, 1984. ISBN: 0–87353–210–4. (Teacher) Discusses computing from standpoints of its challenge, impact and perspective as a diagnostic tool. Discusses programming as a means of teaching mathematics.

Computing and Mathematics: The Impact on Secondary School Curricula (Edited by J.T. Fey.) Reston, VA: National Council of Teachers of mathematics, 1984. ISBN: 0–87353–212–0. (Teacher). Report of a major conference. Topics include mathematics and the new information technology, impact of computing on algebra, geometry, calculus and discrete mathematics and algorithmic methods and prospects and strategies for change in school mathematics.

Creative Geometry: Investigations on the Microcomputer. (Teacher's Guide) (Bell, A. et al., South Notts Project) Nottingham, U.K.: Shell Centre for Mathematical Education, University of Nottingham, University Park, Nottingham, NG7 2RD, n.d. Discusses activities in creating geometric patterns and designs by creating and using programs like those in LOGO.

Descartes' Dream: The World According to Mathematics. (Davis, P. et al.) Boston: Houghton Mifflin, 1986. ISBN: 0–395–43154–9. (Teacher) Discusses the nature of mathematics, the impact of computing on mathematics, the impact of both on society. Eclectic in style, highly readable.

Everybody Counts: A Report to the Nationl on the Future of Mathematics Education. (Board on Mathematical Sciences [and] Council (U.S.).) Washington, D.C.: National Academy of Sciences, 1989. ISBN: 0–309–03977–0 (Teacher) Discusses mathematics, mathematics education and their relation to society. Relevant to Canadian educators.

Estimation and Mental Computation: 1986 Yearbook. (Edited by H.L. Schoen et al.). Reston, VA: National Council of Teachers of Mathematics, 1986. ISBN: 0–87353–226–0 (Teacher) A collection of articles that provide rationale and strategies for developing and evaluating estimation and mental computation skills. See for example:

"A Do–It–Yourself Estimation Workshop" (King, J.A.), pp. 162–170. Describes a number of classroom estimation activities.

"Benchmark: Number Patterns and the Development of Expert Mental Calculation" (Hope, J.A.), pp. 72–73. Illustrates how understanding number patterns can facilitate mental computation.

"Estimation and Mental Arithmetic with Percent" (Ailinger, G. D. et al.), pp. 141–155. Provides discussion of, and strategies (with examples) for applying estimation and mental calculation to percent problems.

"Estimation in Measurement" (Coburn, T. G. et al.), pp. 195–203. Describes a number of classroom measurement estimation activities.

"Evaluating Computational Estimation" (Reys, R.E.), pp. 225–238. Discusses the difficulty in testing estimation skills and provides some guidelines.

"Mental Calculation: Anachronism or Basic Skill?" (Hope, J.A.), pp. 45–54. Provides rationale for developing students' mental computation abilities.

"Reasons for Estimating" (Usiskin, Z.), pp. 1–15. Cites a number of reasons (with illustrative examples) in answer to "Why estimate?"

"Teaching Computational Estimation: Concepts and Strategies" (Reys, B.J.), pp. 31–44. Discusses (a) the development of number concepts, and (b) the development of estimation strategies.

"Teaching Computational Estimation: Establishing an Estimation Mind-Set" (Trafton, P.R.), pp. 16–30. Discusses (a) the development of an awareness for, and an appreciation of, estimation, and (b) the development of number sense.

Algebra Before & After: 1. (Gregory, J.) Palo Alto, CA: Creative Publications, 1987. ISBN: 0–88488–641–7 (Student) A workbook of student developmental and practice activities related to topics in polynomials (exponents, writing expressions, multiplying and factoring), writing and solving linear equations and systems of equations, coordinate geometry and graphing, square roots and the Pythagorean Theorem. Emphasis is on pattern development and generalization.

Algebra Before & After: 2. (Gregory, J.) Palo Alto, CA: Creative Publications, 1987. ISBN: 0–88488–642–5 (Student) A workbook of student developmental and practice activities related to topics in radicals and rational exponents, polynomials (multiplying, factoring, number pattern generalizations) rational expressions, relations and functions (graphing, zeros), and quadratic equations. Emphasis is on pattern development and generalization.

Classroom Ideas from Research on Secondary School Mathematics. (Dessart, D. & Suydam, M.) Reston, VA: National Council of Teachers of Mathematics, 1983. ISBN: 0–87353–207–4. (Teacher) Presents a variety of ideas for teaching algebra and geometry in the secondary school. Discusses research on teaching, cognitive development, attitude and affective factors, problem solving, the use of calculators and computers, and testing and prediction.

Imaginative Ideas for the Teacher of Mathematics, Grades K–12: Ranucci's Reservoir. (Ranucci, E.R., Edited by M.A. Farrell.) Reston, VA: National Council of Teachers of Mathematics, 1988. ISBN: 0–87353–257–0 (Teacher) A variety of topics covered in this collection of articles. Often suitable for extension activities. Referenced:

"Of Shoes–and Ships–and Sealing Wax–Of Barber Poles and Things," pp. 36–39. Describes application of trigonometry to baseball.

Learning and Teaching Geometry, K–12: 1987 Yearbook. (Edited by M.M. Lindquist et al.) Reston, VA: National Council of Teachers of mathematics, 1987. ISBN: 0–87353–235–X (Teacher) Contains a number of articles that deal with topics such as issues in teaching geometry, problem solving activities and relating geometry to other mathematics topics.

Math Projects for Young Scientists. (Thomas, D.A.) New York: Franklin Watts, 1988. ISBN: 0–531–10523–7 (Student) Discusses how to choose, research and present a mathematics project, and briefly describes 96 suggested student projects. Designed for the capable, interested student of mathematics. Varying degrees of background mathematics sophistication required.

Mathematics: A Human Endeavour (2nd ed.) (Jacobs, H.R.) New York: W.H. Freeman, 1982. ISBN: 0–7167–1326–8. (Student) An innovative textbook exploring a variety of mathematics topics.

Mind Tools: The Five Levels of Mathematical Reality. (Rucker, R.) Boston: Houghton Mifflin, 1987. ISBN: 0–395–46810–8 (Teacher) Explores the nature of mathematics as number, space, logic, infinity and information. Very non–routine but readable discussion of the nature of mathematics.

Mindstorms: Children, Computer, and Powerful Ideas (Papert, S.) New York: Basic Books, 1980. ISBN: 0–465–04629–0 (Teacher) Describes the development of LOGO and its potential use by children to develop mathematical thinking and understanding.

Problem Solving in School Mathematics: 1980 Yearbook. (Edited by S. Krulik et al.). Reston, VA: National Council of Teachers of Mathematics, 1980. ISBN: 0–87353–162–0. (Teacher) Contains a number of articles describing various aspects of problem solving.

Recent Revolutions in Mathematics (Stwertka, A.) New York: Franklin Watts, 1987. ISBN: 0–531–10418–4 (Teacher) Provides a readable overview of recent activity in the field of mathematics. Topics include the geometry of the universe, the science of prediction, number theory, fractals, chaos and catastrophe theory and computers.

Student Math Notes. (Edited by E.M. Maletsky) Reston, VA: National Council of Teachers of Mathematics, 1987. ISBN: 0–87353–244–9. Contains a number of articles with interesting non-routine student activities. See for example: "Pi: The Digit Hunt" (Coffield, P.), pp. 39–44. Presents a number of historical notes and activities regarding the approximation of pi.

Teaching Mathematics: A Sourcebook of Aids, Activities and Strategies (2nd ed.) (Sobel, M. et al.) Englewood Cliffs, NJ: Prentice Hall, 1988. ISBN: 0–13–894148–3 (Teacher) A sourcebook for secondary mathematics teachers. Covers a wide variety of topics.

Teaching Statistics and Probability: 1981 Yearbook. (Edited by A.P. Shulte et al.). Reston, VA: National Council of Mathematics, 1981. ISBN: 0–87353–170–1 (Teacher) Provides discussion on rationale, samples of existing programs, classroom activities, teaching and learning specific topics, applications, statistical inference, Monte Carlo techniques, using computers and an extensive bibliography and list of projects. See for example:

"Experiential Statistics and Probability for Elementary Teachers" (Juraschek, W. A. et al.) pp. 8–18. Describes various activities for statistics and probability.

"In All Probability, a Microcomputer" (Kellogg, H.M.), pp. 225–233. Provides discussions and computer programs on simulating, for example, Monte Carlo estimation of area and Buffon's needle problem.

"Misconceptions of Probability: From Systematic Errors to Systematic Experiments and Decisions" (Shaughnessy, J. M.), pp. 90–100. Describes various misconceptions of probability and some teaching activities to help change these erroneous beliefs.

"Projects" (Klitz, R.H. Jr.), pp. 239–246. Briefly describes 34 statistics and probability student projects, many of which are suitable for this senior high level.

"Simple Graphical Techniques for Examining Data Generated by Classroom Activities" (Maher, C.A.), pp. 109–117. Describes some class activities involving stem-and-leaf and box plots, and their application to probability problems.

"The Student Price Index" (Lenoski, C.G.), pp. 73–83. Describes in depth a year-long project in which students kept detailed records of their expenditures and created a student price index along the lines of a consumer price index.

The Art of Problem Posing. (Brown, S.I. et al.) Philadelphia: Franklin Institute, 1983. ISBN: 0–89168–052–7 (Teacher) Provides strategies for and numerous examples of problem posing, designed to encourage open-ended problem solving.

The Ideas of Algebra, K–12: 1988 Yearbook. (Edited by A.F. Coxford et al.). Reston, VA: National Council of Teachers of Mathematics, 1988. ISBN: 0–87353–250–3 (Teacher) Covers a number of topics related to school algebra, including ideas and issues, readiness, equations and expressions, problem solving, computer and calculator use, and teaching ideas. See for example:

"Children's Difficulties in Beginning Algebra" (Booth, L.R.), pp. 20–32. Discusses adolescents' (mis)understandings of algebra.

"An Integration of Equation-Solving Methods into a Developmental Learning Sequence" (Bernard, J.E. et al.), pp. 97–111. Discusses an approach to teaching linear equation solving based on developmental learning. Several examples provided.

"Conceptions of School Algebra and Uses of Variables" (Usiskin, Z.), pp. 8–19. Discusses four conceptions of algebra and the meaning of variable in each case. Different conceptions imply different purposes for algebra.

"Developing Algebraic Representation Using Diagrams" (Simon, M.A. et al.), pp. 136–141. Discusses complementarity of algebraic (linear) and diagrammatic solutions to problems.

"Proportionality and the Development of Prealgebra Understanding," (Post, T.R. et al.), pp. 78–96. Discusses the meaning of proportional reasoning, representation of proportional relationships by $y = mx$ and methods of solving proportion problems.

"Teaching Algebraic Expressions in a Meaningful Way" (Chalouh, L. et al.), pp. 33–42. Discusses the meaning of variable and ways to introduce this concept.

The Mathematical Experience (Davis, P. et al.) Boston: Birkhauser, 1981. Penguin Books ed. (1983) ISBN: 0–14–022456–4. (Teacher) Explores widely the nature of mathematics: history, schools of philosophical thought, symbolic nature, views of some contemporary mathematicians and teachers, teaching and learning and uses. Insightful, written in highly readable style.

The Nature of Mathematics (4th ed.) (Smith, K.J.) Monterey, CA: Brooks/Cole, 1984. ISBN: 0–534–02806–3 (Student) Innovative, wide-ranging textbook. Explores the nature of a number of mathematics topics, such as inductive and deductive reasoning through patterns and sets, calculators and computers, numbers, algebra, geometry, and statistics and probability.

Thinking Mathematically (rev.ed.). (Mason J. et al.) Wokingham, Eng.: Addison-Wesley, 1985 ISBN: 0-201-10238-2 (Teacher) A structured, activity-based book of discussion and open-ended questions designed to help develop ability to think mathematically. Attends to the emotional aspects of working with mathematical problems. Very readable and engaging.

(B) Journals

Computers in Education Moorhead Publications, Ltd., 1300 Don Mills Road, North York, Toronto, ON, M3B 3M8. (Teacher) Provides information on microcomputers and software. Canadian focus. Articles, items discuss computing and programming uses in education, including mathematics education. Published 10 times a year.

delta-K. Mathematics Council of the Alberta Teachers' Association. (Teacher) (Address correspondence to J.B. Percevault, editor, 2510 - 22 Avenue South, Lethbridge, Alberta, T1K 1J5 Articles include discussions of issues in mathematics education, general approaches and styles in mathematics teaching and learning, and specific teaching ideas.

"Mathematical Modeling using Spreadsheets" (Burnett, J.D.), pp. 13-19. Discusses a spreadsheet-with-graphing capability approach as a pedagogically valuable means of investigating equation solving and function analysis.

delta-K. Special Edition: "Mathematics for Gifted Students." Vol. 27, No. 3, April 1989. Presents articles on special topics in mathematics, plus lists of more than 350 problems with solutions and more than 140 books of mathematical puzzles, problems, instruction and applications.

For the Learning of Mathematics. FLM Publishing Association, 4336 Marcil Avenue, Montreal, Quebec H4A 2Z8. (Teacher) Articles deal with a variety of mathematics-related topics, including issues in mathematics education, philosophy of mathematics and research on teaching and learning mathematics. Intended for a wide range of mathematics educators. Published three times a year.

Mathematics Teacher. National Council of Teachers of Mathematics. 1906 Association Drive, Reston, Va 22091 (Teacher) A journal intended for the secondary mathematics teacher. Scope includes mathematics education issues, teaching ideas, innovative secondary level mathematics and resource reviews. Published 10 times annually. See for example:

(I) Volume 81, 1988:

(a) January (No. 1)
"Problem Solving: The Third Dimension in Mathematics Teaching" (Gadanidis, G.), pp. 16-21. Discusses three dimensions of teaching mathematics: facts and skills, understanding, and problem solving. Encourages holistic teaching. Provides examples of non-routine problems.

"Using Percent Problems to Promote Critical Thinking" (Osiecki, B.), pp. 31-34. Describes ideas for teaching percent such as having students check sales flyers, write sales problems, and create class flyers, posters and activity sheets using a graphics computer program.

(b) March (No. 3)
"Concrete Versus Abstract in Teaching Algebra" (Blais, D.M.), pp. 187-188. Describes method of introducing concept of slope through reference to the physical world (skiing). Useful illustrations.

(c) April (No. 4)
"Games, Geometry, and Teaching" (Bright, G. W.), pp. 250–259. Describes games designed to strengthen students' geometry skills and understanding of properties of geometric figures.

"Slope as Speed" (Metz, J.R.), pp. 285–289. Provides set of activities designed to develop students' ability to interpret linear graphs.

"Visual Thinking and Number Relationships" (Bennett, A.B., Jr.), pp. 267–272. Describes variety of examples in which number relationships are visually represented.

(d) May (No. 5)
"Discovery with Cubes" (Reys, R. E.), pp. 377–381. Provides a set of activities in which students discover patterns in cubes of various sizes.

(e) September (No. 6)
"On the Meaning of Variable" (Schoenfeld, A.H. et al.), pp. 420–427. Discusses the different meanings of variable, and offers teaching suggestions.

(f) October (No. 7)
"I have a Number..." (Moody, M.), pp. 550–551. Describes whole class activity designed to help strengthen mental calculation and communication skills.

"Thinking the Unthinkable: The Story of Complex Numbers (with a Moral)" (Kleiner, I.), pp. 583–592. Provides a brief history of the development of complex numbers.

"Using Linear Functions" (Wallace, E. C.), pp. 560–566. Provides a set of activities in which students plot points on coordinate grid and interpret graphs of linear relationships representing "real-life" situation.

(g) November (No. 8)
"Exploring Data with Box Plots" (Bryan, E.H.), pp. 658–663. Provides a set of box-and-whisker activities.

"Integrating Spreadsheets into the Mathematics Classroom" (McDonald, J.L.), pp. 615–622. Describes use of spreadsheet in geometry measurement, function relationships, systems of equations, quadratic equations, powers and roots, and trigonometry.

(h) December (No. 9)
"Explorative Writing and Learning Mathematics" (Keith, S.Z.), pp. 714–719. Describes use of writing assignments in the mathematics class. Provides examples of writing assignments and techniques, and benefits of their use. Reprinted in Appendix. C.

(II) Volume 80, 1987

(a) January (No. 1)
"Polygons Made to Order" (Troccolo, J. A.), pp. 44–50. Provides a set of activities in which students draw regular polygons. Reinforces angle measurement and geometric concepts.

(b) February (No. 2)
"Box Technique for Factoring" (Crisp, T.A.), pp. 115–118. Describes alternative approach to factoring trinomials (a ≥ 1) involving a three-by-three grid or "box."

(c) April (No. 4)
"Flexibility and Algebraic Problem Solving" (Cohen, M.P.), pp. 294–295. Provides non-routine examples of algebra problems.

(d) October (No. 7)
"Exploring Baseball Data" (Olson, A.T.), pp. 565–569, 584. Demonstrates application of stem-and-leaf and box-and-whisker plot techniques to baseball data.

(e) November (No. 8)
"Revisiting the Interior Angles of Polygons" (Wills, H. III), pp. 632–634. Describes various approaches to deriving the formula for the sum of the interior angles of a regular polygon.

(f) December (No. 9)
"Realistic Examples in Elementary Statistics" (Barbella, P.), pp. 740–743. Provides example of applying statistical organizational methods of frequency distribution and stem-and-leaf plots to classroom test data, and interpreting results.

"Skiing the Slopes" (Gustafson, D.), pp. 733–739. Provides set of activities that develop concept of slope first through steepness (of ski hills) and then by graphing on the Cartesian plane.

(III) Volume 79, 1986

(a) January (No. 1)
"Expanding the Limits of the Calculator Display" (Sherzer, L.), pp. 20–21. Describes a method of using the calculator to determine the period of repeating decimals that exceeds the display capabilities of the calculator.

"Factoring Trinomials" (Sherzer, L.), p. 9. Describes alternative approach to factoring trinomials together with underlying theorems that may be proven as class activities.

"Manipulating Percentages" (Wiebe, J. H.), p. 23. Describes two manipulative approaches to solving percent problems.

(b) April (No. 4)
"Secret Codes and Systems of Equations" (Tucker, B.F.), pp. 256–258. Describes novel, motivational approach to solving systems of equations graphically or algebraically.

(c) May (No. 5)
"The Binomial Grid" (Juraschek, B. et al.), pp. 337–339. Describes using a pictorial grid approach to factoring trinomials.

(d) September (No. 6)
"Using Writing to Learn Mathematics" (Nahrgang, C.L. et al.), pp. 461–465. Describes use of journal writing in mathematics class to enhance student learning. Reprinted in Appendix C.

(e) November (No. 8)
"Estimating with 'Nice' Fractions" (Trafton, P. R. et al.) pp. 629–634. Provides set of activities designed to develop students' estimation skills with fractions.

(f) December (No. 9)
"A New Solution to an Old Problem–Solving Word Problems in Algebra" (Thaeler, J.S.), pp. 682–689. Describes a general model for solving word algebra problems requiring a linear solution in one variable.

(IV) Volume 77, 1984

(a) January (No. 1)
"Make Up a Story to Explain the Graph" (Woodward, E. et al.) pp. 32-34. Describes, using examples, class activity of students' writing then discussing their interpretations of graphs.

(V) Volume 76, 1983

(a) January (No. 1)
"Improving Students' Comprehension of Word Problems" (Denmark, T.), pp. 31-34. Offers suggestions for teaching activities to improve students' comprehension of word problems. Activities include cloze exercises, comparing problems, estimating and reading for details.

(b) April (No. 4)
"Mathematics and the Imagination" (Jacobs, H.R.), pp. 245-249. Provides non-routine problems.

(c) May (No. 5)
"Finding Patterns in Linear Relations" (Maletsky, E.M. et al.), pp. 341-344. Presents a set of activities that provides practice in plotting points in the Cartesian plane, and develops skills in interpreting and describing graphical patterns and writing equations of the form $y = mx + b$ based on the interpretation.

(d) September (No. 6)
"More Calculator Capers" (Schmalz, R.), pp. 414-416, 421-422, 437. Provides a set of activities designed to permit the exploration of number patterns with the aid of a calculator.

(e) October (No. 7)
"What are these Things called Variables?" (Wagner, L.S.), pp. 474-479. Discusses various aspects of the meaning of the concept of variable, and offers some teaching suggestions.

(VI) Volume 75, 1982

(a) October (No. 7)
"Repeating Decimals, Geometric Patterns and Open-Ended Questions" (McGinty, R. et al.), pp. 600-602. Describes activity in which students can display in a geometric pattern the period of repeating decimals. Suitable for project or class display.

(VII) Volume 74, 1981

(a) February (No. 2)
"Analysis of a Truck Driver's Square Root Algorithm" (Carmony, L.), pp. 144-148. Describes a method of approximating square root based on the relationship of the given number to the closest perfect squares smaller and larger than the number. The relationship between the two (consecutive) perfect squares suitable for investigation.

"$7.5 + 7.5 \neq 15$" (Snook, H.M.), pp. 114-116. Provides insight into percent with the illustration that two consecutive annual salary raises of 7.5% do not yield the same salary as that based on a 15% raise followed by a 0% raise.

(VIII) Volume 73, 1980

(a) October (No. 7)
"I Have.... Who Has...?" (Giambrone, T.), pp. 504-506. Describes a whole class activity designed to strengthen skills of translating a spoken sentence into an algebraic expression, and vice versa.

"Problem Solving: Look Beyond the Right Answer" (McGinty, R.L. et al.), pp. 501-503. Suggests that complete problem solving involves such components as decision making, the quality of the product, reasonableness of the results, implications for use, practicality and values. These, along with the numerical solution represent a melding of the mathematical, social and ethical components of a situation.

(b) November (No. 8)
"Constructing Meaning for the Concept of Equation" (Herscovics, N. et al.), pp. 572-580. Describes the development of young people's understanding of the concept of equation, conceptual difficulties and an approach to teaching the concept.

School Science and Mathematics. School Science and Mathematics Association, Inc., 126 Life Science Building, Bowling Green University, Bowling Green, OH 43403. (Teacher) Articles discuss issues in mathematics and science education, research on teaching and learning mathematics, science and teaching ideas.

The Computing Teacher International Council for Computers in Education, University of Oregon, 17787 Agate Street, Eugene, OR, 97403-9905 (Teacher) Emphasizes teaching and learning about computers, teaching with computers, teacher education and the impact of computers on curricula. Published nine times a year.

APPENDIX C

USING WRITING TO LEARN MATHEMATICS

C.L. Nahragang and B.T. Petersen

Writing has a place in a mathematics class. It provides a new and interesting mode of communication between the student and teacher, as Watson (1980) reported:

> *This two-way conversation has become beneficial to the class. The students realized I hear them and care. They seem to have looked inside themselves and to have seen what they could do to help solve their mathematical problems. Many of their grades improved.*

Written assignments in mathematics classes also afford students the opportunity to organize their thoughts and, at the same time, improve their writing skills (Johnson 1983.)

The purpose of this article is to show that writing can be used in mathematics to enhance learning. The act of writing gives students the opportunity to formulate, organize, internalize, and evaluate concepts. Emig, who has done extensive investigation of the thinking processes involved in writing, argues that writing "represents a unique mode of learning – not merely valuable, not merely special, but unique" (Emig 1977.) The idea that writing is a powerful aid to learning has also found much support in cognitive psychology. For example, Bruner (1966) states, "the more we know about the manner in which it [writing] can aid thought...[the more it] leads me to put language at the center of the stage in considering the nature of intellectual development."

Our experience indicates that the most effective method of using writing to help students learn mathematics is through the use of journals. These in-class writing exercises offer students the opportunity to work informally and personally on mathematical concepts, using their own language and real-world experiences. They can plan with a variety of concepts without the fear of the formal evaluation of their writing and their mathematical skills. Our experience in college-level classes, from algebra through calculus, makes us feel confident that these short but powerful writing exercises enhance learning. The journal goes beyond rote learning and challenges the students to use intellectual skills. Myers relates writing to individual learning: "When doing this simple activity [writing], students will be forced to relate information from the lecture to what they already know and to organize and synthesize it so that the concept becomes their own" (Myers 1984.)

The Journal

The journal is a diary-like series of writing assignments. Each assignment or entry is a short written response to an instructor's question, statement, or set of instructions. All responses are written in prose rather than in the traditional mathematical style of numbers and equations. The students are directed to use an expressive writing style so that their responses are spontaneous and all their thoughts are recorded. Figure 1 is an example of a journal entry made by a student in technical writing in response to the following assignment: "Describe a real life, nonmathematical analogy to the rule $a^{-n} = {}^{1}/a^{n}$."

Journals have two basic functions. First, they allow students to proceed at their own rate and to converge on an understanding of mathematical concepts using their own experiences. Second, they provide teachers a unique diagnostic tool; the writings of students immediately reveal areas of confusion and expose misunderstandings of mathematical concepts.

Journals were introduced into our mathematics classes at Michigan Tech with a minimum of difficulty. These classes were conducted over a ten-week period. Each class included about ten unannounced quizzes that accounted for 20 percent of a student's final grade. By substituting twenty journal-writing sessions for these quizzes, we gave our students two opportunities a week for writing in their journals. However, the flexibility – one of its advantages – of the journal-writing technique permits the instructor to devote extra assignments to more difficult mathematical concepts.

Reprinted with permission from
Mathematics Teacher December 1986,
Copyright by the National Council of
Teacher of Mathematics

$a^{-n} = \frac{1}{a^n}$ is an inverse rule so, if any power is raised to the neg. it becomes a fraction, also making it positive.

You inverse what it says, you switch it around.

So what in life is switched around or inversed (which means the same thing?)

What can I turn around in my life – yet keep it the same; to mean the same? (yet it is non–mathematics)

- *relationships*
- *feelings*
- *communication – the way I say something or communicate*

The english language and communication could easily apply to this rule. We may change or turn phrases around yet keep their meaning the same.

Figure 1

Most journal entries require three to seven minutes, depending on the type of assignment. Our experiences have shown that as students become more familiar and comfortable with this technique, their responses become better organized and thus easier to read. In all classes, we found that students' experiences in writing in journals led to definite improvements in their ability to organize their responses.

The method of evaluating journals become an important question. Initially, grading each journal seemed to be most appropriate, with the average of all journal grades being a part of the final grade for the course. We immediately realize that this method restricted the potential of this learning tool. By assigning grades, we were indirectly telling the students how they should process concepts, which was defeating one of the important attributes of journal writing – individualized learning and discovery. However, students seemed to deserve some type of credit for their efforts. After considering several alternatives, we decided to give one or two points toward the next major examination on the basis of the students' comments about concepts., The most appropriate responses to mathematical concepts were given the maximum extra credit allowable. A lack of effort received no credit. In addition, we often added appropriate comments (e.g., "excellent," "good," "incomplete," or "please revise.") We found that students appreciated the teacher's comments as much as they did the extra credit. Finally, all mathematical errors were corrected. Although these errors were a secondary consideration, they were brought to the students' attention.

Reprinted with permission from
Mathematics Teacher December 1986,
Copyright by the National Council of
Teacher of Mathematics

APPENDIX E

An Analysis

Journal entries demonstrate individual learning activities. Various types of assignments were made that coincided generally with the levels specified in Bloom's taxonomy (Bloom, Hastings, and Madaus 1971.) During a section on factoring, students were asked to do the following journal assignment: "Discuss the following statement: 'Factoring and finding a product are reverse processes.'" The objective of this particular assignment was to have students collect ideas, internalize the concept of factoring, and examine its significance in finding a product. This task was assigned early in the study of factoring and finding products of polynomials. Observe how each of the following students begins to understand the process of factoring in a personal frame of reference. Student A wrote:

Factoring and finding a product are reverse processes. They result as reverse processes but the way you go about it isn't reverse because the opposite of multiplication is division. The equations are interchangeable though and can be called.

At first, this response appears to be nonsense. Notice, however, the student's attempt to link multiplication ad division to finding a product and factoring. This link is important because it gives this student a bridge to familiar concepts, multiplication and division.

Even in the response of student B, a marginal student, we can see the cognitive value of writing:

Factoring and finding a product are reverse processes. Factoring I guess is to make finding the answer much easier. I really am not sure how much it helps because I can't see much point in it. I guess that it puts the problem in simpler terms.

First, the student restates the assignment and then works toward a basic but correct evaluation. The importance of this student's response can be seen in the last sentence. The student discovers one of the main purposes of factoring, to simplify an expression so that it can be used in further operations.

Student C shows an understanding of the mechanics of the two processes but demonstrates no insight into the concept or purpose of factoring.

I believe that factoring and finding a product are reverse procedures. When you find a product of two binomials that is multiply:

$(x + 2)(2x - 1) = 2x^2 + 3x - 2$
When you factor a trinomial you end up with two binomials:

$2x^2 + 3x - 2 = (x + 2)(x - 1)$

This response also illustrates a difficulty with evaluating statements. The more equations and mathematics we see in a response, the less we know about students' understanding of a concept. For this reason we always encourage our students to answer with words and sentences rather than with equations.

In the following example, notice how student D collects ideas on finding a product and factoring, then, in the third sentence, attempts to answer self-generated questions, with a satisfactory conclusion.

Combining factors to find a product is an application of algebra that allows factors to be combined into either monomials, polynomials, or trinomials, or whatever. Factoring is the process of taking monomials, polynomials, binomials, trinomials, or whatever and reducing them to the terms that multiply together to form them. Thus, factoring is taking polynomials apart, while finding the product is putting them together. Actually the two are very similar in the processes used. One uses primarily multiplication, the other division.

Reprinted with permission from
Mathematics Teacher December 1986,
Copyright by the National Council of
Teacher of Mathematics

The student collects, internalizes, and finally evaluates. For the student, the journal is a progression toward the understanding of a concept. For the teacher, the journal is a record of a student's thinking and thus an excellent diagnostic tool.

Given the same assignment, some students focused their thoughts on isolating a common factor. For this method of factoring, students sometimes confused division with the isolation of a common factor. They did not grasp the idea that division is actually contained within the process of isolating a common factor. The success or failure of students in recognizing this difference is clearly demonstrated by the responses of students E and F, respectively:

Factoring is sort of dividing ($3x^2 + 6x$) when you factor out the 3x you get $3x(x + 2)$. To multiply $3x(x + 2)$ you do the opposite and you get $3x^2 + 6x$ again. So finding a product and factoring are the same simply reversed.

When factoring you remove all common factors from an equation. These are removed usually by dividing them out of the whole equation. Now to get back to the original equation from the factored one, you have to multiply the factored equation by the numbers and varible, etc., that were removed. This shows that factoring and finding the product....

Student E doesn't separate the two ideas as well as student F. In the second sentence student F clearly understands that division is a part of the process of isolating the common factor.

A subsequent assignment in the same class required students to translate the concept of finding a product and factoring to a nonmathematical event. Students responded to the following prompt, "Think of a nonmathematical relationship that is analogous to the process of finding a product and factoring." Student G gave one of the better responses:

One relationship similar to finding a product vs. factoring is that of taking a carburetor apart and then putting it back together. When the carburetor is together it is difficult to clear and repair. Therefore it is disassembled to make it easier to work with. This is analogous to factoring.

When the parts of the carburetor are in working condition, they must be put back together before they will work as desired. This is analogous to finding a product.

This journal assignment differs from the other assignments because students need to compare nonmathematical experiences to what is being learned in class. The response of student G demonstrates an overall understanding of factoring and finding a product by comparing these processes to the disassembly and assembly of a carburetor. He understands their relationship to each other and is able to connect the two processes. His analogy also illustrates that factored and product forms have different mathematical uses. In this assignment the student had to collect information from both mathematical and nonmathematical experiences and then connect them logically. This example shows that in one short journal a student can use a variety of intellectual skills, such as synthesis, interpretation, translation, analysis, and evaluation. We believe that with continued use of these skills, students grow both in the understanding of mathematical concepts and in the ability to express that understanding.

Conclusion

We adapted the technique of journal writing to instruction in mathematics classes. We attempted to promote an improved internalization of mathematical concepts and to see how journal writing could be incorporated in these classes. This study examined students' test scores, attitudes, and writing (the latter by the Writing Apprehension Test) as well as student and teacher evaluations. The data have not established any strong relationships among the various attributes examined. An examination of journals, however, clearly indicates that students used their journals to think about solving problems

Reprinted with permission from
Mathematics Teacher December 1986,
Copyright by the National Council of
Teacher of Mathematics

associated with mathematical concepts. In addition, most of the students' responses to a questionnaire indicated that students viewed journals as a worthwhile inclusion in mathematics classes.

Any conclusion derived from this study must be viewed and extended cautiously. They are specific to a particular isolated setting. The study has led to guarded conclusions and has generated many more questions than it has answered. The investigating team hopes that these efforts are more of a "first step" rather than a "stopping place" and that further study will establish more definitive relationships between writing and mathematics.

Bibliography

Bloom, Benjamin S., J. Thomas Hastings, and George Madaus. *Handbook on Formulative and Summative Evaluation of Student Learning*. New York: McGraw-Hill Book Co., 1971.

Bruner, Jerome. *Toward a Theory of Instruction*. Cambridge: Harvard University Press, 1966.

Burmeister, Lou. *Reading Strategies for Middle and Secondary School Teachers*. Reading, Mass.: Addison-Wesley Publishing Co., 1978.

Burton, Grace M. "Writing as a Way of Knowing in a Mathematics Education Class." *Arithmetic Teacher* 33 (December 1985): 40–45.

Emig, Janet. "Writing as a Mode of Learning." *College Composition and Communication* 28 (May 1977): 122 - 28.

Johnson, Marvin. "Writing in Mathematics Class: A Valuable Tool for Learning." *Mathematics Teacher* 76 (February 1983): 117–19.

Myers, John. *Writing to Learn across the Curriculum*. Bloomington, Ind.: Phi Delta Kappa Educational Foundation, 1984.

Selfe, Cynthia, Bruce Petersen, and Cynthia Nahrgang. "Studying the Effects of Journal Writing on Students in a College-Level Mathematics Course." In *Writing in the Disciplines – Research into Practice*. Upper Montclair, N.J.: Boynton/Cook, forthcoming.

Watson, Margaret. "Sharing Teaching Ideas: Writing Has a Place in a Mathematics Class. "*Mathematics Teacher 73* (October 1980): 518–19.

Reprinted with permission from
Mathematics Teacher December 1986,
Copyright by the National Council of
Teacher of Mathematics

EXPLORATIVE WRITING AND LEARNING MATHEMATICS

S.Z. Keith

Short explorative writing assignments can transform the mathematics classroom into a dynamic and exciting learning laboratory. In explorative writing, students explore their knowledge about a topic by writing what they know about it in their own language. Then, to refine their ideas further, they share reactions with other students and the teacher. In spite of the significant benefits of using such writing assignments, the teacher may feel uncomfortable with the more interactive environment, with the challenge of designing writing assignments, and with figuring out what to do with the results. This article illustrates how explorative writing assignments can help to expose and identify learning problems and offers some assignments that address these problems. This article briefly mentions some techniques that I have found helpful for working with the results and comments on the benefits of working with these assignments.

How Writing Assignments Expose Learning Problems

I use writing assignments continually in my classes with students of all levels. In one class of trigonometry students, a typical assignment had the immediate purpose of addressing the difficulties many students have in transferring from the visual to the verbal mode. The question was, "What is a reference angle?" An illustration and adequate definition of the concept are shown in figure 1.

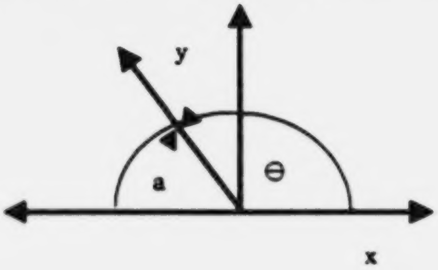


Fig. 1. An Illustration and adequate definition of a reference angle.

DEFINITION: A reference angle for a given angle Θ is an acute angle x that is formed by the X-axis and the terminal side of Θ (x is a reference angle for the angle Θ.)

Whereas all the students could compute reference angles, only a few could adequately define the term. The students' difficulties fell into the following patterns:

1. Some students merely gave an example (which of course was a start:)

   - An angle like if you have 400 degrees on the unit circle your reference angle would be 40 degrees.

2. Others struggled for precision or failed to recognize the conditions of necessity and sufficiency that the definition requires:

   - Another angle that is the same as your first angle but in simpler terms

   (The words *same*, *first*, and *simpler* are vague, and what the "simpler terms" are is the essence of this definition.)

Reprinted with permission from
Mathematics Teacher December 1988,
Copyright by the National Council of
Teacher of Mathematics

- The acute angle with the same terminal side

- An angle closer to the x-axis than the original angle

3. Some tried to define simply by stating how the construction is used:

   - The angle that has to measure up supplementary with the angle given that tells which quadrant it is in

   - The angle that makes it easier to compute the trig functions for the first angle

4. Frequently students either were vague or said as much as possible, with the objective of maximizing partial credit on homework or a test:

   - An angle for which measurements of less than 90 are made in relationship to the x-axis only. The angle is smaller than the original and has equivalent trig functions, like 30, 60, etc., etc.

5. Surprisingly, very few students drew pictures, and when they did, the drawings were neither labeled nor referenced. Although drawing pictures may not be considered a part of writing definitions, students find it much easier to discuss or write about a concept if they have learned to draw an accurate picture first. Most students first grasp a concept visually or tactilely rather than verbally.

Another type of difficulty students have lies in using conventions of notation. Asking students to define a polynomial over the real numbers produces surprising results, even in higher-level calculus classes:

- A polynomial is $ax^2 + bx + c$.

  (This is only a second-degree polynomial. This student may never before have realized the need for the notation, a_1, a_2,....)

- f(x) is a polynomial over the reals if $f(x) = a^1{}_x + a^2{}_x + \ldots$

  (This student has remembered only that some notation is "up" and some "down")

Often the students presume that the reader already knows the answer and so an explanation or analysis is unnecessary. The assignment to describe the procedure for computing the determinant of a x × 3 matrix generated the following responses:

- Take the row-1, column-1 position and multiply it by the determinant of the 2 x 2 matrix....

- Do not use the whole expansion, use co-factors, which is just telling what sign we give it. A plus or a minus times the minor.

The situation becomes more critical still when the question is not merely to state a definition or an algorithm but to write a proof. Then the problem becomes the students' lack of understanding of what constitutes a proof. The examples that follow are from an upper-level abstract algebra course:

- **Question**: Prove that if H and K are normal subgroups of a group G, H K is a normal subgroup of G.

Reprinted with permission from
Mathematics Teacher December 1988,
Copyright by the National Council of
Teacher of Mathematics

Answer: Since H and K are subgroups, H K is normal by a theorem.

- Question: Prove that 1cm (m, n) = mn/gcd(m, n.)

Answer: We must have that 1cm(m, n) - mn/gcd(m, n) because assume that this is not true. But that can't be because we know that 1cm(m, n) = mn/gcd(m, n.) Therefore this is a true statement.

Much energy in college teaching is dedicated to teaching advanced students to do proofs – questioning how to do them and wondering whether a proof is actually possible in certain situations. The algebra students whose work is shown in the foregoing may not belong in an upper-level course; they have reached this point only by mastering some well-digested problems. Their lack of understanding underscores the need for writing assignments at the earliest levels. Furthermore, the failure of many of our students successfully to create proofs or to write expositions raises questions about what they successfully construe or read and about the development of their learning skills. Frequently, students completely avoid reading the text and get by with methodically working exercises.

Activities That Address the Problems

Most of my explorative assignments are in-class summaries or assignments to be completed overnight; they are given to help the students understand a mathematical concept. They include the following types of exercises:

1. *Summaries*. Summaries can reveal where a class stands or serve as an introduction to a concept.

    - Right now, what is your notion of a "function?" Be as precise as possible: we say "y = f(x) is a function of x if...."

    - What's wrong with this logic?

      (Example follows in a reducing-fractions problem)

    - What is a "radian," and why do we prefer this notion to that of "degree?"

2. *Visual image translation*. These experiences integrate visual and verbal understanding.

    - Describe the graph of this function as if you were explaining it to a friend over the telephone.

    - How are the graphs of $y = {}^1/x^2$ and $y = x^2$ related? How could you predict the behavior of the second from that of the first?

    - The class is divided into pairs. One student describes a graph in language that is as accurate and precise as possible while the other draws the graph. No peeking allowed on either side.

3. *Synopsizing tactics for solving a problem*. These exercises enhance students' awareness of problem-solving techniques.

    - Describe how to combine fractions to obtain one term. Do not merely give an example.

    - What is the difference between combinations and permutations and by what clues do you distinguish problems involving the one or the other?

4. *Giving an algorithm*. Students practice describing steps in algorithmic thinking.

Reprinted with permission from
Mathematics Teacher December 1988,
Copyright by the National Council of
Teacher of Mathematics

- Write out an explicit procedure that helps you to solve distance–time–rate problems.

5. *Giving a definition or stating a theorem.* Such verbalizations enhance the precision of students' terms and theorems.

6. *Communicating thoughts to a specific audience.* These exercises give students the experience of translating one explanation into another.

   - Many of you did not understand the concept on page 12 of the text. Rewrite that page as you would have liked to have seen it written.

   - In a few pages, write a "crib sheet" for your best friend, who has fallen seriously behind in this course. List important concepts, definitions, and theorems and briefly explain any methods of this unit on which the class might be tested.

7. Inventing a problem. Students learning to create their own illustrations through such exercises.

   - You live in a family with 3 sisters, 4 brothers, 2 dogs, and 17 cats. Create several problems that deal with combinations and permutations involving those numbers. Give answers and explain what allowed you to recognize that the problem was to be solved by the techniques you used.

8. *Generalizing a concept.* These exercises help to strengthen students' grasp of the overview of mathematics.

   - Mathematicians write definitions when a clear need arises. In your own study of polynomials, what definitions might you write to simplify talking about polynomials? Where could your definitions be useful?

   - Where in "real life" have you come across "absolute value?"

9. *Group projects.* Such projects enhance confidence and productivity by having students work collaboratively. These activities allow students to exchange ideas and to pursue the development of an idea in greater depth than they can when working alone. Since students enjoy this activity, it is a good way to introduce writing assignments.

   - Pass out a few transparencies and markers and have groups compete in writing an accurate statement of a theorem or an explanation of a procedure. Show the results on the overhead projector.

   - Have the groups anticipate a theorem. For instance, "A squirrel runs into a box and out again. It may enter through two doors and exit through three, so it can pass through the box in as many as six different ways. A very important principle is at work here – one that doesn't involve squirrels. Can you state it in its greatest generality?" Or, "Now that you have seen my explanations for finding maxima and minima visually, can you explain how the first derivative can indicate where a relative maximum or minimum occurs? If you can, you will have stated the *first derivative test*."

Techniques and Benefits of Using Writing Assignments

Giving writing assignments, of course, does not in itself automatically generate effective teaching and learning; the teacher must respond to the writing. It is useful to approach writing as part of a process leading to completion in the future rather than as a final product; this approach shifts the perspective from what is wrong to how it could be made more accurate. Attempting to achieve this shift has led me to use several techniques.

Reprinted with permission from
Mathematics Teacher December 1988,
Copyright by the National Council of
Teacher of Mathematics

1. *Discussion.* I put the students' efforts on transparencies, and show them on the overhead projector, encouraging discussion or giving my own reactions. In conformity with the NCTM's *Curriculum and evaluation Standards for School Mathematics*, which state that mathematics should be considered as a form of communication, every effort is made to understand each student's attempt. Any inclinations on the part of students to ridicule or be embarrassed generally subside when it becomes clear that I am trying to understand their thought processes and how their definitions might be improved. Something interesting and worthwhile is usually evident in every student's attempt, and, interestingly, students' answers often go awry in similar ways. The reactions of other students are useful in explaining what was intended; most of us have found that students sometimes have a mysterious ability to interpret the teacher's intent to other students.

2. *Peer evaluation.* Papers can be exchanged and evaluated according to specific criteria (see fig. 2 for a sample worksheet.) Such peer evaluation gives students an opportunity to see papers from a perspective closer to that of the teacher.

3. *Revision.* After the need for precision has been shown through peer evaluation, students may be asked to rewrite their definitions. Frequently, the entire exercise is repeated on another day. Students may think they understand a concept when the teacher shows the corrections on their work, but teachers can create no better learning opportunity for students than having them reconstruct their own answers in the correct way.

4. *Anticipation.* Quite often, the writing assignments generate lively discussion and can be used to introduce a topic; for instance, the assignment to define a degree can be followed by a discussion of the usefulness of radians.

These exercises are short and well worth the time they take. They teach strategies for maximizing understanding; they develop trust and improve participation and collaboration in and out of the classroom. Most students appreciate the chance to be heard and understood. From their point of view, the elegance of the text or the lectures may merely be obfuscating. Students are generally receptive to the discussion of class writing projects and to doing extra work at home because they can see the payoff in their enhanced understanding of the material. Students report that they feel they read and retain material more easily as mathematical conventions become familiar to them.

In the test setting, accuracy and formal quality are of course expected, but the students develop this quality by themselves as a result of their in–class experiences. Test results are frequently improved because detailed notice of a student's work sharpens the student's attention to detail.

I have found that I do not need to grade grammar and mechanics. My objective is to get closer to how students think, and continually correcting spelling and grammar tends to drive students into using a smaller vocabulary and taking fewer risks. In the students' writing generally, their control of grammar and mechanics is closely linked with their ability to use logic. The issue is simply whether they are adequately communicating with me, and the explicit grading, when done at all, is based on this criterion and goes very quickly.

Above all, the teacher can emphasize the fact that the writing of definitions, theorems, and later, proofs is a delightful experience. The aspect of elegance in our mathematical language can and should be shared by our students at every level.

Reprinted with permission from
Mathematics Teacher December 1988,
Copyright by the National Council of
Teacher of Mathematics

DEFINITION AND THEOREM WORKSHEET

DEFINITION WORKSHEET: NAME:________________________

DEFINE:

5 – The student has given a complete definition, with clear and accurate vocabulary and notation, appropriate mathematical vocabulary and minimum redundancy.

4 – Minor problems with clarity, notation, vocabulary or wordiness are evident.

3 – The basic concept is clear (student seems to know what is being defined,) but student exhibits significant confusion in notation or vocabulary.

2 – The basic concept shows significant confusion, with major problems in notation or vocabulary.

1 – The definition does not relate to the concept in question.

THEOREM WORKSHEET: NAME:________________________

STATE:

5 – The student has given a complete theorem, with clear and accurate vocabulary and notation, a complete hypothesis and conclusion, appropriate mathematical vocabulary and minimum redundancy.

4 – Minor problems arise with clarity, notation or vocabulary; or minor incompleteness is found in the hypothesis or conclusion.

3 – The basic concept of the theorem is clear (student seems to understand the theorem), but significant confusion or error is seen in vocabulary or notation.

2 – The basic concept of the theorem shows significant confusion about the hypothesis or conclusion, with major errors in notation or vocabulary.

1 – The theorem does not relate to the concept in question.

Reprinted with permission from
Mathematics Teacher December 1988,
Copyright by the National Council of
Teacher of Mathematics

Bibliography

Burton, Grace M. "Writing as a Way of Knowing in a Mathematics Education Class." *Arithmetic Teacher* 33 (December 1985); 40–45.

Commission on Standards for School Mathematics of the National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics*. Working draft. Reston, Va.: The Council, 1987.

Hillocks, George, Jr. "The Responses of College Freshmen to Three Modes of Instruction." *American Journal of Education* 89 (1981): 373–95.

Johnson, Marvin L. "Writing in Mathematics Classes: A Valuable Tool for Learning." *Mathematics Teacher 76* (February 1983): 117–19.

Watson, Margaret. "Writing Has a Place in a Mathematics Class." *Mathematics Teacher 73* (October 1980): 518–19.

Reprinted with permission from
Mathematics Teacher December 1988,
Copyright by the National Council of
Teacher of Mathematics

APPENDIX D

GUIDELINES FOR EFFECTIVE MEDIA INTEGRATION

There is a growing collection of media resources that support the expectations of the Senior High Mathematics program. As with any resource, films and videos should be selected with due consideration for the expectations of the program and the needs of your class. Specific films and videos are noted throughout the Teacher Resource Manual.

The following questions should be considered when selecting media resources for use in the classroom.

- Does it support the expectations of the program?
- Does it meet individual student needs and learning styles?
- Does it reinforce, enrich or extend the program?
- How appropriate is the timing?
- Does the medium demonstrate abstract ideas?
- How well does the medium fit into existing curricula and teaching/learning needs?
- How clear are the educational aims and objectives?
- What kind of preparation is required?
- How are teaching and learning evaluated?
- What forms of follow-up are required?

Resource/Media Centres

There are 12 resource/media centres in Alberta that carry films and videos that support the Senior High Mathematics program. Each centre publishes its own catalogue, listing the resources in its collection. These centres operate as libraries, lending out audiovisual materials for specified time periods. For more information contact the local resource/media centre:

(A) Regional Resource Centres

Zone 1
Zone One Regional Film Centre
P.O. Box 6536
10020 – 101 Street
Peace River, Alberta
T8S 1S3
Telephone: (403) 624–3187

Zone 2/3
Central Alberta Media Services
(CAMS)
2017 Brentwood Boulevard
Sherwood Park, Alberta
T8A 0X2
Telephone: (403) 464–5540 or 467–8896

Zone 4
Alberta Central Regional Educational Services
(ACRES)
County of Lacombe
Box 3220
5140 – 49 Street
Lacombe, Alberta
T0C 1S0
Telephone: (403) 782–5730

Zone 5
South Central Alberta Film Federation
(SCAFF)
Westmount School
Box 90
Wheatland Trail
Strathmore, Alberta
T0J 3H0
Telephone: (403) 934–5028

Zone 6
Southern Alberta Regional Film Centre
(SAFRC)
McNally School
P.O. Box 845
Lethbridge, Alberta
T1J 3Z8
Telephone: (403) 320–7807

(B) Urban Media Centres

County of Strathcona
Learning Resource Service
2001 Sherwood Drive
Sherwood Park, Alberta
T8A 3W7
Telephone: (403) 464-8235

Calgary Separate School Board
Instructional Materials
6220 Lakeview Drive S.W.
Calgary, Alberta
T3E 6T1
Telephone: (403) 246-6663

Edmonton Public School Board
Learning Resources Centre
Centre for Education
One Kingsway
Edmonton, Alberta
T5H 3G9
Telephone: (403) 429-8320

Edmonton Catholic Schools
Curricular Resources
St. Anthony's Teacher Centre
10425 - 84 Avenue
Edmonton, Alberta
T6E 2H3
Telephone: (403) 439-7356

Red Deer Public School Board
4747 - 53 Street
Red Deer, Alberta
T4N 2E6

Telephone: (403) 343-1405

Calgary Board of Education
Education Media
3610 - 9th Street S.E.
Calgary, Alberta
T2G 3C5
Telephone: (403) 294-8540

Medicine Hat School District
Instructional Materials Centre
601 First Avenue S.W.
Medicine Hat, Alberta
T1A 4Y7

Telephone: (403) 526-1323

ALBERTA EDUCATION MATHEMATICS CONSULTANTS

Grande Prairie Regional Office
Hugh Sanders
5th Floor, Nordic Court
10014 – 99 Street
Grande Prairie, Alberta T8V 3N4
Phone: 538–5130
FAX: 538–5135

Edmonton Regional Office
Dick Daly
Devonian Building – West Tower
11160 Jasper Avenue
Edmonton, Alberta T5K 0L2
Phone: 427–2952
FAX: 422–9682

Art Peddicord
Devonian Building – West Tower
11160 Jasper Avenue
Edmonton, Alberta T5K 0L2
Phone: 427–2952
FAX: 422–9682

Red Deer Regional Office
Ron Babiuk
3rd Floor West, Provincial Building
4920 – 51 Street
Red Deer, Alberta T4N 6K8
Phone: 340–5262
FAX: 340–5305

Calgary Regional Office
Pat McLaughlin
Room 1200, Rocky Mountain Plaza
615 Macleod Trail, SE
Calgary, Alberta T2G 4T8
Phone: 297–6353
FAX: 297–3842

Lethbridge Regional Office
Gary Hill
Provincial Building
200 – 5th Avenue, South
Lethbridge, Alberta T1J 4C7
Phone: 381–5243
FAX: 381–5734

RESPONSE FORM

Mathematics 10/13/14 Interim Teacher Resource Manual

YES NO

1. Is this document useful in guiding your involvement in teaching mathematics? ☐ ☐

Comments ____

____

2. Did you find the Activity and Problem suggestions useful? ☐ ☐

Comments ____

____

3. Did you find the Technology Integration and Elective suggestions useful? ☐ ☐

Comments ____

____

4. Did you find that the Comments assisted you in interpreting the Course of Studies? ☐ ☐

Comments ____

____

5. Other comments and suggestions:

____

____

Thank you for your feedback!

You are a ☐ teacher (please specify which course you are teaching) 10 13 14
☐ school administrator
☐ system administrator
☐ other reader (please specify) ____________________

Please send your response to:

Mathematics Coordinator
Alberta Education
Curriculum Support Branch
Devonian Building, 5th. Fl. East
Edmonton, Alberta
T5K 0L2

331